FUNDAMENTALS OF INDUSTRIAL ELECTRONIC CIRCUITS

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Preface

This book is the result of teaching an evening course in Industrial lectronics at the University of Wisconsin, Extension Division, during e past ten years. Since several excellent texts on the subject of elec- onics are available, the addition of another might seem to represent an unnecessary effort. Apart from the fact that no instructor will probably ver find a text entirely to his liking, the selection of a text for evening ourses presents the teacher with a somewhat more difficult problem than or a course given as part of a regular curriculum. The reason for this is hat evening courses are usually attended by people with widely varying ackgrounds. In order to serve the community best, the teacher must hen find a level of presentation that gives the greatest number of his tudents as much knowledge as can be presented within the given time mit. Usually not more than 15 to 20 per cent of those attending are amiliar with differential and integral calculus, and for many even the olving of a quadratic equation represents a major undertaking. The in- tructor, as much as he may regret it, must attempt to present the material vith a minimum of mathematics. The material presented here has been prepared with this thought in mind, and nothing more than a knowledge of the fundamental principles of electrical engineering is required.

The text is in no way meant to replace such excellent books as Herbert J. Reich's "Theory and Application of Electron Tubes" and the book "Ap- plied Electronics," by the M.I.T. staff but it is intended rather as a bridge to these more elaborate texts. It should also be useful to those who do not eel justified in spending the time required to make a thorough study of the subject. More stress has been laid on the fundamental principles than on any specific application. Important concepts are repeated several times n the hope that each repetition will make the reader a little more familiar with them. It is hoped that the text will show the electrical engineer who inished his schooling before electron tubes assumed their present impor- tance that there is nothing mysterious or occult in their behavior and application and that they are circuit elements whose performance can be s accurately predicted as that of a resistor or an inductor. When the undamental properties of electron tubes are once understood, the design, analysis, and maintenance of electronic circuits do not present any greater difficulty than that of other electrical or mechanical devices, and there is certainly no need or reason to call in an expert every time a problem may find its solution by the application of electronic apparatus.

Throughout the book an attempt has been made to use the symbols and

designations employed in leading texts and approved by the technical societies of this country. To the many practical engineers who, like the author, in the practice of their profession daily commit the terrible sin of referring to a unidirectional voltage of constant magnitude as a "d-c voltage," an apology is offered for using the term "direct voltage" to designate such a voltage. However, since the American Standard Definitions of Electrical Terms, approved by the American Standards Association in 1941 and published by the American Institute of Electrical Engineers, sanctions the term "direct voltage" in section 05.20.110, it seemed advisable to use it in the text, regardless of any personal opinion or of what might be used in daily conversation.

WALTHER RICHTER.

MILWAUKEE, WIS.,
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Acknowledgments

Credit for the suggestion that the original mimeographed notes prepared for use with the lectures might be converted into a text belongs to the author's good friend, Mr. Beverly Dudley, who was also kind enough to read most of the material; this resulted in many valuable suggestions.

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Credit is also due to Miss Jean Stoneman, who was instrumental in starting the work on the original notes, and to Miss Etta Sutheimer, whose help was of the greatest assistance in finishing it.

Thanks are also due to the companies generously supplying information required for the preparation of the material, and to the authors of several texts for permission to use illustrations from their books; in each case such use is acknowledged individually in the text.

And last, but by no means least, this may be the proper place to offer an apology to Mrs. Richter, who suffered in silence for the many months when her husband had nothing else on his mind but the completion of the manuscript.

WALTHER RICHTER.

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Fundamentals of Industrial Electronic Circuits

INTRODUCTION

Between the time when Michael Faraday, in 1831, discovered the law of electromagnetic induction, and the time when this most important discovery was put to practical use, a period of approximately fifty years elapsed. In a similar way, the present achievements and developments in the electronic field are based on the work of so-called "impractical" scientists working in their laboratories forty or fifty years ago. It is altogether likely that highly theoretical and impractical work carried out presently by our scientists will in turn form the foundation of very practical things a few decades from now. In the following paragraphs* a short survey of the discoveries that led to the astounding present-day developments in the field of electronics is given.

In 1873, Guthrie found that a negatively charged electroscope discharged when a metal sphere brought to dull-red heat was placed near the electroscope. A positively charged electroscope discharged when the temperature of the sphere was increased. In the light of our present knowledge, this indicated the emission of electricity from a body brought to a high temperature.

In 1882, Elster and Geitel studied the conduction of electricity near heated solids. In their experiments they also studied the influence of the pressure of the surrounding gas, and they actually constructed a two-electrode vacuum tube. They noted the unilateral conductivity of the device but did not propose its use as a rectifier, probably for a very excellent reason—in 1882, there was no alternating current to be rectified! Note that the electronic device for the rectification of alternating current was a good many years ahead of the introduction of ac generators themselves!

In 1883, Edison had trouble with hot spots on the carbon filaments of his electric bulbs. In trying to determine the cause of this phenomenon, he sealed a plate in the same enclosure with the filament. He noticed that a current would flow to this plate when it was connected to the positive end of the filament; this was not the case when it was connected to the negative end. This so-called "Edison effect" is quite often given as the starting point of modern electronics. Edison did not attempt an explanation of this phenomenon nor did he carry out further investigations of it. The report of this effect, however, stimulated others into a detailed investigation of the factors determining this mysterious flow of current.

* The material presented here is condensed from the excellent and more complete historical sketch given in the Introduction to "Theory of Thermionic Vacuum Tubes," by E. L. Chaffee, McGraw-Hill Book Company, Inc., New York, 1933.

Preece, in England, showed that the current was a function of the temperature of the filament, of the distance between the filament and the plate, and of the voltage applied between them, but that the material of the plate was unimportant.

Fleming of the University of London investigated the Edison effect still further. He was the first to suggest its use for the rectification of high-frequency oscillations and obtained a patent covering the application of the Fleming valve as a detector for wireless communication.

All the early experimenters believed that the emission of electricity was due to a chemical action between the material of the filament and the gas still left in the enclosure, a belief that was apparently justified because a decrease in gas always led to a decrease in current.

In 1897, J. J. Thomson measured the mass of the electron and gave the first true explanation of the Edison effect; and Richardson, in 1901, in his work "Emission of Electricity from Hot Bodies," proved that no gas was needed for the conduction of electricity. Much work was done in the first decade of the present century in the study of the means of increasing the emission. Improvements in vacuum pumps greatly aided the progress in this field.

The year 1831, when Faraday discovered that an electric current could be produced by moving a conductor through a magnetic field, will probably remain the most important date for the electrical engineer, but future generations might well be justified in ranking 1907 as a year of equal importance. It was in this year that Lee De Forest put a third electrode or grid in the two-element device known up to this time. Without this grid, tubes were nothing but rectifiers, but this addition made an amplifier out of the device, and all the applications of electronics that have so fired popular imagination became possible only because of Lee De Forest's invention. It is interesting to note that the American Telephone and Telegraph Company and the Western Electric Company quickly recognized the possibilities and started organized research on the subject in 1912; two years later telephone conversation between New York and San Francisco was made possible by the fruits of this research. Van der Bijl of the American Telephone and Telegraph Company and the Western Electric Company succeeded in formulating the mathematical relations of the voltages and currents involved in such a way that the circuit designer, with the aid of a few characteristic values, could predict the performance of the device when combined with other components.

The past two decades have seen many important improvements on the basic tube structure, such as the use of additional control electrodes and indirectly heated cathodes; furthermore, gaseous tubes and photoelectric devices have been the subject of intense research and investigation, with the result that the electronic engineer today has at his disposal a large variety of tubes, each one suitable for particular applications.

CHAPTER I

FUNDAMENTALS OF DIRECT CURRENT; CIRCUIT THEOREMS; POWER RELATIONS

1-1. Scope of Book.—This book deals with the elements of electronic circuits. It is an unfortunate fact that many people quite familiar with ordinary electric circuits have the feeling that for circuits containing electron tubes all the old laws have been abrogated. A glance at the diagram of any electronic circuit, whether a welding control or a radio set, discloses that it contains more resistors, capacitors, chokes, transformers, etc., than electron tubes. Ohm's law is just as valid for a resistor placed in series with a tube as for one used to start a dc motor, and the voltage appearing across an inductance with a change of current is the same whether that change is produced by the action of a tube or by any other means.

1-2. Nonelectronic Components.—Since, as just stated, electronic circuits consist of the same elements as other electric circuits, except for the addition of one new element, the tube, it is clear that one cannot hope to understand the operation of such a circuit without a full understanding of the laws governing the behavior of the nonelectronic components of the circuit, as well as the laws governing the operation of the tube itself. The discussion of resistance, inductance, and the other components of the usual electric circuits properly belongs in general electrical-engineering texts but, in order to help the reader select the phases of these subjects of particular importance for the understanding of electronic circuits, the first few chapters are devoted to their review. An additional reason for doing so is that certain concepts, not found in the usual engineering text, will be discussed.

1-3. Static Charges.—Static electricity, which usually forms the starting point for texts on electricity, is of very little interest to the electrical engineer; it is usually the electric current, or the *flow* of electric charges, by means of which its many tasks are performed. However, since the subject of electronics stresses the motion of minute electric charges, it may be profitable to discuss static charges briefly.

Electric charges have been observed by everyone combing his hair in dry weather or carrying out the well-known fundamental experiments of rubbing a glass rod with silk or an ebonite rod with fur. The charges manifest themselves by attraction or repulsion of the bodies on which they are located. The fact that either attraction or repulsion can take place indicates that there are two kinds of charges, and arbitrarily we say that any electrified body that is repelled by a glass rod rubbed with silk pos-

sesses a charge of positive electricity, while a body that is repelled by an ebonite rod rubbed with fur possesses a charge of negative electricity. These are then altogether unambiguous and easily intelligible definitions of positive and negative electricity.

1-4. Measurement of Charges.—The amount of electricity on a given body may then be measured by the force existing between this body and a like one charged with the same amount and, naturally enough, in any unit system we could say that the unit charge of electricity is the one that repels a like charge at unit distance with unit force. In the centimeter-gram system this is called the "cgs" unit of charge, but this unit is so small that it would be just as impractical for an electrical engineer to employ it in the solution of his problems as it would be for an astronomer to use the inch as the unit of distance. The practical unit of charge is the coulomb. One coulomb is equal to 3×10^9 cgs units of electricity.

1-5. Electric Current, or the Flow of Charges.—As already stated, the electrical engineer is mostly concerned with the *flow* of electric charges. The unit flow of charge will naturally be one coulomb per second, and this flow is of so much importance to him that it has a name of its own: the ampere. The hydraulic engineer's unit for the amount of liquid is the gallon. He, too, is usually more concerned with the rate of flow of the liquid than with the total amount, and his pumps are rated in so many gallons per minute, but he has not seen fit to give this unit a separate name.

Nobody has ever *seen* an electric charge; and by the same token, nobody has ever seen the flow of an electric charge, but just as the charges present on a body manifest themselves by forces appearing between the bodies, so the flow of electric charges manifests itself in various ways. If the ribbon in our toaster suddenly becomes red hot, we know that this is due to an electric current, *i.e.*, to the flow of electric charges through the wire. If a magnet needle placed in the vicinity of a wire deviates from its north-south direction, we again know that this is due to an electric current. Finally, if two electrodes are immersed in water and we notice the formation of hydrogen bubbles on the one and oxygen bubbles on the other, we are justified in saying that an electric current passes through the solution. The latter two manifestations are directional, *i.e.*, the reversal of the current will change the direction of the deflection of the magnet needle or will make the hydrogen bubbles appear on the electrode where the oxygen bubbles appeared before. Now exactly identical effects are produced by the flow of positive charges through a wire, say, from right to left, as by the flow of negative charges in the opposite direction. In order to avoid confusion, it was arbitrarily decided that the direction of current was to be that in which positive charges would flow in producing the magnetic or chemical effect. This turned out to be an unfortunate decision because later experiments showed that most of the conduction of current represents the motion of negative charges. This knowledge was a result of further

investigation into the nature of electric charges. Experiments in this field led to the recognition of two very important facts: (1) It became apparent that negative charges very easily exist by themselves, *i.e.*, separate from a substance, while positive charges as a rule cannot exist by themselves but are usually found in combination with mass. (2) The negative charges are of molecular nature, which means that, in an attempt to divide them into smaller and smaller amounts, a certain minimum charge was finally reached and any attempts to split this minimum or unit charge further were failures. This unit negative charge or building stone, so to speak, is the electron. The relation between it and the coulomb mentioned above is that it takes 6.25×10^{18} electrons to make 1 coulomb or, expressed differently, the charge represented by one electron equals 1.6×10^{-19} coulomb. Although it was also found to exhibit the qualities of mass, it does not consist of any of the elements known at present. An electron has a mass 1,800 times smaller than the lightest of all atoms, the hydrogen atom. It is a very tiny thing too, its radius being 2×10^{-13} cm.

1–6. Nature of the Electric Current.—Research in the structure of matter has shown that the molecules of all matter are made up of positive and negative charges, but it is beyond our purpose to pursue this thought any further. Let it be said only that the atoms, *i.e.*, the smallest building stones of matter, form a system similar to our solar system, with positive charges bunched together at the center and with a number of electrons spinning around the core in a way similar to that in which our planets spin around the sun. For the electrical engineer, the only important fact is that the electrons spinning in the outer orbits of the atoms may be knocked out of their orbits fairly easily; they are referred to as free electrons. Scientists and physicists do not seem to be any too sure about this matter, however. By one source, the number of free electrons in a given substance may be stated as one free electron for every atom, while in another reference the number is given as one free electron for every 2,000 to 3,000 atoms. The fact that electrical engineering had accomplished many things before the nature of the electron was ever known is sufficient proof that this phase of the subject, although interesting, is not too important.

The flow of electric charges through a wire is now thought of as consisting of a motion of the free electrons discussed above. The speed of this motion is quite slow, amounting to a few inches to a few feet per second, which seems to be in contradiction to the value of 186,000 miles per sec with which we are all familiar. There is no contradiction, however, as will be seen presently. If we purchase a piece of copper wire, there will be in it billions and billions of free electrons. If we want a hydraulic analogy, let us suppose that a 100-ft length of garden hose at the time of purchase was already filled with water. Now let it be connected to a faucet, and the faucet opened. Quite obviously, water will begin to issue

from the free end of the hose almost immediately, but the water issuing at the first instant certainly will not be that which just came out of the faucet. When we say that the speed of electricity is 186,000 miles per sec, we mean that if current begins to flow into a wire 186,000 miles long, it will be only 1 sec later that current will flow at the far end, but the electrons or charges moving at the far end are not those which were pushed into the near end of the wire. It may take days and weeks for these to arrive at the far end. Just as in the case of the garden hose, the time will obviously depend on the diameter of the hose and the amount of the flow of liquid. In an ac circuit where the current reverses quite rapidly, it is obvious that the total motion of the charges may amount to only an inch or so, back and forth, and that in such a circuit one would still have, even after years of service, the original electrons purchased with the piece of wire.

When the student tries to analyze the behavior of electric circuits by comparing them with hydraulic circuits, it is very important to realize that in the hydraulic analogy he will always have to visualize a *closed* circuit completely filled with liquid.

1-7. Voltage, or the Cause of Current Flow.—An electric current, as we have seen, consists of a motion of charges, and the question now arises how the free charges in a wire, for instance, can be put into motion. To do this, a voltage must be applied. Voltage has been compared to pressure in hydraulic circuits because obviously, if two containers under different pressure are connected by a pipe, liquid will flow from the container of higher pressure to that of lower pressure. Although this may be a satisfactory analogy, it certainly cannot be considered as an explanation of what an electric voltage is. Texts on physics define voltage or potential difference between two points as the work necessary to move the unit positive charge from one point to the other; but this seems to be again more a definition than an explanation. Let us then state simply that a voltage exists between two points if, upon being connected by a conducting path, charges move from one point to the other. Thus, if we purchase a 6-volt storage battery or a 45-volt radio battery, we have a device with two terminals, which, if connected by a conducting path, causes a motion of charges. Let us call these terminals A and B for an instant. If upon connecting A and B by means of a conducting path, such as a piece of wire, we find that positive charges move from A to B, or negative charges from B to A, we say that A is positive with respect to B. Or we may connect the terminals to two flat metal plates and place an electric charge, such as a charged pith ball or an electron—if we can find a pair of tweezers with which to pick up an electron—between the two plates. If a positively charged pith ball moves from A to B, or a negatively charged one from B to A, we say that A is positive with respect to B. The unit of this voltage or potential difference is the volt. This is the practical unit

of voltage and again bears a certain relation to the unit of voltage used by the physicist, just as the unit of electric charge derived by him from fundamental relations bears a certain relation to the practical unit, the coulomb.

1-8. Direction of Current.—If two points with a potential difference between them are connected by a conducting path, current will flow from the positive to the negative terminal. Remember that current was arbitrarily defined as the motion of positive charges. Only if the actual charges in motion are positive will the direction of their motion coincide with the direction of the conventional current. In a wire or in a vacuum tube the charges in motion are electrons, or negative charges. Their motion is therefore in a direction opposite to that of the conventional current. In an electrolyte or in a gaseous tube, on the other hand, there are usually also positive charges in motion, which then move in the direction in which we conventionally place the arrow indicating current flow. This is one of the most confusing phases of the subject of electric current. What we really mean when we say that the current in a wire connected to a battery flows from the positive to the negative terminal of the battery is that this is the direction in which it *would flow* if it consisted of positive charges. We are leaving it to the physicist to determine whether in a particular case we deal with positive or negative charges.

1-9. Resistance.—The amount of current flowing in a conducting path connected to two points with a potential difference between them depends on the nature of the conducting path, and the characteristic describing its ability or inability to conduct current is called the "resistance" of the path. Quite logically, a path will have unit resistance when a voltage of 1 volt acting on it will cause a current of 1 amp, *i.e.*, the motion of 1 coulomb per sec. This unit is called the "ohm," in honor of the man who discovered the proportionality between voltage and current through a given conducting path. Similarly, a hydraulic engineer might say that a given pipe would have unit hydraulic resistance if a pressure difference of one unit (1 lb per sq in., or 1 ft head, for instance) between the ends of the pipe caused a flow of 1 gal per min, or 1 cu ft per min (whatever the unit of flow may be).

1-10. Representation of Voltages by Heights.—Positive values are usually pictured as being above negative values (see, for instance, the scale of an ordinary thermometer), and we can therefore say that the current flows from the point of higher potential to the point of lower potential when the two points are connected by a conducting path. This at once suggests again the analogy with hydraulics where liquids flow from a point of higher pressure (or level) to one of lower pressure (or level). The concept of associating voltages or potential differences with differences in level or height is an extremely helpful one and will be used extensively in this book. The operation of a circuit can be visualized with remarkable clarity if in the

drawing of its diagram an attempt is made to indicate voltages as vertical distances, with the positive end located above the negative end. The following examples should help to make this point clear.

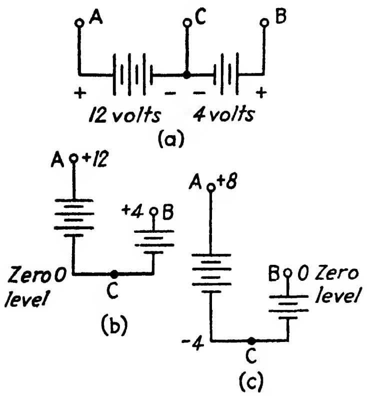

FIG. 1-1.—Two batteries connected as shown in (a) will be represented in the manner indicated in (b) and (c). This method of representation indicates the relative voltage level of the various points.

Suppose that in the diagram of a certain circuit we find a 12-volt and a 4-volt battery shown connected as in Fig. 1-1*a*. The two batteries are obviously bucking each other, and the voltage between terminals *A* and *B* will therefore be 8 volts, but it requires the scrutinizing of the polarity marks in the diagram to come to this conclusion. In this book such a connection will be shown as in Fig. 1-1*b*. Terminal *C* is common to both batteries, but both *A* and *B* are positive with respect to *C* and are therefore shown above *C*, *A* by a larger amount than *B*. A glance at this diagram yields immediately and without any mental effort the information that *A* is positive (*i.e.*, "above") with respect to *B* by an amount equal to the difference between the voltages of the two batteries. The diagram shows another important feature: the choice of one point as reference point for all potentials. If the circuit to be studied is grounded, the point of grounding is usually chosen as the reference point, and zero potential is assigned to it; if the circuit is not grounded, any point may be chosen as the zero level. In Fig. 1-1*b* the terminal *C* was chosen as zero level which makes points *A* and *B* +12 and +4 volts, respectively. If, for any reason, it had been more convenient to choose *B* as the zero level, the potentials of *A* and *C* would become +8 and −4, respectively, as shown in Fig. 1-1*c*. Note that the choice of the reference point does not in any way affect the potential difference *between* the various points. In both cases terminal *A* is 8 volts positive with respect to *B*. At this point it may be well to warn the man working with diagrams of power circuits not to become confused by the markings usually found in them. The two bus bars of a 220-volt dc system are quite often marked in the diagram +220 and −220. In this book, two points marked in this manner have a potential difference of 440 volts with respect to each other, while the two lines of a 220-volt system would be marked 0 and +220,

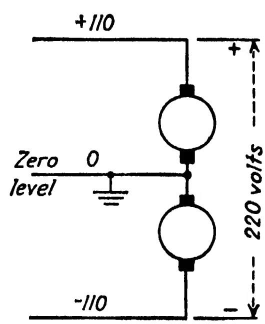

FIG. 1-2.—A three-wire system with the neutral grounded has one terminal above and one terminal below the ground level.

or 0 and -220, depending on whether the negative or the positive terminal of the system was chosen as the reference point. As a matter of fact, if the system happened to be a three-wire system, with 220 volts between the two outside wires and the neutral grounded, the two lines would be marked $+110$ and -110. This is shown in Fig. 1-2.

As a third example, consider the diagram of a Wheatstone bridge in Fig. 1-3. Let the switch S be open. Owing to the current flow established by the battery in the two vertical branches, A and B will be at levels somewhere between O and $+E$. When the well-known condition for bridge balance is fulfilled, both points will be at the same level. Closure of switch S will then connect two points that are at the same electrical level, with the result that no current will flow through the instrument.

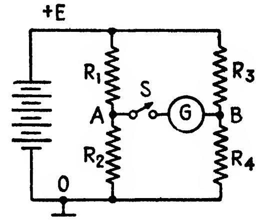

FIG. 1-3.—In a Wheatstone-bridge circuit no current will flow through the instrument if points A and B are at the same level, with or without the instrument in the circuit.

1-11. Ohm's Law.—It was stated earlier in this chapter that the characteristic describing the degree to which a conducting path resists the passage of current is the resistance of the path. We shall now discuss the relation between voltage, current, and resistance, known as "Ohm's law." This law states that the current flowing through a conducting path is proportional to the voltage existing across the two terminals of the path and inversely proportional to the resistance of that path. Everybody is prone to assume that this is so by necessity. It should be pointed out that the amount of liquid passing through a pipe of given cross section and length is *not* proportional to the pressure existing between the two ends of the pipe but follows a rather complicated relation. Therefore, the discovery of the proportionality by Ohm, in 1827, was a most important one, especially when one considers that he did not have at his disposal the simple and exact instruments that we have now come to accept as common tools of the electrical engineer.

Ohm's law, in all its simplicity, does bear a somewhat closer scrutiny, and special attention should be paid to the fact that it can be stated in three ways. The three forms are as follows:

$$i = \frac{e}{R} \tag{1-1}$$

$$e = iR \tag{1-2}$$

$$R = \frac{e}{i} \tag{1-3}$$

In the following, the three equations are expressed in words, and their careful study is highly recommended:

(1–1). The current i, which flows in a path of resistance R when connected to two points with a potential difference e between them, is equal to this potential difference divided by the resistance.

(1–2). If in a path of the resistance R the current i is observed flowing, then there is a potential difference between the two ends of the path equal to the product of the current and the resistance.

(1–3). If in a conducting path connected to two points with a potential difference e a current i is observed, then the resistance of this path is equal to the ratio of the voltage divided by the current.

Sometimes it is found more convenient to use, instead of the value R, the reciprocal of it. This is called the "conductance" of the path and is usually designated by the symbol g. With this value the three forms of Ohm's law given in Eqs. (1–1) to (1–3) take the following forms, respectively:

$$i = eg \tag{1–1a}$$

$$e = \frac{i}{g} \tag{1–2a}$$

$$g = \frac{i}{e} \tag{1–3a}$$

It may be desirable to call particular attention to Eqs. (1–3) and (1–3a). Equation (1–3) states that the resistance of a path is equal to the ratio of voltage and current. Obviously, if we can adjust the flow of current in a path or paths between two points to a value of 1 amp, then the voltage existing across this path is numerically equal to the value of the resistance. In other words, the resistance of a path is numerically equal to the voltage existing across it with a current of 1 amp flowing through it. It is left to the reader to reason out the equivalent statement concerning the conductance: The conductance of a path or paths is numerically equal to the current flowing through it when a voltage of 1 volt is applied to it.

1–12. Resistances in Series and in Parallel.—With the relations developed in the above equations, it becomes easy to prove that the equivalent resistance of several resistances connected in series is simply equal to their sum. All that one has to remember is that in the case of a series connection of resistances there must naturally be the same flow of current in every one of them. This means that the total voltage across all of them must be equal to the sum of the individual voltages. In a similar way, when several resistances are connected in parallel across two points with a given potential difference, then this potential difference sends a current through each branch. The total current flowing is equal to the sum of the individual currents, and the ratio of the potential difference to the sum of all currents would, according to Eq. (1–3), permit us to calculate the value that a single resistance would have to have to take as much current as the parallel branches. In this case the reader will easily see that the use of

conductances is more convenient; in the case of parallel paths, the total conductance is simply equal to the sum of the individual conductances. The simplest case of two parallel resistances is the one shown in Fig. 1-4, and it is well worth while to learn the formula governing this simple case. The single resistance by which the parallel combination of R_1 and R_2 can be replaced, *i.e.*, the single resistance taking as much current as these two branches in parallel, has the following value:

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

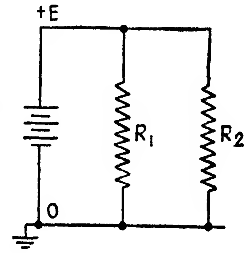

FIG. 1-4.—Two parallel resistors can be replaced by a single one taking as much current as the two.

As an exercise, the reader should prove this formula with the aid of Ohm's law.

1-13. Kirchhoff's Laws.—If a number of resistors and sources of potential are arranged in any arbitrary manner, they will form what is known as a "network." The study of networks is of the utmost importance to the electrical engineer. We shall confine ourselves to the very fundamentals governing this field. The calculation of the currents flowing in the various branches of a network under the influence of one or several voltage sources in the network is usually a very laborious, although not very difficult, task. The fundamental laws used for the solution of this problem were formulated by Kirchhoff and are known as "Kirchhoff's laws." There are two of them: one dealing with currents, the other with voltages. The first one states simply that at a junction of several branches, as shown in Fig. 1-5, all the currents flowing toward the junction must equal in magnitude all the currents flowing away from it. It is hardly possible that anybody could have any doubt about the truth of this statement. To make use of a hydraulic analogy again, let us visualize a complicated network of pipes with several of them, say five, coming together at one junction. If there is any kind of flow in these five pipes, it is obvious that in some of them the liquid must flow toward the junction, in others away from it, and that these two amounts must be equal. This simple relation can also be stated in the following manner: The sum of all currents coming together at a junction equals zero. When this presentation is used, the currents flowing away from the junction are considered as negative currents.

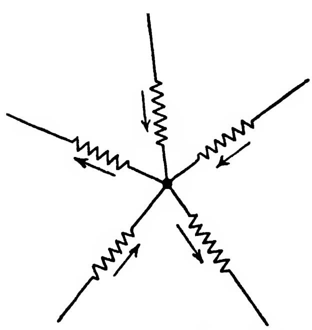

FIG. 1-5.—When a number of currents come together at a junction, the sum of all currents flowing toward the junction must be equal to the sum of all currents flowing away from the junction.

The second of Kirchhoff's laws can be stated in several ways. Most commonly it is stated as follows: The sum of all voltage drops around a closed loop is equal to the sum of all impressed voltages. Before we give the second way of stating it or discuss the relative advantages of the two, the following analogy may be of help. Suppose we found ourselves in a mountainous region with various cities, *A*, *B*, *C*, *D*, etc., located there. Naturally, these cities can be expected to lie on different levels. Suppose we started out from city *A*, making a round trip to *B*, *C*, *D*, etc., finally returning to *A*. Back at *A*, we are obviously at exactly the level from which we started. Regardless of the levels of the other cities, it must be obvious to anybody that we have been going up during some parts of our trip and down during others. We could also state that the sum of all ascending parts of the trip must equal the sum of all the descending parts and, if we were to call a descent a "negative ascent," then we could say that the sum of all ascents must be equal to zero.

If, in Fig. 1–6*a*, a loop in a particular network is shown, then each junction is at a particular voltage level, just as each city in the above analogy

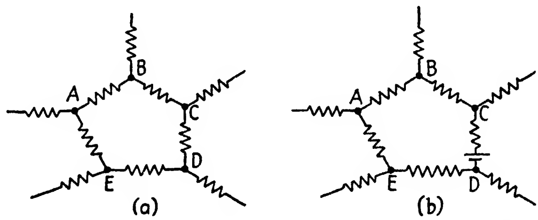

FIG. 1–6.—If we start at any point in a closed mesh, measuring the voltages around the mesh, their sum must be zero because we return to the same level from which we started

was at a particular level. The voltage existing between *B* and *A* is equal to the electrical level difference between those two points and is therefore completely analogous to the amount that we had to climb or descend in the case of cities *A* and *B*. It becomes obvious then, if we measure the five voltages existing from *A* to *B*, from *B* to *C*, from *C* to *D*, from *D* to *E*, and back from *E* to *A*, that this sum will be zero, provided we take into proper consideration the sign of the voltage. Thus, if point *B* was at a higher potential than point *A* (which means that the current in the resistance connecting these two points would flow from *B* to *A*), we would call the voltage of *B* with respect to *A* "positive." If, on the other hand, *C* is at a lower potential than *B* (making the current flow from *B* to *C*), then the voltage of *C* with respect to *B* would be called "negative." It is quite clear that, with the voltages so defined, the sum must be zero.

If one or several of the branches contain voltage sources, such as a battery in branch *CD*, as shown in Fig. 1–6*b*, then the sum of the voltages going around a loop must still be zero. We simply count the voltage of the battery in exactly the same manner as the other voltages. In fact, if

all the resistances and the battery were hidden from view, with only the terminals accessible for measuring purposes, the voltmeter used for making the various measurements of these voltages would certainly not differentiate between a voltage existing across a resistance, due to a flow of current through it, and the voltage existing across a battery. Therefore, we are certainly justified in stating Kirchhoff's second law as follows: The sum of all voltages (voltage drops or voltage sources) around a closed loop equals zero.

If it is desired, however, to differentiate between the voltages existing across resistors due to the flow of current through them and the voltages appearing across voltage sources, then the second form of stating Kirchhoff's law may be preferable. In this formulation it is given as follows: The sum of the voltage drops around a loop is equal to the sum of the voltages of the sources included in the loop. The reader may easily convince himself that from a practical point of view there is no difference between these two methods.

The two Kirchhoff laws are all that is needed for the solution of any network problem. Their application, however, quite often leads to rather cumbersome equations, and investigators of circuit theory have found certain theorems and short cuts that are often extremely helpful in the solution of these problems.

1–14. Principle of Superposition.[1, 2 *]—One of the most powerful tools of which frequent use is made in this book is the principle of superposition, formulated by Helmholtz. Its application can best be explained with the aid of an example. Suppose that we have to deal with a complicated network consisting of many branches of resistances and a number of voltage sources, for instance batteries, and let the voltages of these batteries be designated as E_1, E_2, E_3, etc. The principle of superposition states that the current distribution in such a network can be found in the following manner. Consider all voltage sources short-circuited except E_1 and calculate the current distribution in the network due to E_1 only; next consider all voltage sources short-circuited except E_2 and calculate the current distribution in the network due to E_2. Repeat this performance in a similar way for each voltage source. In this way there will be obtained as many current distributions as there are voltage sources in the network. The principle of superposition then states that the actual current distribution existing in the complete network is simply the superposition of the individual current distributions as determined in the manner just outlined. The principle of superposition will be extremely helpful in the analysis of vacuum-tube circuits because in these circuits there are always several voltages acting, some of them real, others fictitious. The proof of the principle of superposition is based directly on the two Kirchhoff laws. Each current distribution calculated as outlined above on the basis of

* These figures refer to References given at the end of each chapter.

only one voltage acting in the circuit obviously satisfies the two Kirchhoff laws. It is clear that the superposition of several sets of values, where each set adds up to zero, will naturally also add up to zero, which means that the total current distribution so found satisfies the Kirchhoff laws.

1-15. Thévenin's Theorem.[3-6]—Of several other valuable theorems employed by the communication engineer, we shall be satisfied to present only one, which will be of much help in the solution of vacuum-tube circuits. This theorem is due to Thévenin and is of particular value when it is desired to find the current in only one branch of a network. It can be stated in several ways, of which the following is the most common: Open the branch in which it is desired to find the current; calculate the voltage that will appear across the break in this branch; next consider all voltage

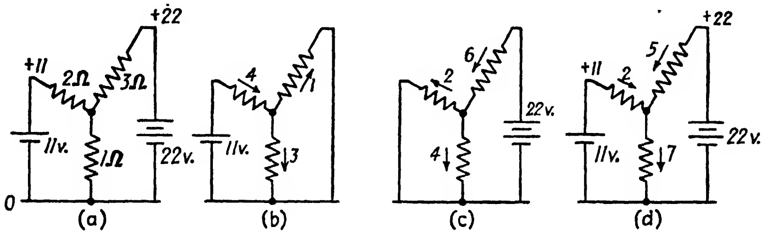

FIG. 1-7.—The currents flowing in a network containing several voltage sources may be found by superposition.

sources in the network as short-circuited or, if they have internal resistance, replaced by a resistance equal to their internal resistance and calculate the resistance measured from one point of the break to the other. Thévenin's theorem then states that the actual current flowing in the branch in question is equal to the open-circuit voltage across the break divided by the resistance calculated in the manner just outlined. The proof of this theorem is rather simple if we use the Helmholtz principle of superposition, but it will not be presented here. Two examples, however, will be given to illustrate the application of the principle of superposition and of Thévenin's theorem. Suppose that it is desired to find the current distribution in the network shown in Fig. 1-7*a*. If the 22-volt battery is replaced by a jumper (short circuit), we obtain a current distribution as shown in *b*; with the 11-volt battery replaced by a short circuit and the 22-volt battery acting alone, the current distribution will be as shown in *c*. With both batteries in the circuit, we have to superimpose the two current distributions. This gives us the final result shown in *d*. The reader will do well to convince himself that this final distribution actually satisfies the two Kirchhoff laws.

In Fig. 1-8*a* a network similar to a Wheatstone bridge is shown. Let it be desired to find the current in the bridge, *i.e.*, the 5-ohm resistor. Applying Thévenin's theorem, we open this branch and calculate the voltage

existing across the break. This can be done easily by applying Ohm's law to the right and left branches. The current in the left branch will be 16 amp, which makes the potential of point A 48 volts (choosing the negative end of the supply voltage as zero level); the current in the right-hand branch will be 9 amp, which places point B at a level of 108 volts. This is shown in Fig. 1–8*b*. Since, with the 5-ohm branch open, there can be no current through this resistor and consequently also no voltage across it [according to Eq. (1–2)], the voltage appearing across the break must be equal to the potential difference between A and B, or 60 volts, with B being positive with respect to A. The next step in the application of Thévenin's

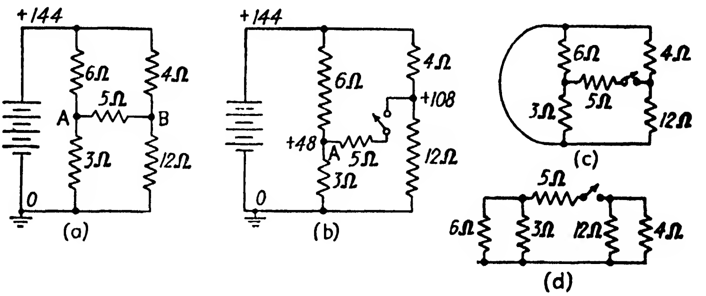

FIG. 1–8.—With the aid of Thévenin's theorem, the current in any branch of a network may be found.

theorem demands that we replace all sources of voltage in the network by short circuits. In our example there is only one voltage. Replacing it by a jumper results in Fig. 1–8*c*. The resistance that we would measure from one terminal of the break to the other can be found easily when the circuit shown in Fig. 1–8*c* is redrawn in the form shown in Fig. 1–8*d*. The resistance will obviously be

$$R = 5 + \frac{3 \times 6}{3 + 6} + \frac{4 \times 12}{4 + 12} = 5 + 2 + 3 = 10 \text{ ohms}$$

The current that flows in the actual circuit in the 5-ohm branch will therefore be 60 volts (the open-circuit voltage appearing across a break in this branch) divided by 10 ohms (the resistance measured from one terminal of the break to the other with all sources of voltage short-circuited), or 6 amp.

1–16. Electric Energy and Power.—When electric circuits were compared with hydraulic circuits, the flow of electric charges was likened to the flow of liquid in a pipe or hose. The cause of the flow of electric current was seen to be due to the existence of a voltage; in the hydraulic circuit the flow of liquid was due to a pressure difference existing between the ends of the pipe or the hose. When liquid is transported from a point of lower pressure (or level) to a point of higher pressure (or level), we require

a pump driven by a motor. This means that work must be done to lift the liquid from the lower to the higher level. On the other hand, when the liquid is permitted to flow from the higher to the lower level, it may be used to drive a water wheel, which means that it can do work on its passage from the higher to the lower level. In the same way, when positive charges are permitted to flow from a point of higher potential to one of lower potential, energy may be abstracted and converted into other forms; for their transportation from a point of lower potential to one of higher potential, work must be furnished from some outside source. Let us consider a simple circuit consisting of a battery across which a resistance is connected. Current will flow from the plus terminal of the battery through the resistance to the negative terminal and within the battery back to the plus terminal (see Fig. 1-9). In the resistance the current enters at the positive and leaves at the negative terminal; the flow of positive charges (if there are any) is from the positive to the negative terminal. In this device electric energy is consumed, and in its place energy of different form, namely, heat, will appear. In the battery, on the other hand, the positive charges enter on the negative and leave on the positive terminal. It acts like a pump in a hydraulic circuit and therefore is a producer of electric energy at the expense of some other form, in this case chemical energy. If instead of a battery we use a generator, then the generator would have to be driven by a motor, and the electric energy represented by lifting the positive charges from a point of lower level to one of higher level would be accomplished by the expenditure of mechanical energy on the shaft of the generator. The important thing to remember is that a device in which the current enters on the positive and leaves at the negative terminal is a consumer of electric energy and that a device in which the current enters at the negative and leaves at the positive terminal is a producer. These distinguishing marks are of the utmost importance in the study of the energy relations in ac circuits where the same device may become alternately a consumer and a producer of electric energy. The reader is therefore urged to give the above statement more than passing attention.

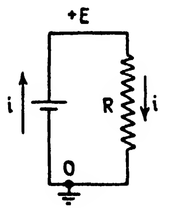

FIG. 1-9.—In a device consuming electric energy, the current flows from the plus to the minus terminal, while in a source of electric energy, the current flows from the negative to the positive terminal.

1-17. Rate of Production or Consumption of Electric Energy.—When a pump has lifted a certain amount of liquid from one level to a higher one, it has done a certain amount of work. Evidently 100 gal of water, for instance, can be lifted from zero level to a 100-ft level by either a small or a large pump, the difference being that the small pump may take several minutes while the large one may lift this amount in a matter of seconds or fractions of a second. The difference between the two pumps is therefore in the rate at which they can perform the required work. In a similar

way, lifting an electric charge from one level to a higher one or letting it flow from a higher level to a lower one always represents the same amount of work or energy, but it is usually the rate at which this work is performed that is of interest to the engineer. The unit of the rate at which electric energy is either consumed or produced, or the unit of electric power, is the watt, and this unit bears a definite relation to the mechanical rates of doing work such as horsepower or foot-pounds per second, etc. A device across which a potential difference of 1 volt exists and through which a current of 1 amp is passing is consuming or producing electric energy at the rate of 1 watt. The power is obviously proportional to the pressure, *i.e.*, the voltage, as well as to the rate of flow, *i.e.*, the current. Therefore the equation for electric power is simply as follows:

$$P = ei \tag{1-4}$$

We saw that a resistance is a consumer of electric power and that the electric energy consumed appears in the shape of heat in a resistance. The electric power or rate of release of energy is seen to be, according to Eq. (1–4), equal to the product of voltage and current but, since voltage and current through a resistance are related to each other by Ohm's law, it is possible to express electric power in terms of current and resistance only, or voltage and resistance only. Thus we arrive at the well-known expressions for the electric power consumed in a resistance or, more correctly, converted into heat.

$$P = i^2R \tag{1-5}$$

$$P = \frac{e^2}{R} \tag{1-6}$$

1–18. Rating of Resistors.—Ohm's law permits us to calculate the current that will flow in a resistor when voltage is applied to it. Whether the heat developed by the conversion of electric power can be dissipated by the resistor without destructive temperature rise is something about which Ohm's law is entirely silent. The specification of a resistor is therefore not complete simply by giving its value in ohms, but the prospective user must also be advised how much power it can dissipate. This value will evidently depend on a number of circumstances. If provisions are made to carry the heat away effectively by blowing air over the resistor or submerging it into a cooling medium, it will be able to dissipate more power without exceeding its temperature limitations than if it is mounted in an enclosed space and possibly surrounded by material not capable of carrying away easily the heat developed in it. The resistors used in electronic circuits are usually rated at some standard wattage values such as ½, 1, 2, 5, or 10 watts. It should be recognized that these values apply to the conditions encountered in the average radio receiver. It is left to

the judgment of the designer to reduce or increase these values when conditions are less or more favorable. In general, it can be said that, as long as the electronic engineer has to use the same components that radio-receiver manufacturers have been using, he will do well to stay a good deal below the ratings given for these components because safety factors in this highly competitive industry seem to be anything but generous.

To illustrate the power relations by an example, a 100-ohm resistor, whether it is rated at 1, or 10, or 100 watts, will always take the same current if connected to, say, 80 volts; this current will be 0.8 amp according to Ohm's law. The power or wattage will be the product of volts and amperes and is seen to be 64 watts. This means that the resistor rated at 1 watt will disintegrate very rapidly because heat is being developed at a rate 64 times as high as the resistor can dissipate. Neither would a 10-watt resistor last very long, but the 100-watt resistor should stand up with a margin to spare.

PROBLEMS

1-1. The charge represented by one electron is 1.6×10^{-19} coulomb. If electrons cross a specified cross section of a conductor at the rate of 5×10^{20} electrons per second, what is the current in amperes?

1-2. In 1 cc of copper there are supposed to be 8.4×10^{22} atoms of copper. Assume that there is one free electron for every 1,000 atoms. If a No. 12 wire (0.081 in. diameter) carries 25 amp, what is the speed of the electron stream in inches per second?

1-3. If the current in Prob. 1-2 were to reverse its direction with a frequency of 60 cps, *i.e.*, $\frac{1}{60}$ sec for a complete cycle, how far would an individual electron move?

1-4. The filament of a 112-type tube requires 5 volts, 0.25 amp for operation. For an experimental setup it is desired to heat the filament from a 225-volt dc source, by putting two 110-volt 40-watt lamps in series with it. What resistance must be put in shunt with the filament?

1-5. Applying Helmholtz's theorem of superposition, find the current distribution in the network given in Fig. 1-10.

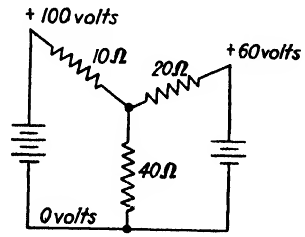

FIG. 1-10.

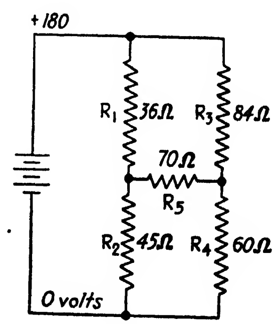

FIG. 1-11.

1-6. Applying Thévenin's theorem, find the current in R_5 of the Wheatstone-bridge circuit shown in Fig. 1-11.

1-7. Design a voltage divider for the four loads L_1 to L_4 shown in Fig. 1-12. The total voltage across the divider is 450 volts, and the total current consumption is to be 100 ma. The loads are to operate with the following voltages and currents:

L_1: E = 220 volts, i = 50 ma

L_2: E = 90 volts, i = 10 ma

L_3: E = 180 volts, i = 20 ma

L_4: E = 100 volts, i = 30 ma

The problem is solved when the resistance values and the wattage consumed in resistances R_1 to R_5 are determined.

1-8. A 3,000-ohm and a 1,500-ohm resistor are connected in parallel; in series with this combination is a 400-ohm resistor; all resistors are rated at 10 watts. Across what voltage can this series-parallel combination be connected without exceeding the 10-watt rating on any of the three resistors? Repeat the calculations if the series resistor is 1,000 ohms instead of 400 ohms.

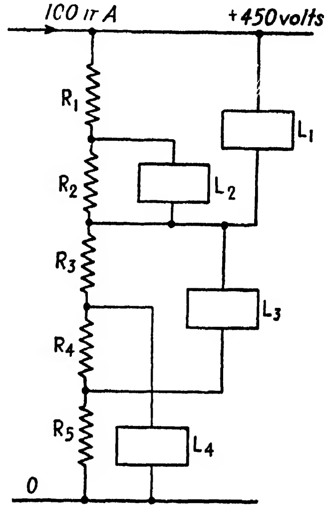

FIG. 1-12.

SUGGESTED ADDITIONAL READING

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CHAPTER II

MUTUAL INDUCTANCE, SELF-INDUCTANCE; CAPACITANCE; RESISTANCE-CAPACITANCE COMBINATION FOR TIMING PURPOSES

2-1. Magnetic Field.—It is assumed that the reader is familiar in a general way with the concept of a magnetic field such as exists in the vicinity of a permanent magnet. Such a field is usually visualized by assuming that so-called "magnetic lines of force" are issuing from one pole of the permanent magnet, finally ending up on the opposite pole. The direction of these field lines coincides with the direction in which a small compass needle would orient itself or in which iron filings would arrange themselves. In a strong magnetic field the lines are considered as more closely packed than in a weak one; we therefore speak of the "density" of magnetic lines. It is perhaps a tribute to the agility of the human mind that we find it easy to make use of such a concept without ever having seen a magnetic field line, for there is no denying the fact that we simply do not know what a magnetic field really is. At present, the answer to the question "What is a magnetic field?" is the lame one "What we find in the vicinity of a magnet."

Oersted and Ampère made the most important discovery that a magnetic field exists also in the vicinity of a wire carrying electric current. This fundamental discovery is the basis of all our electric motors as well as our measuring instruments. Just as two permanent magnets exert a force on each other—attractive or repulsive, depending on their relative positions—so a wire carrying current exerts a force on a permanent magnet or on another wire also carrying current.

2-2. Electromagnetic Induction (Faraday's Law).—If an electric current produces a magnetic field, the question is quite natural whether it is possible to produce an electric current from a magnetic field. This was the problem which Faraday attacked and to which he found the answer in 1831, after several years of experimentation. He found that a voltage appears at the terminals of a loop, or coil, whenever the magnetic field passing through the loop, or "linking" with it, as it is called, changes in strength. In his fundamental experiment he prepared a coil of insulated wire into which he introduced a bar magnet, as shown in Fig. 2-1. It is evident that the magnetic-field lines penetrate the coil, and it is also clear that motion of the bar toward or away from the coil will change the number of

magnetic lines penetrating or linking with the coil. While this change is taking place, *and only during this time,* a voltage will appear across the terminals of the coil. This voltage will be the higher, the faster the change is taking place. Of course, it will exist for a shorter time when the change is taking place rapidly than when it is taking place slowly.

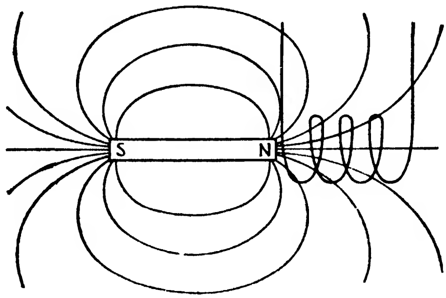

FIG. 2–1.—A voltage will appear across a coil when the number of magnetic lines passing through it is changing.

2–3. Mutual Inductance.—In the preceding paragraph it was assumed that the magnetic field involved was that of a permanent magnet, and the change of the number of lines linking with the coil was due to a change of the relative position of the magnet with respect to the coil; in other words the change of the magnetic linkage was due to a motion of the magnet (or the coil, of course). In the usual dc generator the magnet is stationary and the coil moves; in most ac generators the coils are stationary and the magnet moves. A coil carrying an electric current also produces a magnetic field, as stated above. If another coil is placed near the first one, the magnetic-field lines produced by the first coil may be made to penetrate or link with the second one. If we wish now to change the number of magnetic lines passing through the second coil, we evidently do not have to move the first or second coil but can leave them in a fixed relation and change only the current flowing in the first coil. While the current is changing in the first coil, *and only during this time,* a voltage will appear across the second coil. If two coils are disposed relatively to each other in such a way that a change of current flowing through the one causes a voltage to appear across the other, they are said to possess *mutual inductance.* It is quite logical to define the unit of mutual inductance as that possessed by two coils, when a current changing at the rate of 1 amp per sec in the first coil produces—or "induces"—a voltage of 1 volt in the second. It is true and can be proved mathematically—although it is not at all obvious—that the mutual inductance existing between two coils is of the same value regardless of whether the current flows in coil 1 and the voltage is induced in coil 2, or vice versa. The unit of mutual inductance is the henry.

2-4. Rate of Change of Physical Quantities.—The concept of the rate of change of a quantity with time is of such importance to all branches of engineering that the reader will find it profitable to make every effort to grasp it fully. In mechanical engineering, for instance, the rate at which the speed of a moving body changes is called the "acceleration" of the body. Those familiar with differential calculus know, of course, that if v is the speed of a moving body, dv/dt represents the rate of change of speed, or the acceleration of the body. In a similar way, if the current flowing in coil 1 is designated as i_1, then the rate at which it changes is given by di_1/dt. If coil 2 has a mutual inductance M with respect to coil 1, then the voltage appearing across coil 2 *during the time that the current changes* in coil 1 is given by

$$e_2 = M \frac{di_1}{dt} \tag{2-1}$$

But does a man unfamiliar with differential calculus have to throw up his hands in despair and give up hope of understanding this subject? Just as a carpenter with a spiral screw driver can drive screws more conveniently and faster than with a simple screw driver, so the man equipped with calculus can solve many problems more conveniently than would be possible without this knowledge, but this does not mean that the man without this knowledge is completely lost. In this book we shall replace the symbols of differential calculus by the abbreviation "roc" (pronounced like "rock") which stands for "rate of change." Therefore, roc i stands for "rate of change of current"; roc e means "rate of change of voltage." With this definition, Eq. (2-1) becomes

$$e_2 = M \text{ roc } i_1 \tag{2-2}$$

We have done nothing but write Eq. (2-1) in a different form, but somehow it seems to take the terror out of it for those not familiar with differential calculus.

Let it be stated now that the mutual inductance between the two coils is 0.3 henry, and let the current in coil 1 change from 15 to 17 amp in $\frac{1}{10}$ sec. The rate at which the current changes is evidently 20 amp per sec, and the voltage that will appear across coil 2 *during this* $\frac{1}{10}$ *sec* (or as long as the current keeps on changing at this rate) will be $0.3 \times 20 = 6$ volts. Observe that we are not interested in the *actual* amount of current flowing in coil 1, but only in the *rate at which it changes*. The same voltage would appear across coil 2 if the current in the first coil changed from 87 to 89, or from 503 to 505 amp in $\frac{1}{10}$ sec.

Strictly speaking, the analysis made in the preceding paragraph is correct only if we can assume that the change from 15 to 17 amp occurs at a uniform rate during $\frac{1}{10}$ sec. If the current should change from 15 to 16 amp in less than half the interval while changing from 16 to 17 amp at a

slower rate, then the induced voltage will be *more* than 6 volts during the early part of the interval and *less* than 6 volts during the later part, but the average voltage during the $1/10$ sec under consideration would still be 6 volts.

2-5. Self-inductance.—Suppose that we are dealing with one coil only. It is clear that the magnetic field produced by it will penetrate or link with the coil itself. The law of electromagnetic induction states that a voltage appears across the terminals of an electric circuit *whenever the magnetic field penetrating it changes.* The law does not make any statement as to *how* this magnetic field is produced or how it is changed. Consequently, a voltage appears across the coil during the time that the current through it changes because a *changing current* means also a *changing magnetic field.* This property of the coil, which is analogous to the mutual inductance existing between two coils, is called the "self-inductance" of the coil and is designated by the symbol L. It is measured in the same unit as the mutual inductance, the henry. A coil or electric circuit is said to possess the unit of self-inductance, *i.e.*, 1 henry, when a voltage of 1 volt appears across it *while the current through it is changing at the rate of* 1 *amp per sec.* The formula governing this case is then

$$e = L \text{ roc } i \tag{2-3}$$

Since it is impossible to produce a coil without any resistance, it is usually rather hard to visualize the behavior of a true inductance. When current flows through a resistance, there will be a voltage across it, according to Ohm's law; but when current flows through a true or perfect inductance, no matter how large this current, there will be voltage *only while the current is changing.* Since the behavior of a practical inductance (having resistance) is usually analyzed by considering it as a series combination of a perfect inductance and a separate resistance equal to the resistance of the coil, it is quite necessary to become familiar with the concept of a perfect inductance.

2-6. Polarity of Voltage across Self-inductance.—The question naturally arises as to the polarity of the induced voltage. It may be said that the inductance of a circuit is a characteristic that opposes any change of current in the circuit by the production of a voltage, just as the inertia of a body opposes a change of speed. With this in mind, the polarity of the induced voltage is easy to determine. It is always in such a direction as to *oppose a change* of current in the coil.

Another completely foolproof method of arriving at the polarity of the induced voltage is based on energy considerations. As is well known, a magnetic field represents energy or work. When the current through a coil builds up, this magnetic field also builds up, and it would be against the principle of conservation of energy if no voltage appeared across the coil during the building up of the magnetic field. In other words, we would be

getting the energy of the magnetic field without having to spend watts. While the magnetic field increases, the coil must therefore act as a consumer of electric energy just like a resistance, with the only difference that the consumed electric energy is not converted into heat but into the energy of the magnetic field. Figure 1–9 showed that in the case of a consumer of electric energy the current enters at the positive terminal; therefore, in the case of an inductance the terminal where the current enters is *positive* while the current is *increasing* in strength. On the other hand, when the current is *decreasing*, the magnetic energy of the field becomes less and the inductance gives back this energy as electric power; it becomes a producer or generator of electric energy. In this case the current enters at the *negative terminal*, as shown for the battery in Fig. 1–9. Observe the basic difference between a resistance and an inductance. In the case of a resistance the terminal where the current enters is always positive; for the inductance, the direction in which the current flows is *not* the determining factor for the polarity of the voltage appearing across it. When the current is *increasing*, the terminal of entry is *positive*, as in the case of the resistance but, when it is *decreasing*, the polarity is reversed. When the current is neither increasing nor decreasing, roc i is zero, and Eq. (2–3) tells us that the voltage is zero.

2–7. Ohm's Law of the Self-inductance.—It is of interest, and rather unconventional at the same time, to refer to the relation expressed in Eq. (2–3) as Ohm's law for the inductance. Just like Ohm's law for the resistance, it can be stated in three ways as follows:

$$e = L \operatorname{roc} i \tag{2–3}$$

$$\operatorname{roc} i = \frac{e}{L} \tag{2–4}$$

$$L = \frac{e}{\operatorname{roc} i} \tag{2–5}$$

These three formulas can be expressed in words as follows:

(2–3). When the current through an inductance is changing at the rate roc i, a voltage appears across it equal to roc i times the inductance.

(2–4). If a voltage is applied to an inductance, the current through it changes at a rate equal to the voltage divided by the inductance.

(2–5). If, owing to the application of a voltage, the current changes at a rate roc i, the device is said to have a self inductance equal to the voltage divided by the rate of change of current.

2–8. Self-inductance of Two Coils in Series.—Suppose that in Fig. 2–2 we have two coils with the values of self-inductance L_1 and L_2, respectively, connected in series. The same current is therefore flowing in both coils and, if the current changes, the rate of change will also be the same in both

coils. For example, let $L_1 = 2$ henrys, $L_2 = 6$ henrys. Assume the two coils to be spaced far apart. If the current through the two coils changes at a particular instant at a rate of 10 amp per sec, then we shall have a voltage of 20 volts across coil 1 and 60 volts across coil 2. Now assume that the coils are brought close to each other so that the field of each will pass at least partly through the other coil, the two fields reinforcing each other. There will then exist between these two coils also a mutual inductance. Let it be 3 henrys. Under this condition, there will appear across coil 2 not only the 60 volts due to the self-inductance but an additional 30 volts due to the mutual relation between coils 1 and 2. In a similar way, the voltage across coil 1 will be not only 20 volts but an additional 30 volts due to the current change in coil 2. The total voltage across both coils will then be 20 + 30 + 60 + 30 = 140 volts. The total inductance of the two coils in series is obviously, then, 14 henrys, according to Eq. (2–5), since a current change of 10 amp per sec causes a voltage of 140 volts to appear across the two coils or, expressed mathematically, the total inductance of two coils in series will be

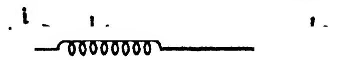

FIG. 2–2.—When a current changing with time flows through two inductances in series, voltages will be induced in each coil also by the action of the current flowing in the other coil.

$$L_t = L_1 + L_2 + 2M \tag{2–6}$$

The conditions outlined above apply, however, only when the magnetic lines produced by coil 1 are reinforcing the magnetic lines produced in coil 2, and vice versa. If we reverse the connection of one coil with respect to the other, then the magnetic lines of one coil will oppose those in the other coil. The voltage in coil 2 would then be 60 − 30 volts, while in coil 1 the voltage would be 20 − 30 volts (which means that the polarity of the voltage changes because of the field originating in coil 2 and penetrating coil 1). The total voltage across the two coils will then be 20 − 30 + 60 − 30 = 20 volts, and the series combination, according to Eq. (2–5), then shows a total inductance of only 2 henrys. Equation (2–6) should therefore be modified as follows:

$$L_t = L_1 + L_2 \pm 2M \tag{2–7}$$

The plus or minus sign depends on whether the coils are connected in such a way that their fields aid or oppose each other. This fact can be made use of for the measurement of the mutual inductance existing between two coils. All that is necessary is to measure the total inductance for aiding and opposing conditions and subtract these two values from each other. Dividing this difference by four gives the mutual inductance, as the reader may easily determine. It can be proved that the mutual inductance M can never exceed the geometric mean of the two inductances L_1 and L_2. In other words, $M \leqq \sqrt{L_1 L_2}$. The ratio $M/\sqrt{L_1 L_2}$, which conse-

quently can never be larger than unity, is a measure of the coupling between the two coils. It is called the "coefficient" of coupling k. This value is of much interest to radio and communication engineers, but is not used extensively by power engineers.

2-9. The Behavior of a Perfect Inductance.—If we could construct a true, resistanceless inductance and applied a constant voltage to it, the current would keep on changing at the same rate forever, as indicated by Eq. (2-4). This case is analogous to a mass on which a force is made to act. If there is no friction, the mass will accelerate as long as the force acts on it. A practical inductance will always have resistance and, as the current increases, an ever-increasing part of the voltage will be needed to drive the current through the resistance. The performance of an actual inductance can best be analyzed by making use of the artifice already mentioned, *i.e.*, by replacing it by two fictitious elements: one, a true inductance without any resistance; the other, a resistance with a value equal to that of the actual coil. The voltage conditions across the two elements can then be analyzed more easily. For instance, let Fig. 2-3*a* show an inductance L having a resistance R. For the study of the circuit the actual inductance is replaced by the series combination shown in Fig. 2-3*b*. Let the voltage E be applied to this combination. At the first instance no current flows since the inductance opposes any sudden change of current. Consequently, there is no voltage across R, and all the voltage E is applied to the inductance. The current will therefore begin to change at a rate dictated by Eq. (2-4), but in so doing more and more voltage will be developed across resistance R, with less and less left across the inductance L. The rate at which the current changes will therefore decrease continuously, and after a long time it will reach a value given simply by Ohm's law of the resistance, *i.e.*, $I = E/R$. The mathematics of this case indicates that the current will change according to an exponential curve. It is suggested that the reader return to this point after a discussion of circuits containing capacitance and resistance, which are of more practical importance to the electronics engineer than the one just discussed.

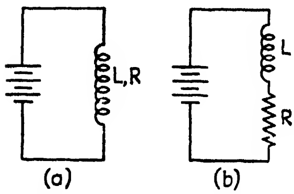

FIG. 2-3.—When a dc voltage is applied to an inductance also having resistance, the analysis can best be made by separating the two circuit elements.

2-10. Capacitance.—Another circuit element of utmost importance to the electrical engineer is capacitance. It is a rather astonishing fact that no other subject seems to cause more confusion than this. There is no reason for this state of affairs because capacitance effects are very easy to understand and visualize. The confusion seems to have its origin in the fact that the subject is usually approached from the point of static electricity; in this book we shall approach it from the point of electric current, *i.e.*, from charges in motion.

Any circuit possesses self-inductance to a larger or smaller degree because there is always a magnetic field associated with an electric current. In a similar way, capacitance effects are practically unavoidable because any two conducting bodies facing each other exhibit what is known as "capacitance."

The larger the surface of these bodies and the closer they are together, the larger is the capacitance. Furthermore, if the space between the conducting bodies is filled with various insulating substances, it is noted that the capacitance effect is higher than with air between the bodies. These general statements will make understandable the construction of air capacitors, such as are used in radio receivers, and of paper capacitors where the two conducting bodies are strips of tin foil and the space between them is occupied by one or several layers of paper. For further details on the construction of capacitors, the reader is referred to any standard text on electrical engineering.

2–11. Voltage and Current Relations for the Capacitor.—So far we have made the acquaintance of the circuit elements resistance and inductance and have learned the relations that exist between the current and voltage across them. We shall now discuss the laws that connect these relations in the case of a capacitor. Everybody is familiar with the action of a storage battery. When we pass a current through—note that the word is "through," not "into"—a storage battery, a voltage builds up across its terminals. In the case of a lead-acid battery, the voltage builds up rather quickly to approximately 2 volts, remains at this value for a long period, and finally rises to about 2.6 volts. Further passage of current through the device does not increase the voltage any further. It is not too difficult to visualize that somebody might succeed in building a storage battery the characteristics of which would differ from that of a lead-acid battery. Thus, we could think of a storage battery where the voltage keeps on rising at a uniform rate as long as current of constant magnitude is passing through it. Well, we do not have to wait for somebody to invent this type of battery: a capacitor is such a battery. In other words, a capacitor is a circuit element across which voltage builds up at a uniform rate as long as a constant current flows through it. The polarity of this voltage with respect to the current is the same as that encountered when an ordinary storage battery is being charged. This means that current entering a completely discharged capacitor at terminal A and leaving at B, as shown in Fig. 2–4, will cause an increasing voltage to appear across the capacitor, A being positive with respect to B. Note that it is just as impossible to charge a capacitor by connecting only one terminal to a source of voltage as it is to charge a storage battery without connecting both wires to the charging source. Current must *flow through the capacitor* to cause a voltage

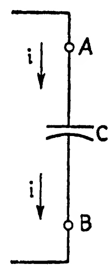

FIG 2-4.—When current passes through a capacitor, the voltage appearing at its terminals will change.

to appear across it. We could spend considerable time discussing what is happening inside the capacitor while this charging takes place; in so doing we would find that the expression "through" is not strictly correct—that it should read "into the capacitor and out of it." The electrical engineer, however—not the physicist—when he observes exactly as much current flowing *into* one terminal of a device as is coming *out* of the other, is justified in saying that the current flows *through* the device, even if he knows quite well that the charges entering one terminal and leaving the other are not the identical ones. The action of a capacitor can be compared to that of the hydraulic device shown in Fig. 2-5. In this figure a cylinder is shown with a tight-fitting plunger held at the center of the cylinder by means of two springs S_1 and S_2 on either side of it. Like the garden hose mentioned earlier, we must consider the device filled with liquid at the time of the purchase. Now suppose that we include this device in a hydraulic circuit by connecting its two ends to the inlet and the outlet of a pump. The whole hydraulic circuit is again considered as closed and filled with liquid. Now it is evident that for every gallon of liquid entering the upper section, a gallon of liquid must leave the lower section; two flowmeters placed at these points will clearly indicate the same amount of flow. With the plunger really tight fitting, however, the liquid leaving the lower section can never be the same as that entering the upper section. For every gallon entering, the plunger is going to be displaced a certain amount downward, compressing S_2 and elongating S_1. A pressure gauge connected to the two sections of the device will indicate that there exists a pressure between the two sections, with the upper section at a higher pressure than the lower one. In other words, the side on which the liquid enters will build up pressure with respect to the side where the liquid leaves. The limit of pumping liquid into the upper section will be reached when the plunger has compressed S_2 solid. Any further amount of liquid pumped into the device will probably lead to the breaking of the plunger and the passage of liquid through it. This compares exactly to the puncture of a capacitor. After a certain amount of current has passed through it (or "into and out of it"), the voltage will reach a value representing the limit for the insulating material between the conducting bodies, and puncture will take place.

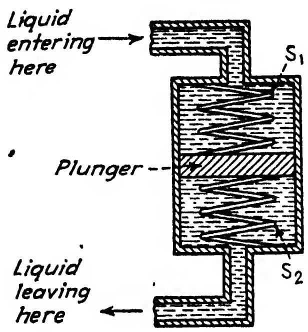

FIG. 2-5.—A hydraulic analogy of a capacitor is given by a cylinder with a tight-fitting plunger held by springs. There will always be just as much liquid leaving through the one pipe as entering through the other.

2-12. Unit of Capacitance; the Capacitor as a Storage Battery.—We have seen that a capacitor is a circuit element across which the voltage rises at a uniform rate as long as a constant current flows through it. With

this definition fixed in our minds, it becomes an easy matter to define the unit of capacitance. The unit of capacitance would logically be possessed by a capacitor across which the voltage rises at the rate of 1 volt per sec when a current of 1 amp flows through it. Since 1 amp is defined as the flow of 1 coulomb per sec, the above definition of capacitance is identical with the one we find in books on electricity. The unit of capacitance is possessed by a capacitor that shows 1 volt across its terminals with a charge of 1 coulomb placed on it. Note and fix in mind that no voltage can exist across a capacitor until a current flow has been permitted through it. The unit of capacitance is the farad. From the point of view of a storage battery, 1 farad is a mighty small storage battery. A charging current of 1 amp would bring it to 6 volts in exactly 6 sec, or a charging current of 3 amp would bring it up to 6 volts in 2 sec; the discharges would be correspondingly short. Nevertheless, this unit is so large that it is a practical impossibility to construct a capacitor of 1 farad. The practical unit is one million times smaller and is called the microfarad. As a matter of fact, the capacitance values encountered in electric circuits are so small that even the microfarad is often found too large and is again divided into one million parts, thus resulting in the micromicrofarad. Let us consider what happens if we pass a current of 1 amp through a capacitance of 1 μf. We have seen that, with a passage of 1 amp through a capacitance of 1 farad, the voltage rises at the rate of 1 volt per sec. Evidently, if the capacitance is one million times smaller, the voltage will rise one million times faster; in other words, it would rise at the rate of 1,000,000 volts per sec. In 0.001 sec it will consequently rise to 1,000 volts. This reasoning makes clear what extremely small storage batteries capacitors really are, but the *size* obviously has nothing to do with the fact that in *principle* the two are alike.

2–13. Ohm's Law for Capacitance.—We have seen that the relation between the current and the voltage across an inductance could be formulated into Ohm's law for the inductance. In a similar way, the relations of these two values in the case of the capacitance can be formulated into Ohm's law of the capacitance. Analogous to Eqs. (1–1*a*), (1–2*a*), and (1–3*a*) where Ohm's law was expressed with the aid of conductance, we can write, for the relations applying to a capacitance,

$$i = C \text{ roc } e \qquad (2\text{–}8)$$

$$\text{roc } e = \frac{i}{C} \qquad (2\text{–}9)$$

$$C = \frac{i}{\text{roc } e} \qquad (2\text{–}10)$$

Expressed in words, these formulas can be stated as follows:

(2–8). When a voltage changing at the rate roc e is applied to a capacitor, the current through it is C roc e.

(2-9). When a current i flows through a capacitor C, the voltage across the capacitor changes at the rate roc $e = i/C$.

(2-10). When a current i through a capacitor causes the voltage across it to change at a rate roc e, the capacitance is $i/\text{roc } e$ farads.

Following the relations outlined above, the reader should have no difficulty in developing the formulas for capacitance values connected in parallel and in series. Several capacitances in parallel can be replaced by one equal to the sum of the individual capacitances. Two capacitances C_1 and C_2 in series, however, are equivalent to a single capacitance given by the relation

$$C = \frac{C_1 C_2}{C_1 + C_2} \tag{2-11}$$

2-14. Summary of Voltage and Current Relations for Resistance, Inductance, and Capacitance.—At this point it may be well to sum up our discussion of resistance, inductance, and capacitance. Suppose that we have a resistance of 1 ohm, an inductance of 1 henry, and a capacitance of 1 farad. Let us assume that at a particular instant a current of 10 amp is passing through each of these elements. Can we determine how much voltage exists across each of the elements? The answer is that only for the resistance can we make a statement pertaining to the voltage existing across it. According to Ohm's law for the resistance, a voltage of 10 volts exists across it at the instant when the current is 10 amp.

The voltage across the inductance does not in any way depend on the actual amount of current flowing through it at a given instant but only on *the rate at which it changes*. If the 10-amp current was, for instance, 9.8 amp $\frac{1}{50}$ sec before the instant in question, and 10.2 amp $\frac{1}{50}$ sec later, the rate of change would be 0.4 amp in $\frac{1}{25}$ sec, or 10 amp per sec. The voltage across the inductance will then be 10 volts. If the change is from 10.2 to 9.8 amp, the polarity of the voltage will be opposite to the first case.

Regarding capacitance, Eq. (2-9) tells us that at the instant when the current through the capacitance is 10 amp the voltage rises (more accurately, changes) at the rate of 10 volts per sec. What actual voltage we shall have across the capacitor is therefore dependent on *how long* this current of 10 amp has been flowing through the capacitor since the time it began to charge. If it was flowing at a constant rate of 10 amp for 2 sec, the voltage across the capacitor would be 20 volts; if it was flowing only 0.1 sec, the voltage would be only 1 volt.

The voltage across a resistance depends only on the *instantaneous value of the current*. Its preceding history or its rate of change is of no interest. The voltage across an inductance depends only on the *rate of change of current* at the instant in question. Neither its actual value nor its preceding history is of interest. The voltage across a capacitance depends only on the *previous history* of the current. Its actual value at the instant or its

rate of change is of no interest except that the actual value gives us the *rate at which the voltage is changing* at the instant in question.

These discussions seem to be rather lengthy, and there may be a tendency to pass lightly over them. Let it be said here that they deserve the fullest attention because a clear understanding of them will make the study of these elements in ac circuits very much easier.

2-15. Series Combination of Resistance and Capacitance.—Since the combination of resistance and capacitance in series forms the basis of almost all electronic timing devices, it may be appropriate to investigate this combination somewhat in detail. In Fig. 2-6 is shown a dc source and a series combination of a resistor R and a capacitor C. Consider the capacitor discharged. Now let switch S be closed. At the very first instant after the closing of the switch, the full voltage E of the battery will appear across the resistor since the voltage across a discharged capacitor is zero. The value of the current flowing in the first instant is therefore given by Ohm's law and is $i_{t=0} = E/R$ which should be read as "the current at the time $t = 0$ equals E/R"; but a current flowing through a capacitor causes a voltage rise to take place. The polarity of this voltage is indicated in Fig. 2-6 and is seen to be in opposition to the voltage applied by the battery. As time goes on the voltage across the resistor R, and with it the current in the circuit, will decrease. At the time, for instance, when the voltage across the capacitor has reached a value equal to half the battery voltage, the current through the resistor will also have dropped to one-half, and the rate of rise of voltage on the capacitor will be only one-half of the value it had at the time $t = 0$. A little thought shows that theoretically the voltage across the capacitor can never reach the value of the supply voltage because, the nearer it gets to this value, the smaller the charging current becomes. This in turn means that the capacitor voltage rises at a lower and lower rate as it approaches the supply voltage.

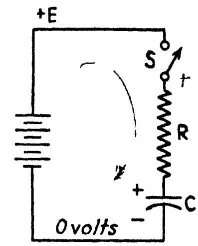

FIG. 2-6.—The charge of a capacitor through a resistor takes time.

2-16. Voltage and Current in *R-C* Combinations; the Time Constant.—It is not a particularly difficult task to plot the voltage across the capacitor as a function of time by using a step-by-step method; as a matter of fact, for a good understanding of the process that takes place, this procedure cannot be too highly recommended. The mathematics covering this case is very simple for those who are familiar with differential equations. Those interested in this phase of the subject should refer to electrical-engineering texts where the subject is treated under the fundamentals of transients. We shall confine ourselves here to presenting the results of the solution of the differential equation governing this case. The mathematical treatment shows that the process of charging will be the slower, the larger the resistor and the larger the capacitor, a result that is quite obvious when

one remembers that a capacitor is really a storage battery. The larger the battery and the higher the resistance through which the charging current flows, the longer the charging will take. It is therefore the product of R and C that characterizes the combination. This product is usually referred to as the time constant of the circuit and is designated as T. The current at any time t is given by

$$i = \frac{E}{R}\epsilon^{-t/T} \tag{2-12}$$

where ϵ is the base of the natural logarithms and is equal to 2.71828. . . . Note that the value E/R represents the current that flows at the first instant after closing the switch and is obviously the maximum current ever flowing during the charging process.

The voltage across the capacitor is given by the following equation:

$$e_c = E(1 - \epsilon^{-t/T}) \tag{2-13}$$

At the time $t = T$, it is seen that the current has dropped to $1/\epsilon$, which is equal to 36.8 per cent of its initial value. The drop across the resistor is then obviously also 36.8 per cent of the applied voltage, which leaves 63.2 per cent for the capacitor.

2-17. Use of the Time Constant.—In general, then, in a time equal to one time constant, the capacitor voltage will be approximately 63 per cent of the charging voltage.

Exactly the same relations apply when we start out with a capacitor charged to a certain voltage, as shown in Fig. 2-7, and discharge it through the resistor R. At the first instant after the switch is closed, the current through the resistor R will be maximum, just as it would be a maximum at the instant when a resistor is connected across a charged storage battery. Its value is, of course, simply equal to the voltage to which the capacitor (storage battery) had been charged, divided by the resistance. Neither a storage battery nor a capacitor can furnish current without its voltage decreasing, and the rate at which the voltage existing across the capacitor decreases when the resistance is connected across its terminals is given by Eq. (2-9). But the rate at which the voltage decreases will not be constant or uniform since the current discharging the capacitor decreases as the voltage of the capacitor (storage battery!) becomes less and less.

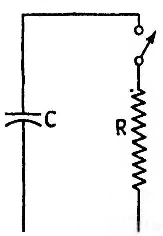

FIG. 2-7.—A charged capacitor upon connection to a resistor will begin to discharge just like a storage battery.

For example, at the instant when the voltage across the capacitor has dropped to one-half its original value, the current through the resistor also has one-half its original value, and the rate at which the capacitor voltage changes will also be one-half the value at which it was changing at the first instant [see Eq. (2-9)]. A complete mathematical treatment

again shows that after an interval equal to one time constant the current has dropped to 36.8 per cent of the initial value and with it, of course, the voltage across the resistor and the capacitor since the two elements in this case are in parallel. The capacitor therefore has lost 63.2 per cent of its voltage. Now the state of affairs existing at this instant could be considered as the starting point for another interval equal to one time constant, at the end of which the capacitor again loses 63.2 per cent of the voltage that existed at the beginning of this interval. Every interval of one time constant therefore means the loss of 63.2 per cent of the voltage that was left at the beginning of the interval. For example, let a capacitor of 2 μf be charged to 200 volts and then connected to a resistor of 3 megohms (3,000,000 ohms). The time constant of this combination is the product of the value of the capacitance and the value of the resistance, or $2 \times 10^{-6} \times 3 \times 10^{6} = 6$ sec. Six seconds *after* the capacitor was connected to the resistor, it lost 63.2 per cent of its original charge, retaining 36.8 per cent or 73.6 volts. After another 6 sec 36.8 per cent of the 73.6 volts, or 27 volts, remain across the capacitor. An additional 6 sec reduces the voltage to 36.8 per cent of 27 volts, or 10 volts. It is again apparent that, theoretically, the capacitor will never discharge completely since the closer it gets to zero, the lower is the current that discharges it. After a period of five time constants, the remaining voltage is 0.67 per cent of the original voltage or, expressed differently, the capacitor has lost 99.33 per cent of the original charge (or voltage). It is therefore common practice to consider the charge or discharge of a capacitor as completed in an interval of five time constants.

It will be of great help to the reader later when he realizes that in any network consisting of dc sources, resistors, and capacitors, the final state of equilibrium will be characterized by zero current in any resistor placed in series with a capacitor. For if any current remained in such a resistor, the voltage across the capacitor would keep on changing, indicating that the final state had not yet been reached. But with zero current finally in all branches containing capacitors, they might just as well be absent from the network; for the purpose of determining the final current and voltage distribution in a dc network, all branches containing capacitors can therefore be considered as absent.

2–18. Solution of *R*-*C* Problems with Graphs.—When the relations outlined in the last few paragraphs are plotted on semilogarithmic paper, a straight line results, as shown in Fig. 2–8. This chart is of the greatest help in the design and analysis of timing circuits and multivibrator circuits, in short, of all circuits, the operation of which is based on a resistor-capacitor combination, and the reader will do well to study its use carefully. An example will illustrate its use.

The statement that in an interval equal to one time constant the voltage across a capacitor changes 63.2 per cent of the total voltage change separat-

ing it from the final state not only holds true in the two simple cases already discussed but applies also when the capacitor has an initial charge,

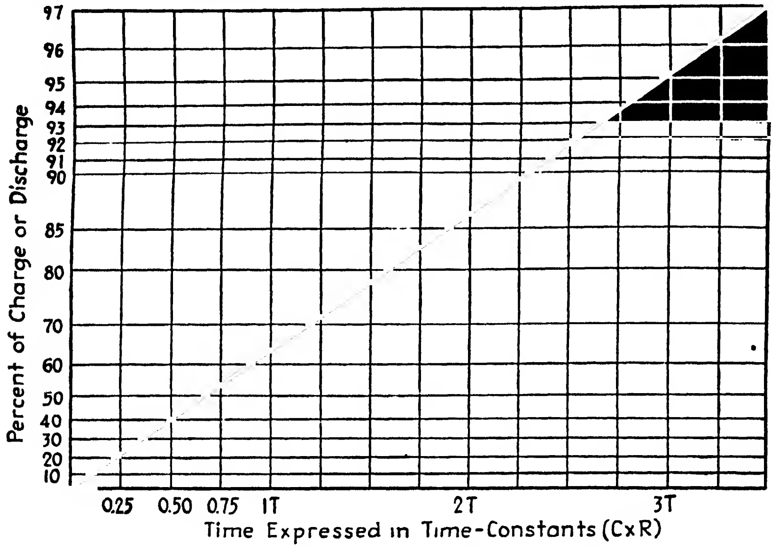

FIG. 2-8.—With the aid of this graph the amount of charge or discharge of a capacitor-resistor combination for any time of discharge can be determined.

regardless of its polarity. Suppose that the positive terminal of a capacitor charged to 30 volts is connected to the negative terminal of a battery of 50 volts. Adhering to the method of showing polarities by levels, we represent this condition as shown in Fig. 2-9. If we choose the negative terminal of the battery as the zero level, or "ground floor," terminal B of the battery will be "up" 50 volts, while terminal A of the capacitor will be "down" 30 volts. Closure of the switch S will therefore place the resistor R across a voltage of 80 volts. The current that begins to flow in the resistor from the point of higher level, which is B, to the point of lower level, namely, A, enters the capacitor at terminal A. As shown in connection with Fig. 2-4, a current flowing through a capacitor makes the terminal of entry progressively more positive, and the potential of point A will therefore begin to travel in the upward direction. How far will this travel go? As long as current flows through R in the direction from B to A, the potential of A will go up; but current through R ceases when A has reached the level of B, which means that there is no longer any voltage across R. Therefore the closure of switch S starts A on an upward swing that will eventually bring it to a level of 50 volts. The chart shown in Fig. 2-8 per-

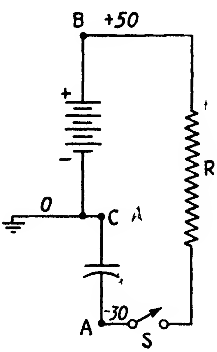

FIG. 2-9.—Point A starts at a level of -30, but after closure of switch S it will wind up after an infinite time at a level of $+50$.

mits us to tell at what point it will be at any given instant. Let us assume that the capacitor has a capacitance of 0.5 μf, that the resistance is 1 megohm, and that we wish to determine at what instant the voltage across the capacitor is just zero. This is obviously the case when terminal A has traveled up 30 volts from it starting level because at this instant it is at the same level as the terminal C. When it started, 80 volts separated it from its final level, and the desired point is reached when it has gone $^{30}\!/_{80} = 0.375$ of the total travel. Reference to the chart shows that to complete 37.5 per cent of the total voltage change requires 0.47 time constant or, in our case, where the time constant is 0.5 sec (product of capacitance and resistance), the time is 0.235 sec. Or suppose that we had been asked to determine at what level point A will be 1 sec after closure of switch S. Since the time constant of the given combination is 0.5 sec, 1 sec represents two time constants, and the chart shows that 86.0 per cent of the total voltage change will be completed. The total change being 80 volts (from -30 to $+50$), 86.5 per cent amounts to 69.2 volts. Starting from a level of -30, A will therefore be at $+39.2$ volts one sec after closure of the switch. Observe that with this method of solving the particular problem the instant when the capacitor is discharged and begins to charge with the opposite polarity is in no way a preferential point to any other. It may also be of interest to see the problem from the storage-battery viewpoint. The problem starts out with two storage batteries in series, connected so that their voltages add; across this series connection is placed the load R. One of the storage batteries (the capacitor) is many million times smaller than the other and is therefore discharged in about $\frac{1}{4}$ sec; from then on it is charged by the larger battery, and the charging current will become zero, when it has reached the voltage of the larger battery.

PROBLEMS

2-1. Two coils have a mutual inductance of 3 henrys with respect to each other. The current through one of them is raised from 20 to 25 amp in an interval of 0.3 sec. If we assume that the change is uniform during this time, what voltage will appear across the second coil?

2-2. An oscillogram shows that the current in a generator field reaches a value of 10 amp in 0.2 sec after connecting it to a dc source of 150 volts. If we neglect the resistance of the field, what value would we obtain for the inductance of the field?

2-3. To build up the pull of a magnetic clutch quickly, it is desired to build up the current to a value of 3 amp in 0.1 sec. The coil has an inductance of 8 henrys and a resistance of only a few ohms, which may be neglected in this particular problem. What voltage must be applied to the coil for the first $\frac{1}{10}$ sec to achieve the desired build-up of the current?

2-4. A coil (a generator field, for instance) with an inductance of 12 henrys and a negligible resistance (say, less than 1 ohm) is connected to a direct voltage of 100 volts. A circuit breaker set to trip at 10 amp is in series with it. Assume that the breaker has a delay of 0.2 sec between reaching the tripping current and opening the circuit.

a. What value will the current have, when the breaker opens?
b. What time will have elapsed between closing the circuit and opening it?

2-5. Given two coils with mutual inductance with respect to each other. The two are connected in series, and it is observed that during a period of 0.1 sec while the current was being changed by 5 amp the voltage across the series combination was 10 volts. Upon reversing one coil and repeating the experiment, the voltage across the two was observed to be 18 volts. What is the mutual inductance between the two coils?

2-6. A coil has an inductance of 3 henrys and a resistance of 100 ohms. It is connected to a source of 200 volts. At the instant when the current has reached a value of ½ amp, at what rate will it be changing?

2-7. An inductance of 5 henrys and a resistance of 10 ohms are connected across a voltage source of 100 volts. At the instant when the current is changing at the rate of 5 amp per sec, how much voltage will there be across the resistance?

2-8. Five microamperes of direct current are flowing into an initially discharged capacitor of 4 μf.

a. To what voltage will the capacitor have charged after 0.4 sec?
b. How long will it take to charge the capacitor to 30 volts?

2-9. An 8 μf capacitor is charged to 300 volts. For $\frac{1}{60}$ sec it is discharged with 15 ma. To what value will the voltage across the capacitor drop?

2-10. A 16 μf capacitor used in a full-wave rectifier circuit (*i.e.*, a discharge period of approximately $\frac{1}{120}$ sec) is charged to 400 volts. If we assume that a voltage fluctuation of 5 per cent is permissible, what load current can be drawn from it?

2-11. A capacitor of 2 μf, initially discharged, is connected to a dc source of 150 volts in series with a resistance of 100,000 ohms.

a. At what rate will the voltage begin to rise?
b. At the instant when the capacitor is charged to 100 volts, at what rate will the voltage be rising?
c. What will be the voltage across the capacitor after ¼ sec?
d. How long will it take to charge it to 135 volts?

2-12. A capacitor of 5 μf is charged to 120 volts and placed in series with a battery of 180 volts. With the plus terminal of the battery connected to the minus terminal of the charged capacitor a resistance of 400,000 ohms is now placed across the combination. What voltage will there be across the capacitor (value and polarity!) after 1, 2, 3, 4, 5, 6, 8, 10 sec?

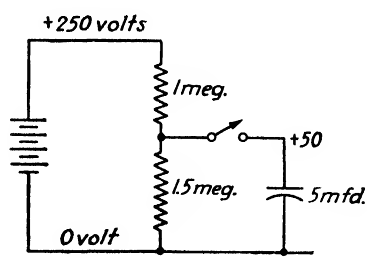

FIG. 2-10.—Diagram for Prob. 2-14.

2-13. If in Prob. 2-12 the connection had been made with opposite polarity, what would the voltage values be at the various times?

2-14. In the diagram shown in Fig. 2-10, let the capacitor be initially charged to 50 volts with a polarity as shown. How much voltage will there be across the capacitor 6 sec after closure of switch *S*? (HINT: Apply Thévenin's theorem to the capacitor branch.)

SUGGESTED ADDITIONAL READING

Henney, K.: "Principles of Radio," Chaps. 5-7, pp. 62-123, John Wiley & Sons, Inc., New York, 1945.

M.I.T. Staff: "Electric Circuits," Chaps. 1 and 2, John Wiley & Sons, Inc., New York, 1940.

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CHAPTER III

ALTERNATING CURRENTS; WAVE SHAPES; AVERAGE AND RMS VALUES; VECTOR REPRESENTATION

3-1. Definition of the Terms "Periodic" and "Alternating Current."—If the value of the current flowing in an electric circuit varies continuously in strength in such a way that the variations repeat themselves over and over again in equal time intervals, the current is called a "periodic" current. The time interval at which the variations repeat themselves is called the "period" of the periodic current and is usually designated by T. If the current not only varies in strength but also reverses its direction during the time T and if, owing to this reversal, the total motion of charges during one period T is zero (or, expressed differently, if the average value of the current during the interval T is zero), then the periodic current is called an "alternating current." A periodic current can be considered as the sum of a direct current and an alternating current. The current in vacuum tubes usually varies in magnitude only, not in direction. It is therefore a periodic current but can be considered as consisting of the two components just mentioned.

3-2. Dc Component, Frequency, and Period.—Suppose that the current values at each instant of a particular periodic current are plotted against time and result in a graph as shown in Fig. 3-1. As already stated, the

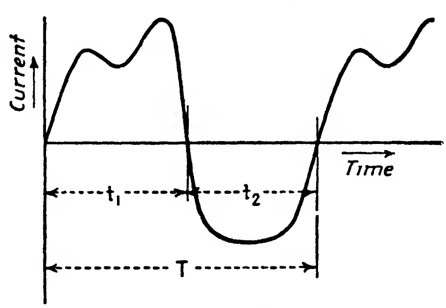

FIG. 3-1.—The graph of a periodic current, it will be an alternating current if the areas above the X axis are equal to the areas below it.

time after which the values repeat themselves is the period T, and the graph of the current during this time is called one "cycle" of the periodic current. The current is seen to flow in one direction during the time t_1, while it is flowing in the opposite direction during the rest of the cycle. t_1 does not necessarily have to be equal to t_2. Just by glancing at the graph

we cannot be sure whether this particular current has a dc component or whether it is a true alternating current. To find this out, we have to determine the area under the graph during the time t_1 and also during t_2. If these two areas are equal in magnitude, then the periodic current has no dc component. A little thought will show that the areas of a current graph represent the amount of charge moved during a particular interval.

It is usually found more convenient to use the reciprocal of the period T as a means of characterizing the alternating current. This value is designated as f and is called the "frequency" of the alternating current. The relation between these two values is as follows:

$$f = \frac{1}{T} \tag{3-1}$$

$$T = \frac{1}{f} \tag{3-2}$$

The truth of this statement does not require any further proof. If the period of one cycle is, for instance, 0.1 sec, then the number of cycles in 1 sec, *i.e.*, the frequency, is obviously 10.

3-3. Average Value of an Alternating Current.—The average value of a true alternating current is evidently zero because, by definition, the amount of charges moved in one direction is equal to the amount moved in the other direction. Nevertheless, reference is made quite often to the average value of an alternating current. This is found to be of importance in the analysis of rectifier problems. By it is meant the average current during the fraction of a cycle in which the current flows in one direction only. More will be said about this phase in connection with particular wave shapes.

Let us consider an alternating current with an average value of 2 amp during the positive and negative half cycles. What heating will take place if this current passes through a resistance of, say, 3 ohms? We know that a direct current of 2 amp would produce a power of 12 watts in such a circuit since i^2R in this case is equal to $2^2 \times 3 = 12$. One would certainly be tempted to assume that, if the average value of an alternating current during the two half cycles is 2 amp, the power would be the same as if we were dealing with a steady current of 2 amp. This is not the case, however, and a simple example will show strikingly how easy it is to become confused on this subject. Suppose that wiring is to be provided for a dc motor taking a current of 28 amp, but that the load on the motor varies periodically in such a way that the current just given flows for 5 sec, followed by an idling period of 5 sec, during which the current drops to 2 amp. The graph of the current is shown in Fig. 3-2. The average current is evidently 15 amp. This is the value that a highly damped dc ammeter would indicate, or which even an ordinary dc instrument would indicate, if the "On"

and "Off" intervals were shortened from 5 sec to, say, $\frac{5}{100}$ sec. Consulting standard wire tables, we find that a No. 14 rubber-covered wire is rated at 15 amp. But would this wire be adequate for the installation, *i.e.*, would it become just as hot and not hotter if the current were a steady 15-amp current instead of the given duty cycle? Suppose that the wiring between the point of supply and the motor has a resistance of 0.1 ohm (which for No. 14 wire would be the case if the motor were located about 20 ft from the point of supply). With a steady current of 15 amp the power lost in the wiring will be $15^2 \times 0.1 = 22.5$ watts, which will cause the temperature rise of the wire. With the duty cycle as outlined, the wattage during the 5 sec when the current is 28 amp will be $28^2 \times 0.1 = 78.4$ watts, while during the 5 sec when the current is 2 amp it will be $2^2 \times 0.1 = 0.4$ watt.

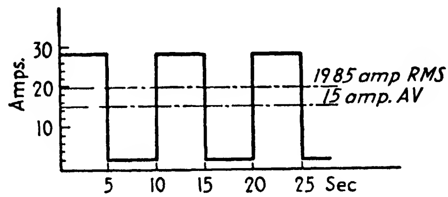

FIG. 3-2.—Average and rms values of a unidirectional current.

The average wattage is (since the two intervals are equal) the average of the two wattage values, or $(78.4 + 0.4)/2 = 39.4$ watts. The average wattage is therefore about 75 per cent higher than with the steady current of 15 amp! The temperature rise will also be 75 per cent higher, and we are therefore certainly exceeding the rating of the No. 14 wire. Now, if it is wrong to use the average value of the current, in this case 15 amp, as the basis of selecting our wire, what should we use? We have seen that our actual current develops an average wattage of 39.4 watts in a resistance of 0.1 ohm. We may now ask what steady current would develop the same wattage. If we call this value x for an instant, it must satisfy the equation $x^2 \times 0.1 = 39.4$, or $x^2 = 394$, and $x = \sqrt{394} = 19.85$ amp. The current as shown in Fig. 3-2, when flowing through any resistor, develops as much heat as a steady current of 19.85 amp although its average value is only 15 amp! The heat-developing value of the current is called the "rms" value, or effective value of the periodic or alternating current. It can be proved mathematically that the rms value of any current can never be smaller than the average current.

3-4. Determination of Average and Rms Values of an Alternating Current.—If the example just given is well understood, the determination of the average and rms values of an alternating current of any wave shape should not be difficult. Let it be required to determine these two values for the alternating current shown in Fig. 3-3, which represents an alternating current varying in steps. A smooth curve can always be replaced by such

steps for graphical analysis. The accuracy of the results obtained by such a replacement will, of course, be the higher, the smaller the steps. If the frequency is 10 cycles, the time for one period is $T = 0.1$ sec, and each step represents 0.01 sec. A glance at the wave shape indicates that the areas above and below the time axis are equal so that we are dealing with a true alternating current. The average value of the current during either the positive or the negative half wave can be determined as follows:

$$I_{av} = \frac{0.01 \times 2 + 0.01 \times 4 + 0.01 \times 6 + 0.01 \times 4 + 0.01 \times 2}{0.05} = 3.6 \text{ amp}$$

In order to determine the rms value of the current, we must determine the average wattage (because this gives us the heating effect) and then

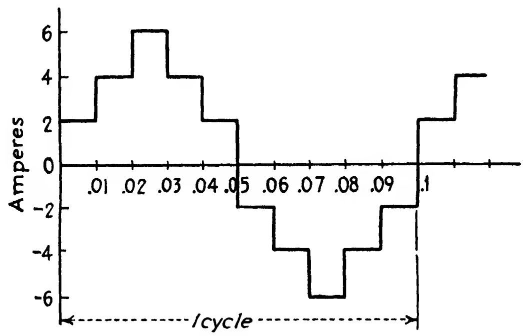

FIG. 3-3.—Determination of the average and rms values of an alternating current varying in steps.

find the value of the steady direct current that would produce the same wattage. If we imagine the current shown in Fig. 3-3 as flowing through a 1-ohm resistor, the wattage during the first 0.01 sec will be $2^2 \times 1 = 4$ watts; during the second 0.01 sec in a similar way (by squaring the current) 16 watts; during the third 0.01 sec 36 watts; etc. During the negative half cycle the current has reversed its direction, but that does not mean that refrigeration now takes place instead of heating. Besides, the square of a negative value is still positive! The average wattage during either the positive or negative half cycle is therefore $(4 + 16 + 36 + 16 + 4)/5 = 15.2$ watts. This is evidently also the average wattage over the whole cycle. What direct current will produce 15.2 watts in a 1-ohm resistor? It must be $x^2 \times 1 = 15.2$, or $x = 3.899$. The rms or effective value of the current shown in Fig. 3-3 is therefore 3.899 amp and the average is 3.6 amp.

The procedure of arriving at the rms value also indicates why it bears this name. We had to *square* the individual current values, then find the *mean* of these squared values (which in this case was 15.2), and finally extract the *square root*. The value so found is therefore the *root of the mean of the squares*, or the root-mean-square, abbreviated rms.

The considerations presented in the last few paragraphs dealt with the average and rms values of a periodically varying current. It is clear that similar considerations apply also to a voltage that varies periodically. The rms value of a voltage is the steady direct voltage that would produce as much heat, if connected to a resistance, as the periodically varying voltage under consideration.

3-5. Alternating Currents of Sinusoidal Wave Shape.—Of particular interest to the electrical engineer are alternating currents and voltages, the wave shape of which when plotted against time is a sine wave. As a matter of fact, it can be proved that any periodically varying current or voltage, no matter what the wave shape (such as shown in Figs. 3-1 and 3-3), can be considered as the result of superimposing upon each other a number of sine waves of different frequencies and amplitudes. The study of the properties of alternating currents of sinusoidal wave shape is therefore of the utmost importance to the electrical engineer.

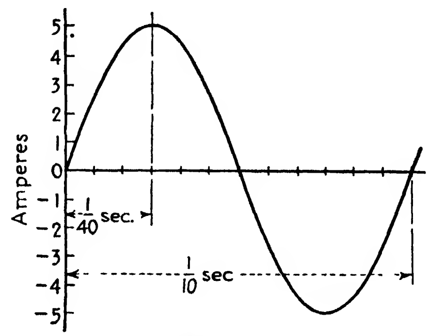

FIG. 3-4.—The graph of a sinusoidally varying alternating current with an amplitude of 5 amp and a frequency of 10 cycles per second.

How can we construct the graph of a current that varies sinusoidally with time 10 times per second and has a maximum value of, say, 5 amp? There are various ways of solving this problem. The most inconvenient but the most accurate method makes use of trigonometric tables. When employing this method we proceed as follows. Referring to Fig. 3-4, we see that if the frequency of the current is 10 cycles, the period will be $\frac{1}{10}$ sec and the current will rise from zero to its maximum value in $\frac{1}{40}$ sec, decrease to zero in another $\frac{1}{40}$ sec, build up to the maximum in the opposite direction in another $\frac{1}{40}$ sec, and return to the starting value during the last $\frac{1}{40}$ sec. Since the various ascending and descending branches of the sine wave are alike, our problem is solved when we can accurately give the values during the first quarter of a period. Reference to trigonometric tables shows that the sine function of an angle rises from zero to a maximum value of unity while the angle changes from 0 to 90 deg. The maximum of our wave, occurring after one quarter cycle, must therefore correspond to 90 deg. We now divide the quarter cycle into a convenient number of parts, such as 3, or 6, or 9. In the first case each part would represent 30 deg; with 9 parts, each would obviously represent 10 deg. We then look up in the trigonometric tables the sine of 10 deg, 20 deg, etc., finishing up with the sine of 90 deg, which is equal to 1. These values are then multiplied

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with the maximum value of the current, in our case 5 amp, and plotted at nine equally spaced intervals in the quarter cycle.

3-6. Vector Representation of Alternating Currents.—It would indeed be a long and laborious method if every time we were to deal with an alternating current we had to proceed as outlined in the preceding paragraph. The early workers in this field hit upon an extremely clever method of avoiding the necessity of plotting a sine wave every time. Its principle is outlined in Fig. 3-5. Let M be a motor with the shaft facing the reader. Into this shaft is fastened, at right angles to it, a rod or strip 5 in. in length. To the right of the motor and far away, we have a light source, producing a parallel beam of light rays, such as a searchlight or the sun itself. To the left of the motor is a projection screen, which in Fig. 3-5 is seen by the reader edgewise and which therefore appears only as a line. On this screen there will evidently appear the shadow of the motor and the rod fastened to the shaft. The shaft will appear as a horizontal line, the rod as a vertical line. Now consider what happens when the motor is running. The shadow of the shaft will remain stationary, but the shadow (or projection) of the rod will change continuously in length and will be alternately above and below the shadow of the shaft. The shadow will have a maximum length of 5 in. at the instant when the rod is in the vertical position and will be practically zero in length (for a very thin rod) when in the horizontal position. For any other position, the length of the shadow will evidently be 5 in. multiplied by the sine of the angle α, shown as the length OP in Fig. 3-5. But this is exactly the relation we have used for constructing the sine wave in Fig. 3-4, and we therefore recognize that the instantaneous values of a sinusoidally varying alternating current or voltage can be considered as the instantaneous lengths of a shadow thrown by a rod and rotating with a frequency equal to that of the alternating current. The length of the rod must be made equal to as many units as the maximum value, also called "amplitude," of the alternating current or voltage. When the shadow is *above* that of the shaft, the values of the alternating current are positive, and vice versa. The word "rod" or "stick" is not quite scientific enough and is therefore replaced by "vector." When we read, for instance, that in a particular case "the vector of the voltage leads the vector of the current by 30 deg," we may translate it into "the rod (or strip) representing the voltage is fastened to the shaft of the motor so that it is 30 deg ahead of the rod representing the current."

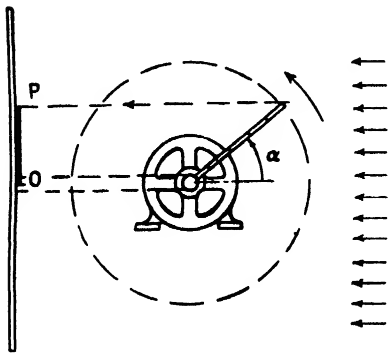

FIG. 3-5.—A vector is nothing but a stick, fastened to the shaft of a motor and throwing a shadow on a screen.

3-7. The Visualization of Alternating Voltages by Heights.—At this point we may profitably examine how our concept of visualizing voltages as vertical levels will fit the case of an alternating voltage and what relation it will bear to the vector concept. The reader is urged to study this carefully because use will be made of the relations discussed here in the chapters on rectifiers.

Let us assume that the voltage appearing across the secondary winding of the transformer shown in Fig. 3-6 has an amplitude of 100 volts. We are asked to plot this voltage, taking the instant when the voltage is zero as the starting point. After one quarter cycle the voltage will then be 100 volts; at this instant one of the terminals is positive with respect to the other. Let us say that A is positive with respect to B. This means, of course, that B is negative with respect to A. How should we plot the voltage? Negative or positive? Now, when we are told that a storage battery has a voltage of 6 volts—nobody ever says that it is *minus* 6 volts—we are assuming that the prospective user will make use of the markings provided by the manufacturer to assure that it is connected properly. But the polarity of an alternating voltage changes continuously, and simply to draw a sine wave with 100 volts amplitude and call it the voltage appearing across the secondary is ambiguous until we state whether it represents the voltage of terminal A with respect to terminal B, or vice versa. If it is only a question of the heat that this voltage produces in a given resistor, then such an accurate statement is not needed; if this source of alternating voltage is to be placed in series with another alternating voltage, however, it is absolutely vital to know just what our sine wave, or our vector, represents. If somebody gave us two batteries, one of 10, the other of 6 volts, but without any terminal markings, we could not predict whether a series connection would give us 16 or 4 volts. Therefore, wherever accuracy of statement is important, we should mark voltages as e_{AB}, which means the potential of point A with respect to B, or as e_{BA}, which means the potential of point B with respect to A. It is quite clear that in all cases, whether direct current or alternating current, $e_{AB} = -e_{BA}$. In other words, when point A is 60 volts up with respect to B, e_{AB} would be $+60$; at this instant point B is of course 60 volts down with respect to A. In other words, e_{BA} is -60.

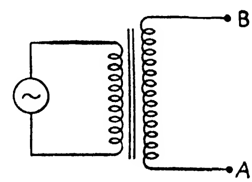

FIG. 3-6.—The alternating voltage between two points is not described properly until it is stated which one of the two points is to be considered as reference point.

In Fig. 3-7 the sine wave is marked e_{BA}. This means we have chosen terminal A as the reference point, and the graph indicates that after one quarter cycle point B is positive, or up, with respect to A. The concept can be made even more forceful by "nailing" the chosen reference point to the ground, as shown in this figure. Point B may then be visualized as "bobbing merrily up and down"; while the graph is positive, point B is

chasing up into the attic and down again, but, during the negative half cycle, it is exploring the basement. (Remember that A is at the ground, or street level!)

If we make use of the vector representation, as shown in Fig. 3–7c, the position of the end point of the shadow thrown by the rotating rod, or vector, can be considered as the instantaneous level of terminal B with respect to A or, in this case, with respect to ground to which A is connected.

The same remarks about accuracy of statement also apply to the current. i_{AB} should mean the current flowing from A to B, and vice versa. If a sine wave representing a current flowing between two terminals A and B is marked i_{AB}, then while the graph shows positive values, the current is flowing from A to B; while the sine wave has negative values, the

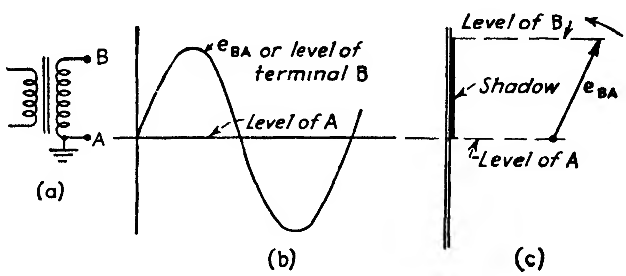

FIG. 3–7.—No ambiguity exists when we state that the graph or the vector represents the voltage of terminal B with respect to terminal A. If the graph or vector were simply called the secondary voltage of the transformer, such ambiguity would exist.

current is flowing from B to A. Ohm's law reads then $e_{AB} = Ri_{AB}$, which has now a much deeper meaning. It not only gives numerical values but also states that when the current flows from A to B, terminal A is positive with respect to B. The reader is invited to express in words the following statement (which follows from the fact that $e_{BA} = -e_{AB}$): $e_{BA} = -Ri_{AB}$.

What have we gained by representing an alternating current with a 5-amp amplitude by a vector five units long? If we wish to know the instantaneous value of the current at any moment, we still have to determine the length of the shadow thrown by the vector at this instant. Why go through the trouble of vector representation?

3–8. The Sum of Two Alternating Currents or Voltages.—Suppose that two alternating currents, one with a 5-amp amplitude and one with a 3-amp amplitude, are flowing in two wires coming together at a junction point as shown in Fig. 3–8. Will the current after the junction be a sinusoidally varying current with the amplitude 8? It is seen, of course, that at *any given instant* Kirchhoff's law must hold true. The instantaneous value of the current in the third wire must equal the sum of the two instantaneous values of current in the two individual wires. It is just as easily recognizable that the amplitude would be 8 amp only in the case that the currents in the two wires reach their maximum at exactly the same instant. If this

is not the case, the amplitude of the combined current will be less than the sum of the amplitudes of the two individual currents. Let the two currents indicated in Fig. 3–8 be two alternating currents of sinusoidal wave shape, as shown in Fig. 3–9. It is seen that the current with the 5-amp amplitude reaches its maximum before the current with the 3-amp amplitude does. The current after the juncture cannot have an amplitude of 8 amp because at the instant when i_1 is 5 amp, i_2 has not reached the 3-amp maximum, and vice versa. The only way to find the total current is to add the corresponding instantaneous values of the two currents. If this is done (a very commendable exercise for the reader), it is found that the resulting current is a sine wave again with an amplitude of approximately 7 amp and passing through zero later than i_1 but earlier than i_2. The mathematical treatment of this case, found in any text on electrical engineering, will also prove that the sum of two sine waves is another sine wave and will permit the calculation of the amplitude of the resulting wave as well as its phase relationship with respect to the two original waves.

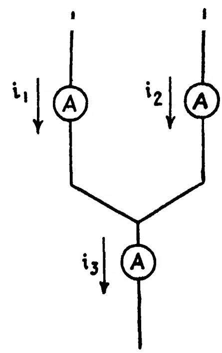

FIG. 3–8.—At any given instant the sum of the two currents i_1 and i_2 must, of course, be equal to i_3; but this no longer holds true for the amplitude or effective values if i_1 and i_2 are alternating current.

Shall we have to go through the laborious point-by-point process or through the mathematical calculations every time we have to find the sum of two alternating currents? This is where the vector representation really comes to our aid, as will be seen by the following. The 5-amp current can be represented by a rod 5 in. long fastened to the motor shaft of Fig. 3–5,

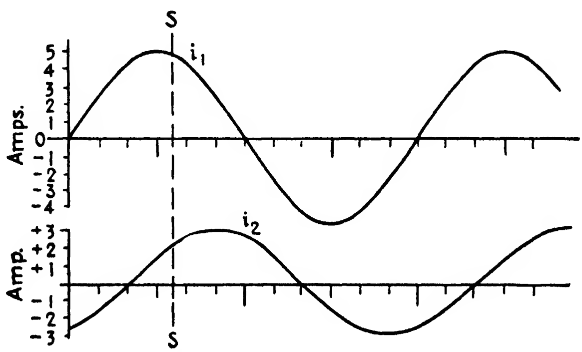

FIG. 3–9.—To find the total current i_3 in Fig. 3–8, we would have to add the instantaneous values of the two currents, i_1 and i_2, as shown for point S.

while the 3-amp current will be a rod 3 in. long. However, these rods must be fastened to the motor shaft, not in line with each other because that would cause the shadows thrown by them to reach their maximum value

at the same instant, but displaced in such a way that their shadows at any instant have the values given by the two sine waves shown in Fig. 3–9. This is evidently the case when the 3-in. rod is fastened to the shaft 60 deg behind the 5-in. rod. What is "behind" and what "ahead" depends of course on the direction of rotation of the motor shaft. It has been agreed to consider all vector diagrams as rotating in a counterclockwise direction, and the proper relation between the two vectors is that shown in Fig. 3–10.

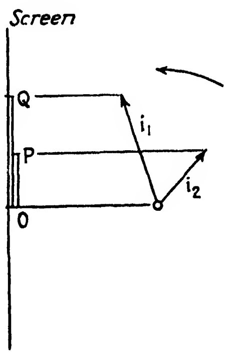

FIG. 3 10.—The instantaneous value of the current i_3 in Fig. 3–8 can be found by adding the shadow lengths OP and OQ, thrown by the vectors representing the currents i_1 and i_2.

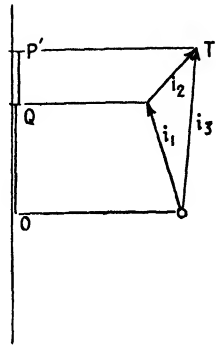

FIG. 3 11.—Instead of adding the lengths of the two shadows, we may move one of the vectors parallel to itself, as shown here. The sum of the two individual shadow lengths is then seen to be equal to the shadow thrown by a new vector i_3, obtained as shown here.

Remember that vector diagrams must always be visualized as rotating with a speed equal to the frequency of the alternating value that they represent, and that the diagram that we see must be considered as a snapshot of these rotating sticks at a given instant. At the instant represented in Fig. 3–10 the lengths of the shadows thrown by the two vectors are OQ and OP, respectively. This represents the instantaneous values of the two alternating currents; the corresponding instant is approximately indicated in Fig. 3–9 by the line SS, as the reader will easily recognize. The sum of the two currents at this instant can therefore be found by adding the two shadow lengths thrown by the two vectors at this instant.

Now the length of the shadow produced when a parallel beam of light strikes a rod will not change when the rod is moved parallel to itself; therefore, instead of measuring the lengths OP and OQ of the two shadows, we can unfasten the rod representing i_2 in Fig. 3–10 and move it parallel to itself, into a position as indicated in Fig. 3–11. Since the length QP' in this

figure is now equal to the length OP in Fig. 3–10, the total current at this instant is given by the length OP'. This is evidently true regardless of the instantaneous position of the motor shaft to which the vectors are fastened in Fig. 3–10. Therefore, the distance of the shadow thrown by the end point of the combination of the two vectors i_1 and i_2, as shown in Fig. 3–11, from point O (which may be considered as the shadow thrown by the shaft) represents at any instant the value of the sum of the two currents, *i.e.*, the current i_3 in Fig. 3–8. Obviously, a new, single vector, the end point T of which would coincide with the end point of the combination of the two individual vectors, would throw at any instant a shadow of the same length; therefore, this new vector represents the sum of the two currents i_1 and i_2. We now recognize that the vector representation of alternating currents is a powerful tool for the electrical engineer since it permits the determination of the sum of two or more alternating currents with any phase relationship relative to each other. It is clear that the same method is applicable when two alternating voltages, instead of currents, have to be added.

3–9. Average and Rms Values of a Sinusoidally Varying Current.—We have learned how to determine the average and rms values of a current varying in steps, as shown in Fig. 3–3. The same method is applicable when we wish to determine these values for a sinusoidally varying current. We divide the sine wave into a convenient number of vertical strips of equal width (the greater the number of strips, the more accurate the result). The average height of these strips for a half wave gives us what is usually referred to as the average value of the sine wave. (As already mentioned, the actual average of a whole ac wave is zero since there are just as many negative strips as there are positive ones.) Those familiar with integral calculus do not have to go through this lengthy process but can obtain the desired result by a simple integration. It will be found that the average value of an alternating current of sinusoidal wave shape is given by the following:

$$I_{av} = \frac{2}{\pi} \times \text{amplitude of current} = 0.638 I_{max} \tag{3-3}$$

To find the rms value of the current, we have to square the current values represented by the strips, determine the mean or average of these squares, and finally extract the square root. The value thus obtained gives us that steady direct current, which would produce as much heat as the actual alternating current of sinusoidal wave shape. If this is done, we find

$$I_{rms} = \frac{1}{\sqrt{2}} I_{max} = \frac{\sqrt{2}}{2} I_{max} = 0.707 I_{max} \tag{3-4}$$

Remember that Eqs. (3–3) and (3–4) hold true only for sinusoidal wave shape of the current.

The same relations hold true for the amplitude and the average and rms values of an alternating *voltage* of sinusoidal wave shape.

3-10. Rms Value and Vector Representation.—Vector diagrams have some physical meaning only as long as the length of the vector is made equal to the amplitude of the alternating quantity it represents. Under this condition the length of the shadow thrown by it, *i.e.*, the distance of the shadow thrown by its end point from the shadow of the axis (or "shaft") of rotation, represents the instantaneous value at a given instant. Thus, if a voltage with an amplitude of 40 volts is connected in series with a voltage having an amplitude of 30 volts of the same frequency but reaching its maximum just at the instant when the first-named voltage passes through zero, then the drawing of the vectors will disclose that the amplitude of the resulting voltage is 50 volts. By multiplying all three values by 0.707, we shall find the rms values, *i.e.*, the values that a meter would read. Evidently then, if we wish to find the voltage resulting when an rms voltage of 80 volts is placed in series with an rms voltage of 60 volts, the latter 90 deg displaced against the former but both of the same frequency and of sinusoidal wave shape, we could draw the vector diagram immediately with rms values instead of converting at first to amplitudes. This is the usual procedure. It should be kept in mind that the vector then becomes a symbol only, its projection on a screen not having physical significance any more.

PROBLEMS

3-1. A dc motor has a load causing the current to vary as shown in Fig. 3-12. For 5 sec the current is 60 amp, dropping to 10 amp for the next 2 sec. The load then becomes overhauling and the motor becomes a generator, furnishing 20 amp back into the line for the next 5 sec. What value of current should be used as a basis for the selection of the wire size?

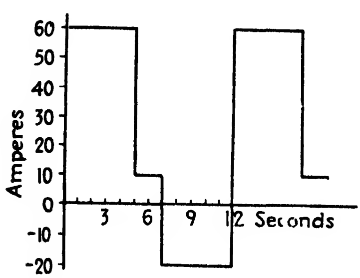

FIG. 3-12.—Graph of motor current in Prob. 3-1.

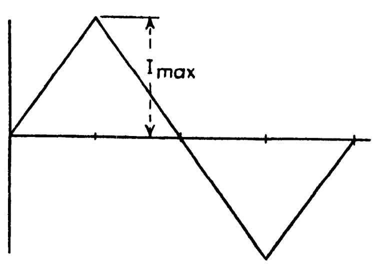

FIG. 3-13.—Wave shape of current in Prob. 3-2.

3-2. Determine the average and rms values of a current with a triangular wave shape, as shown in Fig. 3-13. Division of one quarter cycle into 10 equal parts is suggested.

3-3. Represent by vectors an alternating voltage of a sinusoidal wave shape with an amplitude of 50 volts, and a voltage of the same frequency with an amplitude of 30 volts but lagging the first one by one-sixth of a cycle.

3–4. Suppose that the two voltages of Prob. 3–3 appeared across two devices connected in series. What would be the total voltage appearing across the two devices (amplitude as well as rms value)?

3–5. The two coils shown in Fig. 3–14 are to be considered as coils in an ac generator or as the secondary windings of two transformers, the primaries of which are not shown. Alternating voltages of sinusoidal wave shape appear across the two coils; the amplitude of the voltage appearing across coil 1 is 100 volts, while across coil 2 the voltage has an amplitude of 60 volts. At the instant when terminal *B* is 100 volts (the maximum) positive with respect to terminal *A*, terminal *C* is negative with respect to *D* but is still short of having reached the negative maximum of 60 volts by one-sixth cycle. Terminal *A* is to be connected to terminal *C* as shown. What is the maximum of the voltage appearing between points *B* and *D*?

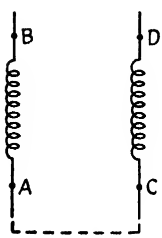

FIG. 3–14.—Connection diagram for Prob. 3–5.

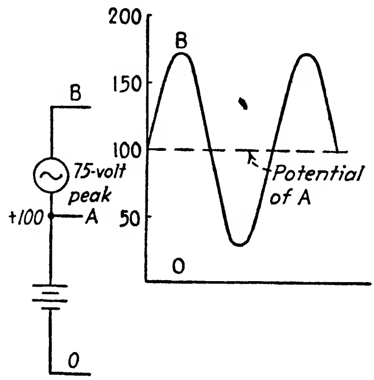

FIG. 3–15.—Connection diagram and wave shape of voltage for Prob. 3–6.

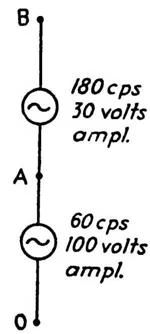

FIG. 3–16.—Connection of generators in Prob. 3–7.

3–6. An ac generator furnishing a sinusoidal voltage with an amplitude of 75 volts is placed in series with a dc battery of 100 volts, as shown in Fig. 3–15. The voltage appearing across the series combination will then, of course, look like the graph in this figure. What would a dc voltmeter read when connected across the two? What would an instrument indicating the rms value of a voltage read when connected in the same manner?

3–7. Two ac generators are connected in series as shown in Fig. 3–16. One furnishes a voltage with an amplitude of 100 volts and a frequency of 60 cps; the second furnishes a voltage with a 30-volt amplitude and a frequency of 180 cps. Plot the total voltage appearing across the two for the following two conditions:

a. When the 60-cycle voltage passes through zero from negative to positive values, the 180-cycle voltage does the same.

b. When the 60-cycle voltage passes through zero from negative to positive values, the 180-cycle voltage passes also through zero but from positive to negative values.

Determine the average and rms voltages for the two cases. (Inspection of the two graphs shows that it is sufficient to carry the determination out for the first quarter cycle of the 60-cycle wave since the remaining quarter cycles are of the same shape.)

SUGGESTED ADDITIONAL READING

See end of Chap. IV.

CHAPTER IV

RESISTANCE, INDUCTANCE, AND CAPACITANCE IN ALTERNATING-CURRENT CIRCUITS; THE TRANSFORMER

4-1. Resistance with Alternating Current Flowing through It.—According to Ohm's law for resistance, the voltage existing across a resistance is always equal to the current flowing at the particular instant multiplied by the resistance. Consequently, if an alternating current is flowing through a resistance, the voltage across it will be of the same wave shape as the current. If, for instance, an alternating current with a 5-amp amplitude flows through a resistance of 20 ohms, the voltage across the resistance will be a sine wave with an amplitude of 100 volts. Obviously, the voltage will reach its maximum value at exactly the same instant as the current does. This condition is expressed by stating that "current and voltage are in phase."

When the current reverses its direction, so does the voltage. The current is therefore always entering at the plus terminal, which means that the resistance is a consumer of electric power at all times (except of course at the exact instant when the current is zero). The wattage is always positive, varying between a maximum value at the instant when the current is a maximum and zero at the instant when the current passes through zero. The average wattage or power, as can be proved easily, is given by the same relations as for direct current, except that the rms values of current and voltage must be used. The average power is given by the following:

$$P = E_{\text{rms}}I_{\text{rms}} = I_{\text{rms}}{}^2R = \frac{E_{\text{rms}}{}^2}{R} \qquad (4\text{-}1)$$

since evidently $E_{\text{rms}} = I_{\text{rms}}R$.

4-2. Inductance with Alternating Current Flowing through It.—Now let us investigate the relation of current and voltage when the resistance is replaced by an inductance. As shown in Eqs. (2-3) to (2-5), there is a voltage across an inductance *only when the current through it is changing.* An alternating current is by nature a current that changes from instant to instant; therefore, when such a current passes through an inductance, there is a voltage across the inductance at every instant, depending on the rate at which the current is changing at that particular instant. Consider a current varying sinusoidally. Evidently, such a current changes fastest at the very instant when it passes through zero. Consequently, when a sinusoidally varying alternating current passes through an inductance, the

voltage is a maximum at the instant when the current passes through zero. On the other hand, when the current has reached its maximum, there is an instant when it does not change; therefore, the voltage across the inductance is zero at this instant. After passing through its maximum, the current begins to decrease. As explained in Chap. II, the inductance changes from a consumer of electric energy to a producer, which means that the polarity of the voltage now reverses. While the current is rising, its direction and the polarity of the voltage across the inductance are the same as for resistance; the terminal of entry of the current is positive. This does not mean that the inductance acts *in every respect* like a resistance during the first quarter of a cycle. In the case of the resistance, the voltage is zero when the current passes through zero; for the inductance we have seen the voltage to be a maximum at the instant when the current is zero.

Let us turn again to the first quarter of a cycle during which the current increases from zero to its maximum value. We reasoned that the voltage across the inductance would have to be a maximum at the instant when the current was zero, and vice versa. What will the values be between these two instants? This question can be answered by two methods. We can carefully plot the sine wave representing the current and then determine graphically the rate at which it changes at any instant during the quarter of a cycle. Multiplying these values of the rate of change of current by the value L of the inductance gives us the instantaneous values of the voltage. If we are familiar with differential calculus, the answer is obtained very much faster, but the result will be the same. We obtain a sine wave again, shifted 90 deg with respect to the current wave. It would be utterly incorrect to draw from this result the conclusion that the wave shape of the voltage across an inductance is always equal to the wave shape of the current through it and is merely shifted by one-quarter of a cycle; a triangular wave shape of current results, for instance, in a square wave for the voltage. Only in the case of a sine wave is the rate of change again a sine wave.

4-3. Rate of Change of a Sinusoidally Varying Quantity.—Having accepted, then, the fact that the voltage across an inductance will be a sine wave when a sinusoidal current flows through it, this voltage will be determined completely if we are able to determine its amplitude. The amplitude of the voltage is equal to the inductance multiplied by the rate of change of current at the instant when the latter passes through zero. Our problem, therefore, comes down to determining the rate of change of a sinusoidally varying quantity at the instant when it passes through zero. This is again a problem easy to solve for those familiar with differential calculus; those who are not will have to take on faith the result, which is

"The maximum rate of change of a sinusoidally varying quantity is equal to $2\pi f$ times the amplitude of the quantity where f is the frequency in cycles per second."

This result is of the utmost importance, and it is well worth while to become thoroughly familiar with it. In the following an attempt is made to give a proof of it for those not familiar with differential calculus.

Let it be required to find the rate at which a sinusoidally varying alternating current of the amplitude I_{max} and the frequency f cps changes at the instant when the current passes through zero. We have seen that the instantaneous values of such a current are given by the length of the shadow (or projection) of a vector with the length I_{max} units, rotating with a speed of f rps. We therefore wish to determine the rate or speed with which this projection grows at the very instant when it passes through zero. From Fig. 4-1 it is clear that at the instant when the vector is in a horizontal position the length of its shadow is zero but that it is growing at

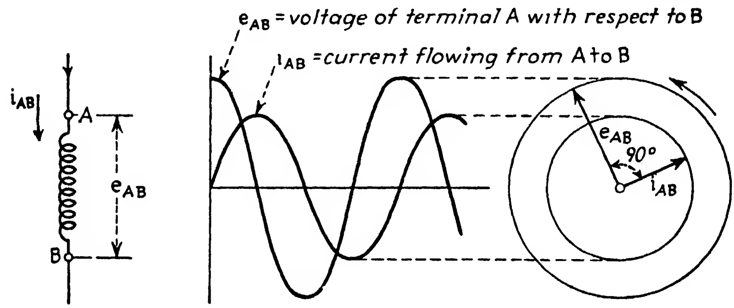

FIG. 4-1.—When an alternating current flows through an inductance, the voltage across the inductance will be a maximum at the instant when the current changes at its fastest rate. Analyze this figure carefully as to the direction of the current and the polarity of the voltage.

this instant with the same speed with which the end point of the vector itself travels. The end point of the vector evidently describes a circle with the radius I_{max} units of length (inches, for instance, if 1 amp is made equal to 1 in.), and it traverses this circle f times per second. The speed of the end point of the vector is therefore $2\pi I_{max}f$ in. per sec. This is also the rate at which the current changes at the instant when it passes through zero. We therefore have

$$\text{max roc } i = 2\pi f I_{max} \tag{4-2}$$

The factor $2\pi f$ is often referred to as "circular frequency" and is designated by the Greek letter ω. Equation (4-2) then becomes

$$\text{max roc } i = \omega I_{max} \tag{4-2a}$$

When this current flows through an inductance of L henrys, the voltage appearing across it at the instant when the current is passing through zero and is therefore changing at the rate given by Eq. (4-2) will be, according to the fundamental law of the inductance given by Eq. (2-3),

$$\begin{aligned} E_{max} &= \text{max roc of current} \times L \\ &= 2\pi f I_{max} L \text{ volts} \end{aligned} \tag{4-3}$$

Please note that the multiplication with $2\pi f$ applies to I_{max}, the result giving us the maximum rate of change of current.

The importance of the concept of the rate of change cannot be overemphasized. In order to be sure that the reader has grasped the relations just outlined, let us find the maximum rate of change of an alternating current with an rms value of 2 amp and a frequency of 20 cps. Applying Eq. (3–4), we find that the amplitude of this current will be $2\sqrt{2} = 2.83$ amp. Equation (4–2) now gives us

$$\text{max roc } i = 2\pi \times 20 \times 2.83 = 355 \text{ amp per sec}$$

The reader is urged to draw a sine wave with 2.83 amp amplitude, letting one cycle be $\frac{1}{20}$ sec, or 50 msec. • Examination of the graph reveals that the current right after passing through zero is indeed increasing at the above rate; at the end of the first millisecond (1/1,000 sec) the graph shows the current to be 0.35 amp, which is a rate of rise of 350 amp per sec.

If this current flows through an inductance of 0.2 henry, for instance, then the voltage that will exist across the inductance at the *instant when the current passes through zero* will be given by

$$E_{max} = \text{max roc } i\, L = 355 \times 0.2 = 71 \text{ volts}$$

The voltage across the inductance will be of sinusoidal wave shape with an amplitude of 71 volts and will therefore have an rms value of approximately 50 volts.

4–4. Inductive Reactance.—In the case of an alternating current passing through a resistance, the maximum voltage across the resistance is given by $E_{max} = RI_{max}$. Comparing this with Eq. (4–3), we are obviously tempted to combine the value $2\pi f$ with L and call it "ohms" because multiplication of the current with this value results in a voltage. This has been done, and the value thus obtained is called the "reactance." It is unquestionably a step that simplifies calculations of ac problems considerably, but it is also a step that often veils the true state of affairs and makes the student lose sight of the principle underlying these phenomena. Sight must never be lost of the fact that the factor $2\pi fL$ cannot be used to obtain the instantaneous values of the voltage. The two waves representing voltage and current are 90 deg out of phase, and, consequently, no proportionality exists between instantaneous values.

If we divide both sides of Eq. (4–3) by $\sqrt{2}$, we obtain

$$E_{rms} = 2\pi fLI_{rms} \tag{4–4}$$

As already stated, the value $2\pi fL$ (a value without any physical meaning) is the reactance and is designated x_L. We therefore have

$$x_L = 2\pi fL = \omega L \tag{4–5}$$

The sine wave representing the voltage across an inductance is seen to have its positive maximum when the current passes through zero; it is, therefore, 90 deg ahead of the current. If we were to represent both current and voltage by vectors, the two would therefore have to be displaced correspondingly. These relations are shown in Fig. 4-1.

4-5. Power Relations of the Inductance.—Figure 4-1 permits us to complete the analysis of the power relations for an inductance, started in Sec. 4-2. The two sine waves representing current and voltage are both positive during the first quarter cycle, and the inductance is therefore a consumer of power. During the next quarter cycle the current is still flowing in the original direction, but the voltage has reversed since the rate of change of current has reversed (it is decreasing instead of increasing). During this quarter cycle the inductance acts like a source of power and gives back all the energy it had stored in its magnetic field. During the next quarter cycle both voltage and current are negative, *i.e.*, of a polarity and direction opposite to that during the first quarter cycle. This means that the inductance is now again a consumer of electrical energy. During the final quarter cycle it becomes again a source. The *average* power or wattage is therefore zero, when taken over a half or a whole cycle, but the *instantaneous* power is not zero.

4-6. Capacitance with Alternating Current Flowing through It.—Before attempting an analysis of the current-voltage relations existing in the case of a capacitor, we call attention to a principle often used in the solution of electrical problems. In Sec. 4-3 we found the voltage that would appear across an inductance of 0.2 henry when an alternating current of 2 amp with a frequency of 20 cps flows through it. These considerations led to the development of Eq. (4-4). Now, if a current of 2 amp causes a voltage of 50 volts to appear across the inductance, we can reverse the reasoning and state that the application of 50 volts to the inductance will cause a current of 2 amp to flow. Mathematically, this is nothing else but solving Eq. (4-4) for the current. But, although it was reasonably easy to develop the relation between the current and the voltage by starting with the current, we would have found it considerably more difficult if we had insisted on starting with the voltage; in other words, if we had asked "What current will flow if we apply an alternating voltage to the inductance?" instead of asking "What voltage will appear across an inductance when an alternating current flows through it?" As long as we can develop the relation between the two quantities, however, both questions are answered, and it is therefore entirely satisfactory to use whichever approach develops the relations in the more convenient manner.

In the case of the capacitance we shall find it easier to use the voltage as the starting quantity for the development of the relation between current and voltage. In other words, this time we ask "What current will

flow through a capacitor when we apply an alternating voltage (of sinusoidal wave shape) to the capacitor?"

Ohm's law for the capacitor was stated in Eqs. (2-8) to (2-10). The first of these tells us that the current flowing through a capacitor is equal to the rate of change of voltage, or roc e, multiplied by the value of the capacitance in farads. With a sinusoidally varying voltage, the rate of change of voltage will be greatest at the instant when the voltage passes through zero. Similarly to Eq. (4-2), it will be given by

$$\max \text{roc } e = 2\pi f E_{\max} \tag{4-6}$$

The current through the capacitor will therefore be of maximum value at the instant when the voltage passes through zero and will be

$$I_{\max} = 2\pi f E_{\max} C \tag{4-7}$$

This equation can be transposed to give us

$$E_{\max} = I_{\max} \frac{1}{2\pi f C} \tag{4-8}$$

The value $1/2\pi fC$ is again seen acting like ohms because multiplying a current by it results in a voltage. The same remarks of caution made with respect to the value $2\pi fL$ also apply to this value, which is called the "capacitive reactance" of the capacitor at the frequency f. It is, therefore,

$$x_c = \frac{1}{2\pi f C} \tag{4-9}$$

The current through and the voltage across a capacitor are shown in Fig. 4-2, which shows also the vector diagram covering this case. We see that

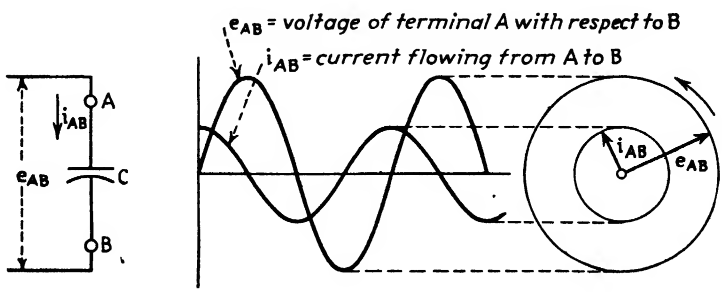

FIG. 4-2.—When an alternating voltage is applied to a capacitor, the current flowing through the capacitor will be a maximum at the instant when the voltage is changing at its fastest rate. Analyze the figure carefully as to the direction of the current and the polarity of the voltage.

this time the *current* is *leading* the voltage by 90 deg, which is just the opposite of the phase relation in case of the inductance, as shown in Fig. 4-1.

4-7. Series Combination of Various Circuit Elements.—In the preceding sections we saw that when an alternating current flows through a resistance the voltage across it is in phase with the current, *i.e.*, it goes through zero at the same instant when the current goes through zero and is at its maximum at the same instant when the current reaches its maximum. When an alternating current flows through an inductance, on the other hand, we saw that the voltage is a maximum at the instant when the current is zero because at this moment the current changes fastest, which is the only thing that determines the voltage across an inductance. In the case of a capacitance, the current was seen to be at its positive maximum when the voltage was passing through zero. The question is now appropriate, "What voltage shall we observe when an alternating current passes through two or three of these elements in series?"

Suppose that in Fig. 4-3*a* an alternating current passes through a resistance of 10 ohms and an inductance of such value that at the particular

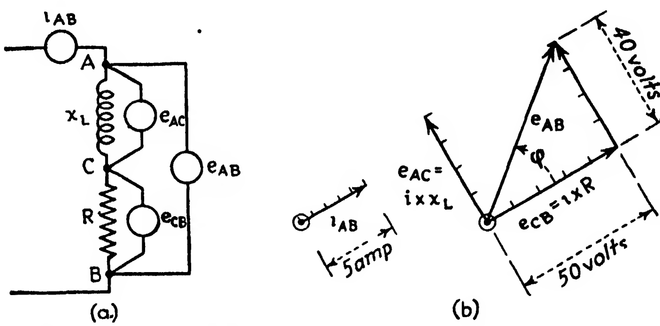

FIG 4-3.—When an alternating current flows through a series combination of resistance and inductance, the voltage across the resistor will be in phase with the current, while the voltage across the inductance will be leading the current by 90 deg, as shown in Fig. 4-1. The total voltage across the combination is found by combining the two vectors representing the individual voltages

frequency the inductive reactance $x_L = 2\pi fL$ amounts to 8 ohms. Let the amplitude of the alternating current be 5 amp. The amplitude of the voltage across the resistance is then 50 volts, and the amplitude of the voltage across the inductance is 40. Now, if these two voltages were in phase, *i.e.*, were to reach their maximum at the same instant, then the amplitude of the voltage appearing across the two would obviously be 90; but we have seen that the voltage across the inductance reaches its maximum when the current is zero and when, consequently, the voltage across the resistance is also zero. Figure 4-3*b* shows the vector diagram applying in this case. In order not to confuse the diagram, the vector representing the current is drawn separately from the vectors representing the voltages. We must therefore imagine the two small circles in the diagram as two shafts running synchronously. It is common practice to show the reference vector—which in this case would be the current since this is the

quantity common to both the resistance and the inductance—in either the vertical or the horizontal position. This practice is deliberately ignored here in order to impress on the reader the fact that vector diagrams are snapshots of rotating rods. It so happens that when snapshot Fig. 4–3*b* was taken the rod representing the current was in the position shown in this figure. The vector representing the voltage across the resistance has a length of 50 in a suitable scale and, since the shadow thrown by it must reach its maximum at the same time as the shadow thrown by the current vector, it must be fastened to the shaft so that it is parallel to the current vector. The vector representing the 40 volts (amplitude) across the inductance must be fastened so that the shadow thrown by it will be a positive maximum just when the shadow of the current vector changes from a negative to a positive value. Evidently this is the case when it is mounted on its shaft 90 deg ahead of the current vector, which places it also 90 deg ahead of the vector representing the voltage across the resistance. As shown in Fig. 3–11, the sum of two alternating values represented by vectors can be represented by a new vector obtained by adding the two individual vectors in the manner outlined in Chap. III. In our particular case, therefore, we have to add two vectors of the length 50 and 40, respectively, under an angle of 90 deg. As evident from Fig. 4–3*b* the length of the new vector is given by the hypotenuse of a right-angle triangle with the two sides equal to 40 and 50, respectively. The amplitude of the total voltage is therefore given by

$$E_{\max} = \sqrt{(I_{\max}R)^2 + (I_{\max}x_L)^2} = I_{\max}\sqrt{R^2 + x_L{}^2} \qquad (4\text{–}10)$$

In Eq. (4–10), the value $\sqrt{R^2 + x_L{}^2}$ obviously again plays the role of "ohms" because by multiplying a current with this value we obtain a voltage. It is called the "impedance" of the combination and is designated as z. The vector representing the total voltage is seen to be ahead of the vector representing the voltage across the resistance (which in turn is in phase with the current vector) by the angle φ. The size of this angle is evidently given by the relation

$$\tan\varphi = \frac{x_L}{R} \qquad (4\text{–}11)$$

In the case of an inductance, the voltage is seen to lead the current; in the case of a capacitance, the condition is reversed. Consequently, when an alternating current passes through a series combination of inductance and capacitance, as shown in Fig. 4–4*a*, then the vector representing the voltage across the inductance leads the vector representing the current, while the vector representing the voltage across the capacitance lags the current vector, as shown in Fig. 4–4*b*. This is seen to be equivalent to a 180-deg displacement between the two voltage vectors.

This means that the instantaneous values of the voltage across these two circuit elements oppose each other at every instant. Thus, if the amplitude of the voltage across the inductance is, for instance, 70 volts while that across the capacitance is 50 volts, then when the two reach their maximum the total voltage across the two amounts to only 20 volts. The phase relationship with respect to the current is determined by the larger of the two. It is quite obvious that, if the inductive reactance $2\pi fL$ equals the capacitive reactance $1/2\pi fC$, the two voltages will be exactly alike and the total voltage across the two will be zero. This condition is called

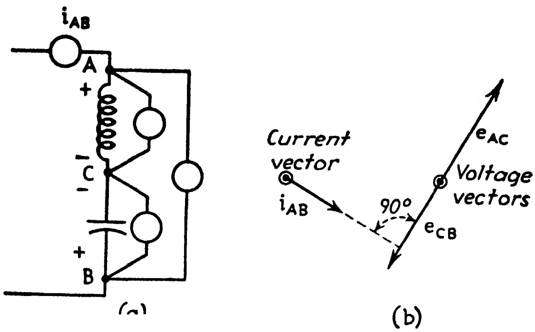

FIG. 4-4.—When an alternating current flows through the series combination of inductance and capacitance, the two voltages will be 180 deg out of phase with each other, this means that they are of opposing polarity at any instant and that the total voltage across both will therefore be smaller than either one of the two individual voltages.

"resonance" and is of the utmost importance in communication engineering. It seems to be a difficult phenomenon to grasp. The same person who will unhesitatingly agree that two 6-volt batteries connected in series may result in a total voltage of either 12 or 0 volts, depending on whether the voltages are aiding or opposing each other, is often unable to visualize that the voltage conditions across an inductance and a capacitance through which the same alternating current passes are exactly identical with those existing when two batteries are connected so as to oppose each other. The subject of resonance will be considered again in greater detail in connection with oscillating circuits.

4-8. Various Types of Transformers.—Another circuit element of utmost importance in electronic circuits is the transformer. Here, too, considerable confusion seems to exist owing to the fact that the subject can be treated in different ways. The method of approach used by the radio engineer is so different from the method used by the power engineer that one sometimes wonders whether the two are talking about the same device.

In Chap. II we investigated the subject of mutual inductance. Any two coils arranged in such a way that they have mutual inductance with respect to each other can be called a "transformer." To the radio engineer, transformers are usually so-called "air-core" transformers; *i.e.*, the

magnetic field produced by the current in one of the coils and linking with the other coil is entirely in air. Quite often he intentionally places the two coils so that only a very small fraction of the magnetic field produced by one coil passes through the other. He then states that the two windings are "loosely coupled." The power engineer, on the other hand, builds a laminated iron core on which he places the two coils. He usually is trying to get a very high degree of coupling, *i.e.*, he would like to have all the magnetic lines produced by the current in one coil linked with the second coil.

Although from a theoretical point of view the degree of coupling or the fact that one transformer may have an iron core while the other has an air core should not make any difference as to the method of treating the device mathematically, since clearly the fundamental laws apply to each case, it has nevertheless been found that certain methods of approach may be more suitable to the one or the other case. For this reason it is sometimes hard to realize that the various kinds of transformers are all brothers under the skin.

In most lines of human endeavor, the borderline cases cause the most trouble. For instance, it is easy enough to separate from a stack of various colored papers the blue and the green ones, but the blue-green ones will give us trouble. In a similar way, there is no question about which method of approach is indicated if we are dealing with the extremely loosely coupled transformer of the radio engineer or the extremely closely coupled transformer of the power engineer. In vacuum-tube circuits, however, we often meet borderline conditions and the proper decision as to which of the two treatments will give the desired results is then not an easy one.

4-9. Statement of Transformer Problem.—When an alternating voltage is applied to a coil, a current flows in it determined by its impedance, as shown in Sec. 4-7. If the magnetic field produced by this coil links more or less completely with the second coil, a voltage appears also across the second coil. The questions now arise, "What will happen if we connect a load, say, for simplicity's sake, a resistance, across the second coil? Will the voltage observed at the terminals of the second coil be the same with or without this load? Will the current taken by the first coil (also called the 'primary') remain the same regardless of what we happen to do to the secondary coil?" These are questions for which the proper analysis of the transformer problem should furnish answers.

It is not our purpose to make a detailed study of the transformer. Those interested in the subject will find it treated with varying degrees of completeness in several good texts. Let it be said only that it is in the degree in which certain phenomena take place in the various types of transformers that their main difference is found. Thus, in a very loosely coupled air-core transformer, the current taken by the primary coil, whether it is amperes or milliamperes, will change only imperceptibly if

we place a resistive load across the secondary or even go to the ultimate condition of short-circuiting it completely.

In the iron-core power transformer, on the other hand, the primary current, with a constant voltage applied to the primary winding, will change considerably when we begin to draw a current from the secondary. A complete short circuit of the secondary would usually lead to the destruction of the transformer by the overheating of the secondary as well as the primary of the device.

4-10. Ideal Iron-core Transformer.—Some of the aspects of air-core transformers will be treated in connection with oscillator circuits. At present we shall confine ourselves to the iron-core transformer, which is of considerable interest in the design of electronic circuits. In Fig. 4-5 is shown an iron core on which are placed two windings with n_1 and n_2 turns, respectively. The iron provides a path for the magnetic lines so much better than the surrounding air that nearly all the magnetic lines produced by a current flowing in one coil will pass also through the other coil. Let us now apply an alternating voltage to one of them (the one where power is applied is usually called the "primary"). If the other coil, the secondary, is open, we might just as well imagine it as absent, and the current that flows under this condition in the primary coil can be calculated if we know the impedance of the coil. Let us assume that the resistance of the winding is negligibly small compared to the inductive reactance, a condition that is usually reasonably satisfied in iron-core transformers. The current flowing when the primary is connected to a source of alternating voltage can then be found with the aid of Eq. (4-4) by solving the latter for I_{rms}.

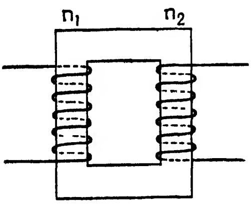

FIG. 4-5.—A transformer consists of two windings with the number of turns n_1 and n_2, wound on an iron core.

4-11. Magnetizing or No-load Current.—Making use of the inductance of the primary for the determination of the current that flows when this winding is connected to a source of alternating voltage (the secondary winding is still considered as open!) is a method of approach with which the electronic engineer must be familiar because the performance of transformers in tube circuits can best be analyzed with the aid of this concept. The power engineer—also the electronic engineer for certain types of transformers—usually does not use this approach. In order to understand the power engineer's approach, the reader will find it profitable to examine the following viewpoint, which applies not only to transformers but to any other kind of electrical equipment or circuit elements.

What happens when we apply a voltage of 20 volts to a resistance of 10 ohms? We know the answer, of course, which is given by the application of Ohm's law: a current of 2 amp will flow. But we could interpret

the phenomenon in another way. We could say that the resistance "defends" *itself against the application of voltage by taking a current of exactly the amount necessary to produce a voltage across it equal to the* applied voltage. In our case it takes 2 amp because this current flowing through 10 ohms will produce a voltage of 20 volts.

What is true for the resistance is true for any piece of electrical equipment and for direct as well as alternating current. Whenever a voltage is applied to a circuit element, it does whatever is necessary to produce a voltage equal to the applied voltage. When a dc motor is connected to the dc line, it speeds up until the generated voltage plus the voltage caused by the current flowing through the armature resistance exactly equals the line voltage. When an inductance of 10 henrys is connected to a dc source of 20 volts, it takes a current changing at the rate of 2 amp per sec because that is the only way an inductance knows how to produce a voltage. A capacitor, when connected to a source of direct current, takes a surge of current until it is charged to a voltage equal to the applied voltage. These examples show that each circuit element reacts to the application of a voltage in its own way, but they all defend themselves by producing a voltage equal to the applied voltage.

When an alternating voltage is applied to an inductance, it therefore takes a current of whatever magnitude and wave shape is necessary to produce a magnetic field, the rate of change of which at every instant induces a voltage in the turns of the coil equal to the applied voltage at that instant. (This is under the assumption that the resistance of the coil is so very low that practically all the voltage must be produced by the changing magnetic field and not by the current flowing through the resistance of the coil.) The power engineer calls this current the "magnetizing current" of the transformer. To produce the required voltage a certain amount of changing field lines is required, regardless of whether the core is air or iron; but to produce this required amount of changing field lines, of course, requires much less current if the coil is wound on an iron core than when it is in air. It is therefore easily understandable that the magnetizing current of a well-designed iron-core transformer is quite small —many thousand times smaller than it would be if we removed the iron core. It is for the same reason that an iron core is employed when it is desired to obtain high inductance values.

In the effort to gain an approximate picture of the performance of an iron-core transformer, it is the usual practice to disregard the magnetizing current completely. The physical meaning of neglecting the magnetizing current is equivalent to saying that it takes an infinitely small alternating current to produce in the core an ac magnetic field of sufficient magnitude to induce a voltage in the primary winding exactly equal to the applied primary voltage. Since the same field that is now inducing a voltage in the primary winding equal to the applied voltage also passes through the

secondary winding, there is also a voltage induced in the latter. If the secondary winding has, for instance, one-half as many turns as the primary, the voltage induced there is one-half the voltage applied to the primary. In general, the two voltages will be in the ratio of the number of turns. We therefore have the following:

$$\frac{E_2}{E_1} = \frac{n_2}{n_1} \tag{4-12}$$

4-12. Effect of the Secondary Current.—If we connect a load to the secondary winding, a current will flow in this winding, which according to the fundamental laws of induction is in such a direction as to oppose the flux change which is taking place and which is producing the voltages in the two windings. But as long as the alternating voltage applied to the primary remains constant, the voltage induced in this winding must remain the same. Consequently, if the current in the secondary is trying to weaken the magnetic field produced by the small magnetizing current in the primary winding, then the primary winding will counteract this effect by taking a current from the applied voltage and cancel exactly the magnetic effects of the secondary current. The magnetic effect of a given coil is determined by the strength of the current and the number of turns. The magnetic effect of the secondary current on the iron core is therefore given by the product I_2n_2 and, if the primary winding is to take a current such as to cancel the magnetic effect of the secondary current, this current I_1 must satisfy the relation:

$$I_1n_1 = I_2n_2 \tag{4-13}$$

which can also be written as

$$\frac{I_1}{I_2} = \frac{n_2}{n_1} \tag{4-14}$$

It should be remembered that these relations apply only to an ideal transformer, *i.e.*, to one where the magnetizing current (the current flowing in the primary with the secondary open) is negligibly small compared to the currents when rated load is connected to the secondary.

For the ideal iron-core transformer we then have two fundamental relations for the primary and secondary voltages and currents: The voltages are seen to be proportional to the number of turns, and the currents are inversely proportional to the number of turns.

We ordinarily associate the term "transformer" with a device whose main function is to step up or step down the voltage. In many vacuum-tube circuits this is exactly the role that transformers play; in other cases, although they still accomplish a transformation of voltage, this feature is not of importance to the electronic engineer, and it is not a certain voltage transformation ratio for which he designs his transformers. This will be seen more clearly in the next section.

4-13. Load or Impedance Matching.—In Fig. 4-6 a source of voltage E is shown feeding a load with the resistance r_L through a resistance r. Let us investigate what resistance the load must have so that maximum power will be consumed in it. The voltage source may, for example, be a little generator driven by a water wheel on a stream, while the series resistance r may represent the resistance of a line going into the woods to a cabin where it is desired to obtain a maximum heating effect. The occupant might wish to heat his cabin with electric power and would be interested in determining the resistance of the heating element that would give him maximum power. The solution is simple for those who are familiar with differential calculus and who will have only to apply the methods of determining the maxima and minima of a function. However, unfamiliarity with that subject does not prevent one from arriving at a clear understanding of the fundamental principle involved. Evidently the higher we make the load resistance, the smaller will be the current, and consequently the smaller will be the loss of voltage suffered in the resistance r. Under this condition we shall obtain a high voltage across the load but a small current. If the resistance of the load is decreased more and more, the current will increase and reach its maximum if the load is to be represented by a short circuit. With a large current there will also be a large loss of voltage through the resistance r. Under this condition we therefore have a low voltage but a large current in the load. In both cases the wattage or power in the load might be quite low. There must be a certain value of the load for which the power will be a maximum.

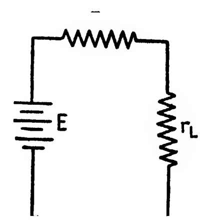

FIG. 4-6.—When a source of voltage supplies current to a load r_L over a resistance r, the load must have a certain resistance in order to obtain maximum power in it.

Let us assume, for instance, that the voltage of the source is 100 volts and the resistance r of the line is 10 ohms. If we make the load resistance 40 ohms, the total resistance in the circuit is 50 ohms, the current 2 amp, and the voltage across the load 80 volts. The wattage in the load is therefore equal to 160 watts. In the table on page 64, calculation of current and wattage for various values of load resistance is shown, and it is seen that for a load resistance equal to 10 ohms the wattage in the load reaches a maximum of 250 watts. Any other load resistance leads to a smaller wattage.

The principle outlined here is stated in many books under a somewhat different guise. It usually is developed in connection with finding the maximum power obtainable from a generator with an internal resistance r_I, and it is stated that for maximum power the load resistance must be equal to the internal resistance of the voltage source. In the example just treated, r was the resistance of the line connecting the voltage source with the consumer. As far as the occupant of the cabin is concerned, the

resistance r of the line might just as well be within the voltage source. The two terminals entering his cabin represent a voltage source with an internal resistance r over which he has no control. *In general, the principle should be stated as follows: If a load is supplied from a voltage source* over a fixed resistance, which either cannot be removed from the circuit or which for some reason it is not desired to remove, then for maximum power in the load its resistance should be equal to this fixed resistance. In the example of the cabin obtaining its power from a distant source, the occupant should therefore use a heating element of 10 ohms resistance if he wishes to obtain the maximum heating effect.

r_L	$r_L + r$	i	$i^2 r_L$
40	50	2	160
30	40	2.5	187
20	30	3.33	222
15	25	4	240
10	20	5	250
8	18	5.55	247
6	16	6.25	234
4	14	7.14	204
2	12	8.33	139
0	10	10.00	0

Suppose now that he has on hand a heating element of 40 ohms resistance and is not able to change it over to the 10 ohms required for maximum power. Is there anything that he can do about this situation? If the voltage source furnishes direct current, he is out of luck and will have to be satisfied with the 160 watts as shown in the first line of the table; but if the source furnishes alternating current, then he can make the 40-ohm element appear like a 10-ohm element and thus fool the two supply wires into giving him the maximum power. This trick is called "impedance matching" and it is one that the electronic engineer is quite often called upon to perform. It is accomplished by the use of a transformer, as the following consideration will show.

In Fig. 4–7 let a load resistance R be connected to the secondary of a transformer, and let this secondary winding have n_2 turns. Assume that the secondary voltage under the given condition is E_2. If the primary winding has n_1 turns and the transformer is reasonably near the "ideal," the voltage E_1 across the primary must then be $E_1 = E_2 n_1/n_2$ since the voltages are proportional to the number of turns. The primary current, on the other hand, is given by the relation that the currents are inversely proportional to the number of turns, *i.e.*, according to Eq. (4–14) we have

$I_1 = I_2 n_2/n_1$. The ratio of current and voltage on the primary is therefore given by

$$\frac{E_1}{I_1} = \frac{E_2}{I_2}\left(\frac{n_1}{n_2}\right)^2 \tag{4-15}$$

But the ratio E_2/I_2 is obviously equal to the resistance connected to the secondary. Now on the primary side we observe the primary current I_1 flowing into the transformer while the primary voltage is E_1. The transformer with the resistance R connected to the secondary side therefore

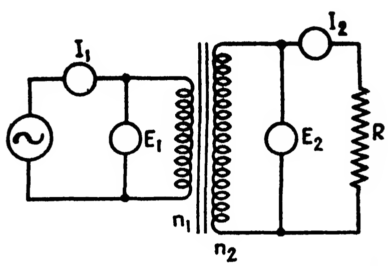

FIG. 4-7.—If the transformer can be considered as ideal, the ac generator is fooled into believing that it feeds a load with a resistance $R' = R\left(\frac{N_1}{N_2}\right)^2$.

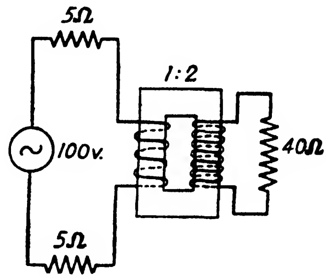

FIG. 4-8.—The action of the transformer makes the load of 40 ohms appear as one with 10 ohms, which will give the maximum power in the load.

appears to the primary voltage E_1 as if it were a resistance R' given by the ratio E_1/I_1. Equation (4-15) can then be written in the form:

$$R' = R\left(\frac{n_1}{n_2}\right)^2 \tag{4-16}$$

This then means that the ideal transformer is a device by means of which a given resistance may be made to look like a resistance of different value.

In the above example it would be desirable to make the available 40-ohm element appear like a 10-ohm element. This is a ratio of 4:1 of the actual resistance to the desired value and, according to Eq. (4-16), the turns ratio of the transformer would therefore have to be $\sqrt{4}$, which equals 2. This is indicated in Fig. 4-8. A transformer with a turns ratio 1:2, as shown there, will make the 40-ohm element appear like a 10-ohm element because it reduces the actual voltage across the 40-ohm element to one-half while the current is twice as high as in the 40-ohm element. Let us check whether the conditions are now the same as if we had a 10-ohm element in the cabin. With an actual 10-ohm element we saw that we would have a current of 5 amp and that there would be a drop of 50 volts across the line, leaving 50 volts across the load. If the conditions with the transformer in the circuit are actually the same, we should then have 100 volts across the secondary. But with 100 volts across the secondary,

the 40-ohm element will take $2\frac{1}{2}$ amp, and $2\frac{1}{2}$ amp on the secondary will give 5 amp on the primary. This is therefore seen to fulfill the condition that, as far as the primary is concerned, it appears to the line like a 10-ohm element. As far as the wattage in the 40-ohm element is concerned, with 100 volts and $2\frac{1}{2}$ amp, this will actually be 250 watts, which we have recognized as the maximum obtainable in the cabin.

In the preceding example, the purpose of matching was to obtain the maximum power. It should be realized that the concept of matching applies in all cases where for any reason a given load must be made to appear as a load of different value. Suppose that it is desired to develop 1,000 watts in a resistance of 10 ohms and that the available voltage is 500 volts. Now in order to obtain 1,000 watts from a 500-volt source we need a resistance that can be determined by solving Eq. (1–6) for R. This will give a value of 250 ohms. We therefore have to make our 10 ohms appear like 250 ohms if the desired result is to be obtained. This is a ratio of 25:1, and the square root of 25 is 5. A step-down transformer with a ratio 5:1 will therefore serve the purpose. It is evident that this particular problem could have been solved just as easily by determining the voltage necessary to obtain 1,000 watts in a resistance of 10 ohms. This would be found to be 100 volts and, since the voltage of the source is 500 volts, it is obvious that a step-down transformer with a ratio 5:1 will be required. In many cases, however, where the loaded transformer is placed in series with other circuit elements, the concept of impedance matching or impedance transformation will provide a more convenient method of analysis than that of voltage transformation.

Owing to the fact that the phase relations between voltage and current existing in the load connected to the secondary of the transformer are quite faithfully reproduced on the primary side, which is easily seen from Eqs. (4–12) and (4–13), the relation given by Eq. (4–16) holds true not only for a resistive load, for which it had been derived, but for any other type of load. Thus, a small capacitor can be made to appear as a large one; in short, the matching transformer changes only the values of voltage and current (always in opposite directions) but does not affect their phase relationship. If a load that takes a current lagging the voltage by 30 deg, for instance, is connected to the secondary of a transformer, then the current taken by the primary of the transformer will also lag the applied voltage by 30 deg. It should be emphasized again, however, that this statement is strictly true only for an ideal transformer and only approximately correct for an actual transformer.

PROBLEMS

4-1. An alternating current of 5-amp rms value and a frequency of 500 cps passes through a series combination of 15 ohms and an inductance of 10 mh (or 0.01 henry).

What are the voltages that will appear across the individual elements as well as the total voltage and their phase relations with respect to each other?

4–2. A resistance of 10 ohms, an inductance of 150 mh, and a capacitance of 0.47 μf are in series across a voltage of 100 volts ac. Find the current, if the frequency of this voltage is 400, 500, 600, 700, and 800 cps. What is the highest voltage across the inductance and capacitance?

4–3. A choke coil has an inductance of 15 henrys and a resistance of 4,000 ohms. Through it flows simultaneously a direct current of 25 ma and an alternating current of 8 ma. (Such a condition is usually encountered when an inductance is operated in the plate circuit of a vacuum tube.) The frequency of the alternating current is 600 cps, and the wave shape is sinusoidal. What would a dc meter read when connected across the choke coil? What would an ac meter read if we blocked out the direct voltage by means of a capacitor? What will an ac meter read if we do *not* block out the direct voltage?

4–4. The same combination of direct and alternating currents as given above flows through the parallel combination of a resistor of 1,000 ohms and a capacitor of 3 μf. (Such a condition is usually encountered in the cathode circuit of a tube.) What are the alternating and direct voltages across it? For simplicity assume first that all the alternating current passes through the capacitor. Convince yourself, however, that in this particular case such an assumption does not lead to an appreciable error.

4–5. A generator with a constant voltage of 200 volts feeds a load over a transmission line that has a resistance of 20 ohms (for instance, two No. 16 wires running for a distance of a little less than ½ mile). Suppose the load on the far end of the line consists of a heating element. What resistance must the heating element have in order to furnish the maximum amount of heating effect?

HINT: Calculate and plot the watts consumed in the heating element, assuming various resistance values for it, such as 5, 10, 15, 20, 25 ohms.

In the case of maximum power in the heating element, what power will the generator deliver?

4–6. Given a transformer with a primary voltage of 440 volts and a secondary voltage of 110 volts, *i.e.*, a turns ratio n_1/n_2 of 4. Assume that the no-load or magnetizing current is negligible. Connect a resistance of 2 ohms across the secondary side, *i.e.*, across the 110 volts. The transformer will now draw also a current from the 440-volt side, of course. What resistance, if connected across the 440-volt supply instead of the loaded transformer, would draw just as much current as is actually drawn by the primary of the transformer? In general, let the transformer have a turns ratio n_1/n_2 and assume a resistance load of R ohms connected to the side with n_2 turns. By what equivalent (sometimes called "reflected") resistance could the transformer and its load be replaced on the side with n_1 turns, as far as the supply line is concerned?

4–7. Problem 4–6 shows that a transformer permits a resistance or any other load to appear as a load of different value (this is, of course, possible only with alternating current). Assume that the value of resistance for the heating element in Prob. 4–5 could not be chosen any more but that the resistance was given as 80 ohms. What transformer would have to be used to make it appear to the line of Prob. 4–5 as the value giving the maximum heating effect?

4–8. A flashlight bulb is rated at 3.6 volts and 0.27 amp. It is supposed to be operated over a transformer from the plate circuit of a vacuum tube. In order to obtain the maximum power from the particular tube, its load must have a resistance of 5,000 ohms. What is the turns ratio of the transformer needed for properly "matching" the bulb to the tube?

4–9. To prevent freezing, a 15-ft run of 1-in. iron pipe carrying water is to be heated electrically by passing current through it. It has been estimated that 500 watts will

be sufficient to accomplish the desired purpose. The resistance of the pipe has been found to be 125 microhms per ft. What "matching" transformer is needed if the available voltage is 220 volts?

PROCEDURE: Calculate the resistance needed to develop 500 watts when the pipe is connected to 220 volts; find the actual resistance of the pipe and then the turns ratio of the matching transformer.

SUGGESTED ADDITIONAL READING FOR CHAPTERS III AND IV

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CHAPTER V

NONLINEAR CONDUCTORS

5–1. Definition of the Term "Linear Circuit Element."—Helmholtz's principle of superposition and Thévenin's theorem are applicable only if the network consists of linear circuit elements. By that we mean a device where current and voltage, or the rates of change of these quantities, are proportional to each other over the whole operating range. An air-core inductance is a linear circuit element, for instance, because the voltage is proportional to the rate at which the current changes [as per Eq. (2–3)], no matter how large the current may be. An iron-core inductance, on the other hand, is no longer a linear circuit element when the iron begins to saturate. When saturation is reached, the same rate of change of current that may have produced 100 volts across the inductance as long as the iron was not saturated may produce only 10 or 2 volts, because the magnetic field no longer increases as rapidly as at the start, in spite of the fact that the current may keep on increasing at the same rate.

5–2. Volt-ampere Characteristic of a Resistor.—A resistor is a linear circuit element because the current is proportional to the voltage, as given by Ohm's law. If we plot the current flowing in a resistor against the voltage applied to it, a straight line evidently results, passing through the origin of the coordinate system, as shown in Fig. 5–1. In this figure the volt-ampere characteristics of several resistors are shown. Since the characteristic of a resistor is a straight line going through the origin, it is sufficient to determine one additional point on the characteristic. For a 40-ohm resistor, for instance, it takes 40 volts to produce 1 amp. This will be one point of the characteristic. For a 100-ohm resistor the characteristic must pass through the point determined by 100 volts and 1 amp. The characteristic of a 150-ohm resistor passes through the point given by 75 volts and ½ amp. The tangent of the angle, which these lines include with the abscissa axis, is given by the ratio $I/E = 1/R$. The *higher*

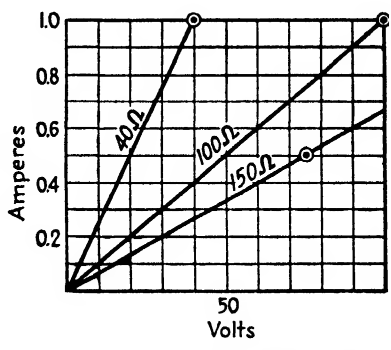

FIG. 5–1.—The volt-ampere characteristic of a resistor is a straight line passing through the origin. Only one additional point is required for the complete determination of the volt-ampere characteristic of a resistance.

the resistance, the *smaller* the angle or the slope of the characteristic This is clearly shown in Fig. 5-1 where the line representing the characteristic of the 40-ohm resistor is much steeper than the line representing the 150-ohm resistor.

5-3. The Mazda Lamp as a Nonlinear Conductor.—We shall see later that vacuum tubes do not have a linear volt-ampere characteristic, and it will therefore be of value to study the methods to be used when it is required to design circuits containing nonlinear circuit elements. The electrical engineer fortunately has to deal most of the time with linear ele-

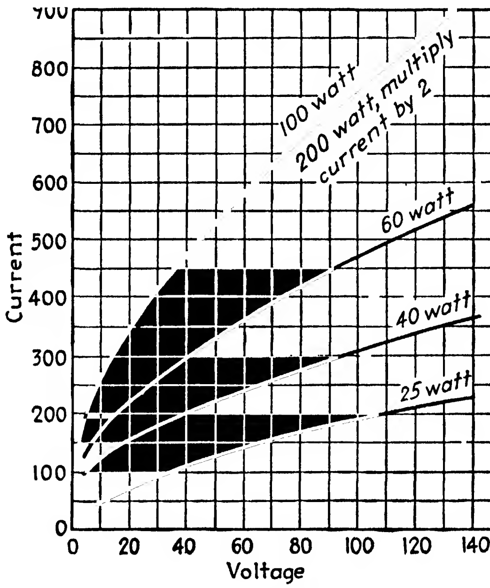

FIG. 5-2.—The volt-ampere characteristics of Mazda lamps are not straight lines because the resistance of the filament changes with the temperature and therefore with the current or voltage

ments. This makes a mathematical treatment of the circuits possible, but there are a number of devices that do not have a linear characteristic. One of the most common nonlinear circuit elements is an ordinary Mazda lamp. Owing to the high temperature reached by the filament in operation, the resistance of it changes with a change of voltage, and under this condition the current will not drop to one-half of its value, for instance, when the voltage applied to the lamp is reduced one-half.

5-4. Methods of Giving the Characteristics of Nonlinear Circuit Elements.—How can information about a device like a Mazda lamp be given in the most convenient way? Three methods are evidently available to convey such information: (1) we can plot the resistance of the lamp against the voltage applied to it; (2) we can plot the resistance against the current flowing through the lamp; or (3) we can plot simply the rela-

tion between current and voltage. Which of these three representations is the most desirable depends on the particular circuit problem to be investigated and also on the individual taste and preference of the worker. In order to conform with the methods to be used later in connection with vacuum tubes, we shall use the last-mentioned, *i.e.*, the relation between voltage and current found by plotting the current values as ordinates against the voltage values as abscissas. In Fig. 5-2 the volt-ampere characteristics of various Mazda lamps are shown.

5-5. Two Resistance Values of a Nonlinear Resistor.—How shall we define the resistance of a Mazda lamp? Although it is obvious that this value will be different for every value of current or voltage, we still would probably say, without hesitation, that for a particular voltage the value of the resistance is simply given by dividing the voltage by the current or, in other words, by the value E/I. However, there is another value that can justifiably be called the resistance of the device, namely, the ratio of a small change of voltage to the small change of current produced by this voltage change. Any argument as to which one of these two values is really the resistance of the device is meaningless; depending on the problem on hand, either the one or the other will prove the more advantageous one to use.

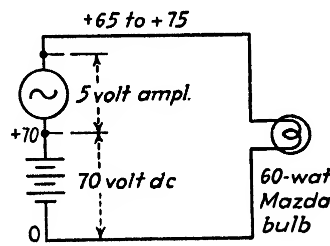

FIG. 5-3.—When a dc and an ac voltage act simultaneously on a Mazda lamp, we get into trouble when we attempt to use the principle of superposition for the calculation of the dc and ac components of the current.

A condition that will be encountered quite often later is one where a relatively large direct voltage and a smaller alternating voltage in series with it are acting on a circuit element. Suppose then that we were asked to investigate what happens when a direct voltage of 70 volts and an alternating voltage of 5 volts peak in series are applied to a 60-watt bulb, as shown in Fig. 5-3. Assume further that we are interested only in finding the ac component of the current, which is obviously due only to the alternating voltage. (It is assumed that the alternating voltage is of such low frequency that the filament temperature can follow the variations of voltage, in which case the current will follow the characteristic shown in Fig. 5-2.) With 70 volts applied to the lamp, the graph indicates that the current will be 0.39 amp; the resistance, if we define it as the ratio of voltage and current as by Eq. (1-3), is then $70/0.39 = 179$ ohms. According to the principle of superposition the current in a circuit containing several sources of voltage can be found by determining the current that each voltage source alone would produce, with all others short-circuited, and then adding all the currents thus determined. Since any alternating current in the circuit shown in Fig. 5-3 can evidently be due only to the presence of the ac generator, we seem to be justified in obtaining it by

simply dividing the alternating voltage by the resistance. The alternating voltage has a peak of 5 volts. By dividing this value by 179, we obtain an alternating current with a peak of 0.028 amp or 28 ma. This means that our actual current should consist of a dc component of 390 ma plus an ac component of 28 ma peak, which would make the current fluctuate between $390 + 28 = 418$ ma and $390 - 28 = 362$ ma. A look at the characteristic shows us immediately that this is faulty reasoning. With the 5-volt peak alternating voltage placed in series with the 70 volts dc, the voltage across the lamp will vary between 65 and 75 volts, and the characteristic shows that for these two voltages the corresponding current values will be 375 and 405 ma, respectively. In other words, the peak alternating current is only 15 ma, and any method that gives us by calculation a different result is obviously incorrect. Now let us try to find what value we have to assign to the resistance if we are to define it as the rate of change of voltage with current. If we want to find the resistance of the lamp, as defined in this manner, at a voltage of 70 volts, we have to increase and decrease the voltage a small value from 70 volts, for instance, increase it to 70.1 and decrease it to 69.9. Then we observe the current change due to this small voltage change, and the ratio of these two values will give us the resistance. This can be accomplished on the given curve only if the region around the 70-volt point is magnified greatly. Now, if we take the 70-volt point on the curve under a magnifying glass with more and more magnification, it is obvious that the section at which we are looking will seem to become straighter and straighter. It also is evident that the angle that the curve makes with the vertical and horizontal direction at the 70-volt point will not change with the magnification of this region. With a high degree of magnification, this angle will clearly be equal to the angle that a tangent drawn to the curve at the 70-volt point includes with the two directions of the axes. If this is true, all we have to do is to draw a tangent to the curve at the 70-volt point, and we can then consider this tangent as a powerful enlargement of the actual curve around the 70-volt point. Now, we were interested in finding the ratio of a small voltage change to a small current change near the 70-volt point on the curve; but for a straight line this ratio will be the same no matter how large or how small we choose the two changes just mentioned. The tangent that we have just drawn can therefore be extended as far as we want across the graph. We can even shift it parallel to itself so that it will pass through the zero point. This new line passing through the zero point can then be considered as the characteristic of a resistance. It is seen that the ratio of voltage and current for this resistance is the same as the ratio of a small voltage change to the small current change produced by it in the neighborhood of the 70-volt point on our actual curve. If this is done, we find a value of about 340 ohms for the point in question. Now, if we use this value for calculating the alternating current due to the superposi-

tion of the 5-volt peak alternating voltage on the steady 70-volt direct voltage, we obtain for the alternating current a value of $5/340 = 0.0147$ amp or 14.7 ma. This value obviously is very close to the 15 ma that we read directly from the curve.

5-6. Static and Dynamic Resistance.—The resistance obtained simply by dividing the actual voltage by the actual current is often called the "static" resistance of a device, while the resistance found in the manner outlined above, *i.e.*, by finding the ratio of a small voltage change to a small current change (written as $\Delta E/\Delta I$) is sometimes called the "dynamic" resistance. It is open to argument whether the choice of this name is a fortunate one. For a true resistance, it is clear that the static resistance is equal to the dynamic resistance for all points of operation.

With 70 volts applied to the 60-watt lamp discussed above, there are then two values of resistance that may be of use in the solution of problems involving this lamp. The static resistance is 179 ohms. With this value we can calculate the current that flows with 70 volts direct voltage applied to the lamp, but if we wish to calculate the alternating current that flows owing to the superposition of a small alternating voltage in series with the 70 volts, then the dynamic resistance of 340 ohms must be used. This is a state of affairs that can hardly be regarded as satisfactory; it prohibits the use of the superposition principle and all other network theorems that are based on linear conductors. Evidently it would be very desirable if it were possible to replace our lamp or any nonlinear conductor, at least for a small region around the desired operating point, by an arrangement of linear circuit elements such that no distinction would have to be made between the steady and the alternating voltages. Whatever the substitution is, we must demand (1) that it takes exactly the same current at the operating point as the nonlinear conductor and (2) that the current changes exactly in the same manner with a change of voltage as in the case of the nonlinear element. We shall now investigate whether such a substitution can be found.

5-7. The Black Box.—Assume that a black box with two terminals is presented to us. Nothing is said about its contents, except that we may place as high as 200 volts across the terminals without doing any damage to whatever may be inside. An electrical engineer confronted with any electrical device would probably take first the volt-ampere characteristic of it, *i.e.*, he would apply a variable voltage to it and observe the current taken by it. A setup suitable to carry out this experiment is shown in Fig. 5-4. Suppose that we vary the voltage

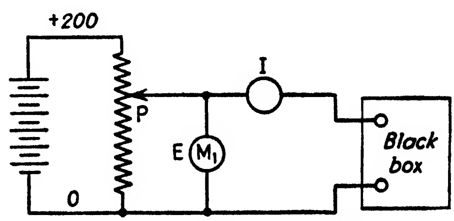

FIG. 5-4.—A two-terminal device should be investigated by taking its volt-ampere characteristics.

applied to the box, indicated by meter M_1, from 200 volts down to 0 volts in 20-volt steps (by moving the arm of the potential divider P) and obtain current values as given in the second row of the accompanying table. In the third and fourth rows are given corresponding values of static and dynamic resistance.

e, volts	200	180	160	140	120	100	80	60	40	20	0
i, amp	5	4	3	2	1	0	−1	−2	−3	−4	−5
R, ohms static	40	45	53	70	120	∞	−80	−30	$-13\frac{1}{3}$	−5	0
r, ohms dynamic	20	20	20	20	20	20	20	20	20	20	

If we plot the volt-ampere characteristic of the black box, we obtain a straight line, as shown in Fig. 5–5. Since the dynamic resistance of a device is given by the angle of the tangent, it is clear that the black box has the same dynamic resistance at any voltage, since the tangent to a straight line is the straight line itself. To any alternating voltage superimposed on a direct voltage the box will therefore show a resistance of 20 ohms; to the direct voltage, it shows a varying resistance. Now, there are two points of particular interest in this table or on the volt-ampere characteristic. We obtain zero current when 100 volts is applied to the device. This means that its static resistance is infinite at this point. We could possibly imagine that there might be within the box selective relays and a compressible carbon pile changing its resistance with the voltage applied to the box, finally opening the circuit when the voltage has reached the value of 100. Such an explanation would fail utterly, however, for voltages less than 100 volts when the current is seen to reverse! Now the fact that we obtain a current of 5 amp with zero voltage applied to the box, *i.e.*, with its terminals shorted, gives us a very valuable clue. There may be a resistance in it, but there must also be a voltage source in it, possibly a battery.

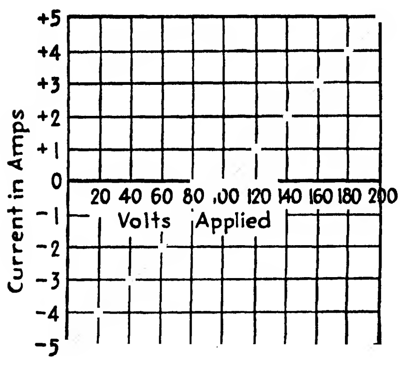

FIG. 5 5.—If the black box shown in Fig. 5–4 has a characteristic as shown here, it certainly cannot contain simply a resistance; if it did, the characteristic would have to pass through the origin of the coordinate system.

5–8. The Substitution for the "Black Box."—Is it possible that we could find a combination of a resistance and a battery that would give us the same volt-ampere characteristic as the actual box does? With 100 volts

applied to it, we obtain zero current. At first we thought that something had opened the circuit at this value but now, since we suspect that there is a battery in the circuit, it might also be that the battery hidden in the box is exactly 100 volts. If this is the case, obviously no current will

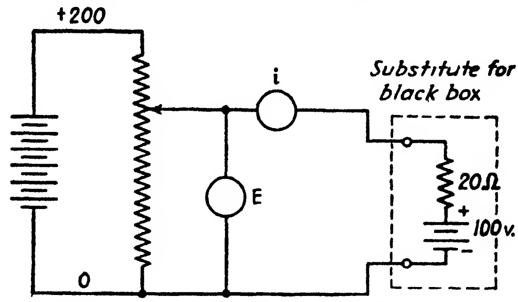

FIG. 5-6.—The reader should convince himself that the series combination of a resistance of 20 ohms and a battery of 100 volts connected as shown would have a volt-ampere characteristic exactly as shown in Fig. 5-5.

flow if the external voltage applied to the box is also 100 volts, since the two voltages will then oppose and cancel each other. But if we obtain a current of 5 amp with the device shorted, then the only conclusion that we can draw is that the resistance hidden in the box together with the battery of 100 volts has a value of 20 ohms. Therefore, if we replace the black box in Fig. 5-4 by a series combination of a battery of 100 volts and a resistance of 20 ohms, the circuit shown in Fig. 5-6 results. The reader should convince himself that with 200, 180, 160, etc., volts applied to this combination the current has the same value as the current taken by the black box with these voltages; in other words, the combination has the volt-ampere characteristic shown in Fig. 5-5. We still do not know what might actually be in the box but, since it behaves exactly like a battery of 100 volts with a resistance of 20 ohms in series with it, then in any circuit where it might be used we could substitute for it this simple arrangement and calculate the performance of the circuit for any desired condition.

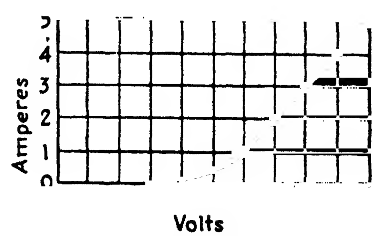

FIG. 5-7.—A device having a volt-ampere characteristic as shown here certainly will not consist of a series combination of a resistance and a battery; but over the range from 120 to 200 volts, it acts like such a combination.

Now assume that the mysterious black box did not have the volt-ampere characteristic, as shown in Fig. 5-5, but that it gave a graph like that shown in Fig. 5-7. Whatever is in the box, it certainly cannot be a simple arrangement of a battery and a resistance any more. But if we were told that the circuit in which it is to be used was such that there was never less than 120 volts or more

than 200 volts across it, then it is seen that between these limits it acts in exactly the same manner as the device with the characteristic shown in Fig. 5-5. Whatever the actual device may be, we can substitute the combination of resistance and battery for it, which will permit the easy calculation of the performance of the device in any circuit into which it may be included.

5-9. Curved Characteristics.—Even if the device has a completely curved characteristic, as shown in Fig. 5-8, we still can say that at a given voltage it acts over a limited range of voltage just like a battery and a resistance in series with it. If 60 volts is applied to a device with a characteristic as shown in Fig. 5-8, the current is 5 amp, and the static resistance is

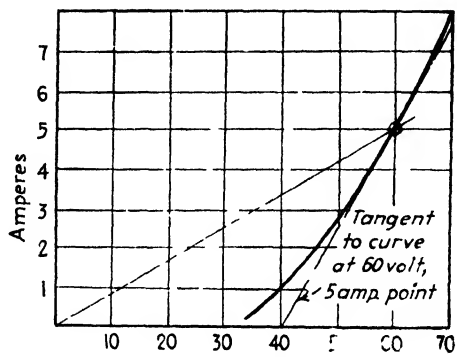

FIG 5-8.—Even if the device has a completely curved characteristic, it may be replaced for a limited range of applied voltage by the series combination of a battery and a resistor

therefore $^{60}/_{5} = 12$ ohms. It is true that a 12-ohm resistor will take exactly as much current as the device in question at exactly 60 volts. But, if the voltage should be changed to 61 or 59 volts on both, the current change would be quite different in the two. The current in the resistor would change along the straight line going through the origin in Fig. 5-8, since this represents the characteristic of a 12-ohm resistor, while the current through the actual device would, of course, change along the curved characteristic. The dynamic resistance of the device at 60 volts is found by drawing a tangent to the curve at the 60-volt, 5-amp point. This tangent intersects the voltage axis at 40 volts and represents, therefore, a dynamic resistance of 4 ohms because, with a voltage change from 60 to 40 volts (20 volts change), the current drops from 5 to zero. The device has therefore at 60 volts a static resistance of 12 ohms but a dynamic resistance of 4 ohms. As already stated, a device with two resistance values is hard to treat mathematically but, applying now the results of our considerations in connection with the black box, we state simply that in the neighborhood of 60 volts the device acts exactly like a battery of 40 volts and a resistance of 4 ohms in series with it. It is easy to see that this is

true. With 60 volts applied to such a combination, we have $60 - 40 = 20$ volts actually across the 4 ohms, which gives us a current of 5 amp, just what the actual device takes. If we place in series with the 60 volts dc an alternating voltage with, say, 5 volts peak, we obtain an alternating current with a peak of $5/4 = 1.25$ amp. The total current will therefore fluctuate between $5 + 1.25 = 6.25$ and $5 - 1.25 = 3.75$ amp. Looking at the actual volt-ampere characteristic, we see that this is very nearly equal to the values we read from the characteristic. To sum up: If a device with a curved characteristic is to be used in a circuit and the voltage fluctuations across the device are kept relatively small so that the tangent to the characteristic does not change its direction appreciably in the range of this voltage fluctuation, then for all calculation purposes the actual device can be replaced by a battery and a resistance in series. The value of the battery voltage is found by intersecting the tangent to the characteristic with the voltage axis, while the resistance is given by the ratio $\Delta E/\Delta I$ of this tangent.

At an operating voltage of 60 volts the device just described has a static resistance of 12 ohms and a dynamic resistance of 4 ohms. Which of the two is the more useful value? (Note that it is not asked which of the two has more right to call itself the resistance at 60 volts.) Let the reader decide this for himself on the following basis: by combining the dynamic resistance with a battery of 40 volts, not only can we produce something that takes exactly as much current as the actual device at 60 volts, but the current will *change* for *small changes* of voltage from the 60-volt point in exactly the same manner as on the actual device. With the 12-ohm static value we cannot produce such a substitution.

5–10. The Three-terminal Black Box.—Suppose that we were given a black box with three terminals, 1, 2, 3, and were told to plot the volt-ampere characteristic of terminals 1, 2, first with terminals 1 and 3 shorted, then with terminal 3 made 2 volts negative with respect to terminal 1, then with 4 volts negative, then with 6 volts negative, etc. Let the results of the investigation be shown in Fig. 5–9. Inspection shows that the characteristics thus obtained are very nearly of the same shape, displaced equal amounts in the horizontal direction. What does this mean as far as our proposed substitution is concerned? For each voltage applied to terminal 3 with respect to terminal 1, the device acts like a resistance and a battery in series with this resistance. Since the characteristics are all parallel to each other, the resistance seems to be the same in all cases, and only the fictitious battery seems to have changed its value in equal steps for every step in voltage applied to terminal 3. We are therefore justified in saying that the three-terminal device hidden in the box acts like a fixed resistance with a battery in series with it, and that this battery assumes different values, depending on the voltage applied to terminal 3 with respect to terminal 1.

Upon opening the box, we find in it a radio tube marked 6J5. Terminal 1 is found to go to the cathode, terminal 2 to the anode, and terminal 3 is found to connect to the grid. The characteristics shown in Fig. 5-9 will

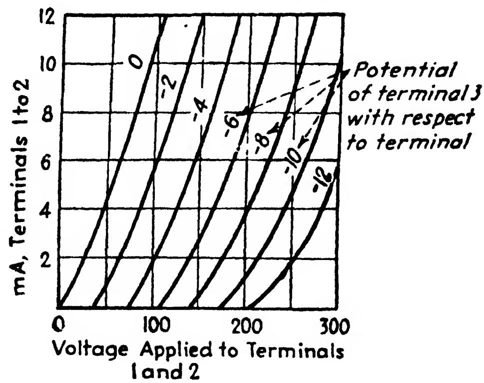

FIG. 5-9.—For any voltage applied between terminals 1 and 3 of this three-terminal device, terminals 1 and 2 exhibit a characteristic suggesting the combination of a resistance and a battery, the value of the latter depending on the voltage applied to terminals 1 and 3.

be found identical with those given by the manufacturer as so-called "plate characteristics" of this tube.

It is not expected that this statement will make the reader now familiar with a vacuum tube. More detailed reference to this discussion will be made later. The only thought he is to consider at this time is that the tube is nothing but a circuit element with a certain volt-ampere characteristic, and that a person familiar with the methods of using volt-ampere characteristics of electrical devices will be able to design circuits containing vacuum tubes and to predict their performance without ever having to know just what makes a tube have its particular characteristic.

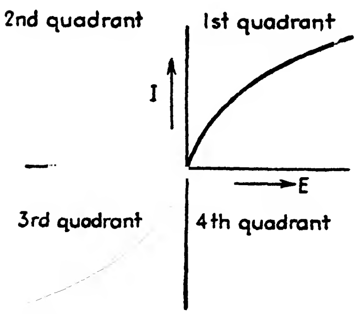

FIG. 5-10.—When the volt-ampere characteristics of a device in the first and third quadrant are identical, the application of an alternating voltage will produce an alternating current having no dc component.

5-11. Volt-ampere Characteristic of a Rectifier.—Coming back to the volt-ampere characteristic of a Mazda lamp—if we reverse the polarity of the applied voltage, the current will obviously also reverse and we obtain another branch of the volt-ampere characteristic extending into the third quadrant of the coordinate system, as shown in Fig. 5-10. If the amount of current passed by a device depends on the polarity of the applied voltage, which means that the volt-ampere characteristic in the third quadrant differs from that in the first quadrant, the device is called

a "rectifier." An ideal rectifier would be one that passes no current whatsoever when the voltage is applied with one given polarity while it passes any amount of current with an infinitely small voltage applied in the opposite direction. Its volt-ampere characteristic would then look as shown in Fig. 5–11*a*. If a rectifier passes no current in one direction but acts like a resistance in the other, it has a characteristic as shown in Fig. 5–11*b*. Such a device could obviously be considered as the combination

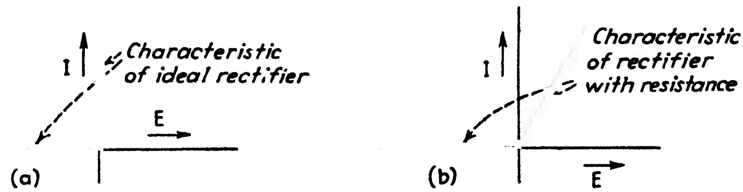

FIG. 5–11. (*a*).—The volt-ampere characteristic of an ideal rectifier coincides with the voltage axis for negative voltages, and with the current axis for positive current values.

(*b*) If the rectifier has resistance, it will, of course, show the characteristic of a resistance for positive voltage.

of an ideal rectifier and a resistance in series with it. Usually, actual rectifiers have characteristics that are curved but, for many practical purposes, the error committed by substituting either an ideal rectifier or a combination of a resistance with an ideal rectifier is not serious.

5–12. Nonlinear Element in Combination with Resistor.—In the preceding sections we have seen that a device with a curved characteristic can be replaced over a limited operating range by a series combination of a battery and a resistance. Calculations of circuit performance based on this substitution, of course, give correct results over only a limited range of voltages. In many cases it is not known from the beginning at what voltage the nonlinear element may be operating in a given circuit, nor may the voltage fluctuations imposed on it by the circuit action be always of sufficiently small magnitude to permit the substitution outlined in the preceding sections.

Suppose we wish to find the current that flows when a 60-watt lamp is placed in series with a 120-ohm resistor across a voltage of 140 volts. If we knew the resistance of the lamp, this would be an easy problem. But since the resistance is different for every voltage, we must know the latter, which in turn cannot be determined without knowing the resistance. We certainly seem to be going around in a circle!

The following method shows how to obtain an answer, even if the process involved is rather time-consuming. In Sec. 4–11 the statement was made that any electrical device defends itself, so to speak, against the application of voltage by taking a current of exactly the magnitude necessary to produce a voltage equal to the applied voltage. Therefore, if we can find the value of current at which the voltage across the series combination

of lamp and resistor is exactly 140 volts, we have our answer. Let us therefore assume various currents and obtain the voltage across the lamp from the curve, the voltage across the resistor by calculation, and then see for which current the two will add up to 140 volts. In the accompanying table this is done for the four current values 0.2, 0.3, 0.4, and 0.5 amp,

i	E_l	E_R	E_{total}
0.2	16.5	24	40.5
0.3	41	36	77
0.4	73	48	121
0.5	113	60	173

given in the first column. The second column shows the voltage across the lamp, taken from the curve. The third column gives the voltage across the resistor obtained by multiplying the current value in the first column with 120 ohms. The fourth column shows the sum of the two voltages. From this table it is seen that with 0.4 amp the voltage across the combination is 121 volts, while with 0.5 amp it is 173. This indicates that with 140 volts applied to the combination the current must be between 0.4 and 0.5 amp, closer to 0.4 than to 0.5. The table is therefore continued for values of current between 0.42 and 0.45.

i	E_l	E_R	E_{total}
0.42	81	50.4	131 4
0.43	84	51.6	135.6
0.44	87	52.8	139.8
0.45	92	54	146

It is seen that for 0.44 amp, the voltage across the lamp is 87 volts, across the resistance 52.8, which adds up to 139.8 volts. We can therefore say that the current is just slightly higher than 0.44 amp.

5-13. Graphical Solution and the Load Line.—Instead of calculating the voltage across the resistor, we could possibly have saved some time by drawing the volt-ampere characteristic of it, which as we saw before, is simply a straight line. In Fig. 5-12 there is shown the volt-ampere characteristic of the lamp and of the resistor in the same graph. We can avoid all calculations in the following manner. It is desired to find that current value for which the voltage across the lamp plus the voltage across the resistor adds up to 140 volts. We therefore draw in Fig. 5-12 a verti-

cal line through the 140-volt point and then, by means of a compass, add the voltage across the lamp to the voltage across the resistor for several trial values of current. In Fig. 5-12 this is shown for a trial value of 400 ma; the voltage e_r across the resistor is transferred by means of a compass to the position shown in the figure. The two voltages are seen to fall short of the desired 140 volts. When we have found the current value for which these two voltages add up to 140 volts, our problem is solved. This

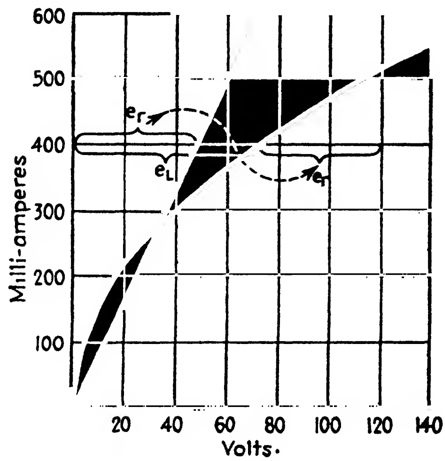

FIG. 5-12.—When a fixed voltage is applied to a series combination of a resistor and a Mazda lamp, the current will adjust itself to such a value that the voltage across the resistor plus the voltage across the lamp will add up to the total applied voltage.

solution is again one of trial and error, although the amount of work compared to the method of calculating is reduced considerably. There is, however, a neat trick available with the aid of which we can arrive directly at the desired result.

Suppose we place over Fig. 5-12 a tracing paper, copying on it the volt-ampere characteristic of the resistor only and the vertical and horizontal axes with their scales. We then take this tracing paper, turn it over as we would turn the page of a book, and line up the Y axis with the vertical line drawn through the 140-volt point while the X axis lines up with itself, as shown in Fig. 5-13. Now, the tracing paper represents the volt-ampere characteristic of the resistor, and it will continue to do so even after turning the figure over. The voltage scale, however, will now read from right to left, its zero point coinciding with the 140-volt point on the lamp characteristic. For a current of 0.3 amp, for instance, the voltage across the lamp is given by the length AB while the voltage across the resistor is given by CD. We now have to find only that value of current for which these two lengths will add up to 140. This is evidently the case for the intersection F of these two lines; for this point the voltage across the lamp will be GF, while the voltage across the resistor is given by HF, the two

adding up to *GH*, which is equal to 140 volts. This process is known as "drawing in the load line" and permits the determination of the exact value of current when a nonlinear conductor is placed in series with a resistor. Vacuum tubes are nonlinear conductors, as already stated. When used to control the current through a load placed in series with

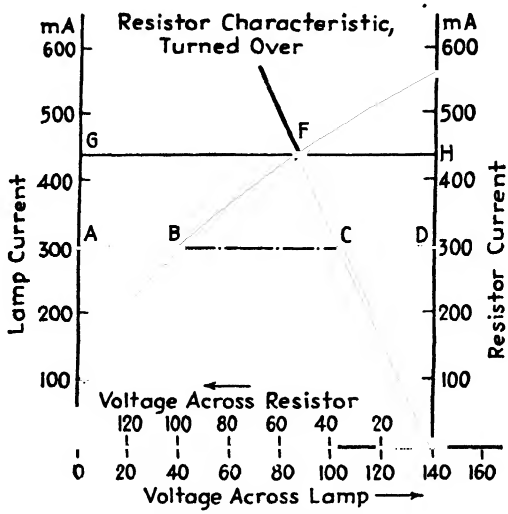

FIG. 5-13.—When the lamp characteristic is plotted from left to right, while the resistor characteristic is plotted from right to left, then at the point of intersection *F* the voltage across the resistor *HF* plus the voltage across the lamp *GF* adds up to the total voltages applied to the combination.

them, the principle of the load line permits the determination of the current changes taking place in the circuit. This will be discussed in more detail later.

PROBLEMS

5-1. What is the static or actual resistance of a 120-volt 40-watt bulb at a voltage of 80 volts?

5-2. What is the dynamic resistance, or the resistance to a small alternating voltage superimposed on the direct voltage, of the same bulb and under the same conditions as in Prob. 5-1?

5-3. You are given a device with the following volt-ampere characteristic:

e, volts	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140
i, ma	0	0	0	0	1	3	5	7	9	11	16	21	26	31	36

Plot this characteristic and from it determine the following values:

a. The static or actual resistance of the device at 40, 60, 80, 100, and 120 volts.
b. The dynamic resistance of the device at the same voltages.

c. What could be substituted for the device if we were told that the voltage applied to the device in the circuit where its use is intended would always be between the limits of 40 and 80 volts?

d. What could be substituted if the applied voltage would never fall below 100 volts and never exceed 130 volts?

5-4. What could be substituted for a 60-watt bulb operating at 60 volts, provided the operation of the circuit in which it is used would not cause voltage changes of more than 10 volts on the bulb?

5-5. A 60-watt lamp and a 200-ohm resistor are placed in series across a voltage of 130 volts. What will the current be through this combination, and what will the voltages be across the lamp and across the resistor?

5-6. A 60-watt and a 100-watt lamp are placed in series across a voltage of 200 volts. What voltages will appear across the two individual lamps?

CHAPTER VI

EMISSION; SPACE CHARGE; EFFECT OF GAS IN A TUBE

6-1. Proposed Scope of Discussion.—In Chap. V it was intimated that vacuum tubes and rectifying devices are simply circuit elements with certain volt-ampere characteristics. The design and the analysis of circuits containing vacuum tubes require only a knowledge of these characteristics and the maximum values of current and voltage that may be permitted. A mechanical engineer engaged in the design of presses can make the selection of a proper driving motor if he has at his disposal the different speed-torque curves of various types of motors. He does not necessarily have to know why the characteristic of a dc shunt motor and that of a series motor differ from each other. In a similar way, the industrial electronic engineer is more interested in having complete information on the characteristic curves of the tubes he wishes to use than in their inner workings. With this in mind, the discussion of this phase of the subject will be kept to an absolute minimum.

6-2. Relation of Electronics to Circuit Theory.—We have seen that an electric current in a copper wire is a movement of electrons due to the application of a driving voltage to the wire. Ordinary electric circuits consist of a closed conducting path consisting of the generator, the transmission line, the load, the transmission line again, finally closing at the generator. Control over the amount of current is exercised either by switches that open the circuit completely or by the insertion of devices that impede the flow of current. (We exclude at the moment such circuits as electroplating baths where the conducting path is not metallic all the way around; furthermore, the phenomenon taking place where the circuit includes a capacitor will not be discussed in view of the material presented in Chap. II.) The advent of electronics added an additional circuit element in which the passage of current is due to the movement of charges through a gap between two electrodes. Electronics has been defined as the field of electrical engineering that deals with the passage of current through a gap between two electrodes. If the charges moving across the gap are electrons only, *i.e.*, negative charges not attached to any matter or substance, the device is called a "vacuum tube" but, if there is a certain amount of gas or vapor in the tube and the passage of current is influenced or augmented by the presence of charged particles of the gas, then we are dealing with a gaseous tube. The volt-ampere characteristics of these two devices are altogether different, as will be seen later.

6-3. Free Electrons and the Flow of Current.—It has been stated that a conductor material is thought of as containing a great many "free" electrons, *i.e.*, electrons so loosely attached to the atoms to which they belong that it takes little effort to pry them loose from their orbits. If this is true, one would assume that we could cut the electric circuit consisting of conductors and that, with the gap thus produced either in vacuum or in a gas, it would not take much voltage to make the electrons jump out of the two conductor surfaces adjacent to the gap, thus maintaining a flow of current in spite of a gap. Everybody knows, of course, that this is not so; as a matter of fact, it is very fortunate that an electric current cannot jump a gap too easily, or else all our switches would be inoperative.

6-4. Electron Affinity.—Although within the conductor electrons are able to move rather freely and, as a matter of fact, even without a voltage applied to the conductor, have a random motion of their own, they seem to be restrained from jumping out of the metal. Evidently then, before we can hope to establish a current across a gap, it is necessary to persuade electrons to jump out of the metal. One could liken the free electrons in a metal to a swarm of bees flying around in a container consisting of a frame covered with tissue paper or the like. The tissue paper has a certain amount of strength, and a bee, to break through, would have to fly against it with a definite speed. In a similar way, electrons moving with a given speed within a metal will not be able to break through the surface—whatever is meant by "surface"—except at a certain speed. The amount of speed necessary to break out depends on the metal itself. Its actual value is of the order of several hundred miles per second, but it is not usually given as a speed but as a voltage. This seems a rather strange way of indicating a speed, but an explanation of this term will be deferred until later. For most metals the barrier that seems to exist near the surface cannot be hurdled by the electrons except at a speed corresponding to from 1 to 6 volts. This voltage is called the electron "affinity" of the respective metal. A mechanical analogy to this condition could be found in an open-top hopper full of marbles kept in motion by some kind of vibratory movement of the bottom of it. Some of these marbles might acquire enough velocity to jump higher than the walls of the hopper. The height of the walls is then seen to be equivalent to the electron affinity of the metal.

6-5. Various Methods of Producing Emission.—Our problem resolves itself, therefore, to speeding up the electrons inside the conductor to a sufficient value so that some of them at least will have enough velocity to break through the barrier. There are several ways to accomplish this goal:

1. Elevation of temperature, which gives thermionic emission.
2. Bombardment by electrons, which gives secondary emission.
3. Light radiation, which gives photoelectric emission.
4. Extremely high electric fields, which give field emission.
5. Radioactive disintegration.

By far the most practical source of electrons, employed in every radio tube, is the one based on elevation of temperature. In most books on electronics this subject is treated in great detail, and the reader interested in this phase is referred to these texts. Every metal will emit electrons when heated sufficiently, the emission increasing very rapidly as the temperature is raised higher and higher. Most of the metals will melt, however, before sufficient emission of electrons takes place; therefore, only metals with a high melting point can be used for this purpose. Tungsten and tantalum, practically the only metals used for this purpose, have to be heated to white heat before a satisfactory amount of emission takes place.

In tubes, the electrode from which electrons are made to emit is called the "cathode," while the electrode to which they fly is called the "anode."

6-6. Oxide-coated Cathodes.—In 1904, Whenelt discovered that he obtained an emission many times as high as for a pure metal when he covered a strip of platinum with oxides of barium, strontium, or calcium. This was the prototype of what is known today as an "oxide-coated" cathode. In place of the platinum, a less expensive alloy is used today. It would seem that all tubes would then make use of the oxide-coated cathode because of its high emission with lower temperature. This is not the case though. The question of the type of cathode most suitable for a particular service is one to which the designer of tubes must necessarily pay a great deal of attention. The application engineer, on the other hand, should not be concerned about it but should be able to assume that the tube designer has chosen the proper cathode material for the service for which the tube is intended. Thus, in tubes meant to operate with a high plate voltage, the cathode is usually made of pure tungsten, tantalum, or possibly thoriated tungsten, because an oxide-coated cathode is more liable to be damaged if the residual gas in the tube becomes ionized. As already stated, these questions do not concern the application engineer, and the tube designer in turn has a right to expect that his tubes will be used within the rated values.

6-7. Directly Heated and Indirectly Heated Cathodes.—The practical arrangement of heating the emitting surface of a cathode can take two basic forms. The first method is simply to pass an electric current through the material that is to emit the electrons or on which the oxide has been deposited. This form of cathode is referred to as the directly heated or filamentary type of cathode. It usually consists of a wire or ribbon of tungsten—if this material itself is to be used as the emitter of electrons—or the above-mentioned alloy on which has been deposited a layer of oxide, and the heating current is passed through it directly. As will be seen later, this type of cathode has certain drawbacks that become especially serious when the heating is done by an alternating current. In 1927, the so-called "indirectly heated" cathode appeared on the market. In this type the

cathode usually consists of a thin-walled nickel tube, the outside of which is covered with the oxide. Inside the nickel cylinder the actual heating element is arranged. This may be in the form of a spiral, or the wire constituting it may be folded back and forth several times. In order to insulate the various parts of the wire from each other, as well as to prevent them from touching the inner wall of the nickel cylinder, the wire is usually covered with a ceramic insulating material, which must, of course, be able to withstand the operating temperature of the actual filament. All indirectly heated cathodes, sometimes also called "unipotential cathodes," are oxide-covered cathodes for the following obvious reason. The heating element of an indirectly heated cathode must operate at a higher temperature than the outside of the cathode because heat must flow from it essentially by radiation to the nickel cylinder surrounding it. The operating temperature of an oxide-coated cathode is low enough so that the filament within it, operating at the higher temperature, is still not in danger of burning out. But if the outer cylinder were of tungsten, which must operate at almost white heat to furnish satisfactory emission, the heating element within it would operate at a still higher temperature, no suitable material being available for this purpose. These few remarks referring to cathodes will be considered as sufficient. The proper preparation and so-called "activation" of cathodes comprise a special field, and the reader interested in this phase will have no difficulty in finding a great amount of information about it in various texts. For the purpose on hand, however, it is only necessary to remember that, for a tube to operate in the way intended by the manufacturer, we shall have to connect the filament or the heater to the rated voltage in order to obtain the necessary emission.

A discussion of the other types of emission will be postponed until devices making use of the respective kinds of emission have been described.

6-8. Electric Field within a Tube.—After having investigated the methods of obtaining emission from the catho let us return now to the gap that we cut in our conducting path. If the tube is a vacuum tube, the gap is now enclosed in an airtight enclosure from which the gas has been evacuated to such an extent that the residual amount does not interfere with the phenomenon under consideration. With the cathode properly heated, there is a swarm of electrons around it. If there is no other electrode within the enclosure, these electrons will fly around inside the tube, many of them landing on the walls of the envelope. If the envelope is glass, or even if it is metal but not connected metallically to anything, the negative charges thus deposited on the wall, as well as the negative charges represented by the swarm of electrons, will exert a repelling influence on those electrons nearer to the cathode, which consequently have a tendency to "fall" right back into the cathode. But if there is a second electrode within the same enclosure and a battery is connected between the heated cathode and the anode with such a polarity as to make the anode positive, then

the negative charges represented by the electrons will fly toward the anode. This is usually explained by stating that the positive anode "attracts" the negative electrons, but this viewpoint is not a very fortunate one. In order to show to what mental puzzles it may lead, let us consider the original cathode-ray tube, also known as "Braun's tube," shown in Fig. 6–1. In the original Braun tube, the anode consists of a disk with a small hole in the center. Most of the electrons "attracted" by the anode strike it, but some of them pass through the small hole and continue their flight until they hit the glass wall on the far end of the tube. If this wall is covered with the proper kind of fluorescent material, light will emit at the point of impact of the electrons. The anode is connected to the positive

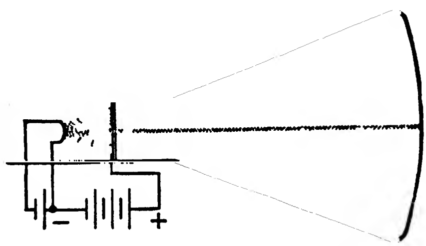

Fig. 6–1.—If the anode "attracts" the electrons as long as they are between cathode and anode, why does it no longer attract them after they have passed through the hole in the anode?

terminal of the battery; it is certainly just as positive on the face away from the cathode as on the surface facing it. (Or is there any question that a voltmeter connected with one terminal to the cathode will read the same voltage, regardless of which side of the anode we may touch with the other terminal?) Now, if an electron is attracted to the anode because the electron is a negative charge and the anode is positive, why does an *electron after having passed through the hole not turn around and come* back to the anode that was so "attractive" as long as the electron was on the other side of it? It is therefore desirable to change our concept of this matter to one that will not lead us astray.

6–9. A Mechanical Analogy of the Electric Field.—Figure 6–2 shows two boards with a rubber sheet clamped and stretched horizontally between them. Let a few marbles be placed on this sheet. Assume the weight of the marbles small enough or the tension of the rubber sheet high enough so that their presence will not deform the sheet to any appreciable extent. Now, if one of the boards is lowered with respect to the other, as shown in Fig. 6–3, an incline will be established and the marbles will begin to roll toward the lower board. Nobody would explain this phenomenon by stating that the lower board is attracting the marbles. If they hit the board, it is for no other reason than that the board happens to be at the foot of the incline. Let us assume that there is a gap in the lower board, as

shown in Fig. 6-4, with a horizontal sheet of rubber stretching out behind this gap. There is then no difficulty in visualizing that the marbles that happen to run through the gap will continue with constant speed if the

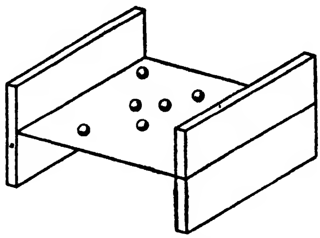

FIG. 6-2.—Marbles placed on a rubber sheet stretched horizontally between two boards will remain stationary.

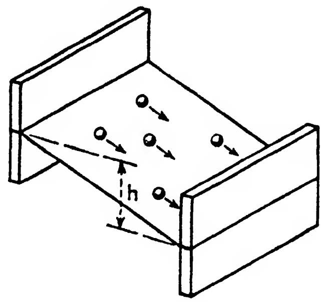

FIG. 6-3.—When one board is lowered, the marbles will run down the incline established by the movement of the board; would you explain the motion of the marbles by saying that the lower board "attracts" them?

rubber sheet beyond the gap is stretched horizontally. If it should be slightly uphill, they will slow down; if it should be downhill, their speed will further increase.

It is exactly this situation that prevails electrically when a voltage is applied between two electrodes. We say that an electric field is established between the two. Thus, if two plates are placed 2 in. apart, facing

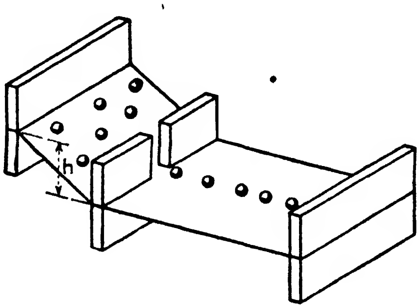

FIG. 6-4.—The mechanical analogy of a cathode-ray tube; whether the marbles after having scooted through the gate will speed up or slow down depends entirely on whether there will be an incline existing beyond the lower board.

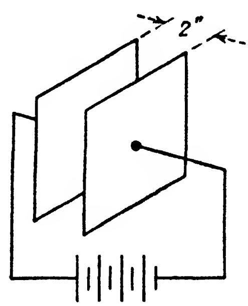

FIG. 6-5.—When a voltage is applied to two electrodes, an electric field will be established between them, analagous to the incline of the rubber sheet in Fig. 6-3.

each other as shown in Fig. 6-5, and a voltage of 1,000 volts is applied to them, an electric field will be established between the two which will be fairly uniform, at least near the center of the plates. By uniform is meant

the following: If we call the potential of the negative plate zero, then for every ¼ in. that we move toward the positive plate we reach a point 125 volts higher in potential; thus, at a distance of ½ in. from the negative plate, the potential will be 250 volts. At a distance of 1 in., *i.e.*, in the center between the two plates, there will be 500 volts, etc. It is true that there would be considerable practical difficulty in measuring this voltage—you cannot very well connect one voltmeter lead to the negative plate and stick the other one to a point between the two plates and expect a reading —but, when falling back on the physicist's definition of voltage as the work necessary to move a unit charge from one point to the other, it is possible to prove that the above statement is true. Such a field where equal voltages exist over equal distances is called a "uniform" field, and in our mechanical analogy, the rubber sheet stretched between the two boards will give a perfect picture of it. For every inch of distance between the two boards, the rubber sheet falls away an equal amount or, expressed differently, the slope is the same at every point. The slope of an incline is defined as the rise or fall per unit length in the horizontal direction. Since the total rise or fall of the rubber sheet is seen to be equivalent to the total voltage applied to the electrodes, then the slope of the rubber sheet is equivalent to the volts per inch or volts per centimeter in the case of the electrical arrangement. This value, *i.e.*, volts per centimeter, is commonly called the "electric-field strength" and is consequently seen to be equivalent to the slope of the rubber sheet. The electric field described above would have a field strength of 500 volts per in. If we were to move the two plates closer together so that they would be only ½ in. apart, without changing the applied voltage, however, then the field between the two plates would increase in strength to 2,000 volts per in. In the mechanical analogy, if we brought the two boards closer together horizontally without changing their vertical displacement, the slope of the rubber sheet would also increase.

6-10. Speed of the Electron in an Electric Field.—The application of voltage between two electrodes is seen to establish an electric field between them, equivalent in the mechanical analogy to the establishment of an incline when two boards with a rubber sheet stretched between them are displaced vertically. Marbles on the incline will accelerate toward the bottom of it, owing to the gravitational field of the incline. Electric charges placed into an electric field will also accelerate: positive charges toward the negative region of the field, negative charges—electrons for instance—toward the positive region of the electric field. In a uniform field the force acting on a charge is the same, no matter where the charge may be located, just as the force acting on the marble is the same, no matter where it may be located on the incline. If a constant force is acting on a body, a constant acceleration takes place, the amount of acceleration depending on the force and the mass of the body on which it acts. Research in the na-

ture of matter has proved that the electron, even if it does not consist of any of the elements known to the chemist, has a definite mass. The mass of an electron is very small, amounting only to 1/1,800 of the mass of the lightest of all atoms, the hydrogen atom. The fact that an electron has mass means that a definite acceleration will take place when it is brought into an electric field. The law governing the flight of electrons between two plates to which voltage is applied is therefore the same as that governing the motion of a marble on an incline, which in turn is known to be the same as that governing the fall of a body in the vertical direction. Another way of looking at this situation is from the point of view of energy conversion. A marble located near the top of the incline has potential energy, or energy of position. When it begins to roll down the incline, it loses this energy of position, but in place of it will appear the kinetic energy of motion. In a similar way, an electron emerging from the cathode into the electric field between cathode and anode has energy of position. On its way toward the anode, it loses this energy of position but gains kinetic energy. The speed of a body after having "fallen through" a vertical distance of h ft—whether the vertical displacement is straight down or on an incline—is given by the formula $v = \sqrt{2gh}$, or roughly $v = 8\sqrt{h}$ ft per sec. In an exactly similar way, an electron after "falling" through a voltage of E volts has a velocity given by

$$v = 5.93 \times 10^7 \times \sqrt{E} \text{ cm per sec} \tag{6-1}$$

6-11. Relativistic Change of Mass.—It becomes now clear why it is possible to designate the speed of an electron in volts, as was mentioned when we discussed emission. For every voltage that the electron has "fallen through" it has a definite speed, given by Eq. (6-1) and, if we find the statement that the electron affinity of a metal is 3 volts, for instance, it means that the electrons within it must have at least a speed equal to the one that they would have after having fallen through 3 volts in order to break out of the metal. It may be desirable to say a word of caution about Eq. (6-1). It would seem that with a sufficiently high voltage the electrons could be brought up to any desired speed. The same remark would also hold true if we had a gravitational field of high value extending in vacuum for a large distance. It would seem that in such a field a mass would accelerate as long as in this field, with no limit on the speed it could reach. But according to the theory of relativity of Einstein, when a mass reaches a speed approaching that of light, it increases in mass in such a way that the actual speed can never exceed that of light. In a similar way, Eq. (6-1) would yield speeds in excess of that of light for voltages exceeding approximately 250,000 volts. In this case the increase of mass of the electron with speed will have to be taken into account. It becomes obvious, however, that the operating voltages of ordinary vacuum tubes

are so low that speed calculated on the basis of Eq. (6–1) will give quite a true picture of the speed of the electrons arriving at the anode.

6–12. Space-charge Saturation.—We likened the action of the cathode to that of an open-top hopper with marbles jumping out. The arrangement of a cathode and an anode with voltage applied between them would now find a mechanical analogy in the picture shown in Fig. 6–6. Suppose that the marbles were emitted at the rate of 100 per second. Would the amount of incline established by pushing the right-hand board downward have anything to do with the number of marbles arriving at the foot of the incline? Evidently not. Every marble emitted will begin its journey down the incline, and although with a steep incline the marbles would roll faster than with a gradual one, nevertheless, after they begin to arrive at the foot it is clear that just as many will arrive per second as started their journey at the top of the incline. We consequently have a right to expect that the amount of current received by the anode will depend only on the amount of electrons emitted by the cathode as long as the anode is even the slightest amount positive with respect to the cathode (*i.e.*, as long as even the slightest "incline" exists). This reasoning is borne out only partly by the experiment. If we start with a relatively high voltage across a two-element tube—also called a "diode"—say, 40 to 50 volts, and reduce it gradually, the current remains substantially constant, as we would expect from the above analysis. The decrease of the incline does not reduce the current. When the voltage has been reduced to a few volts, however, the current begins to drop off with any further reduction in voltage. If the experiment is repeated with a higher filament temperature, the current observed with the higher voltages applied to the anode will be higher, which is again in line with our mechanical analogy, because the higher filament temperature means the emission of more electrons (or marbles) per second. Figure 6–7 shows a number of volt-ampere characteristics of a diode, taken with various filament temperatures. The straight parts of the curves we can explain with our mechanical analogy, but this analogy would tell us that the straight parts should all continue to practically the ordinate axis, because even the smallest voltage should cause all emitted electrons to go to the anode. What is the explanation for the fact that all the straight parts merge into one curve long before reaching zero voltage? With 15 volts applied to the diode, for instance, the same current flows no matter whether the filament is at a temperature t_1, t_2, or t_3, while between 30 and 60 volts applied, the voltage has nothing to say about the current. It is then only the filament temperature that decides the current value.

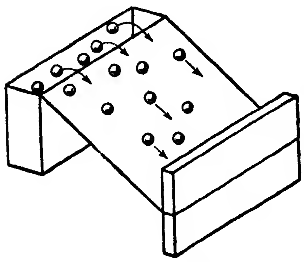

FIG. 6–6.—The cathode of an electron tube may be likened to a box out of which marbles are jumping.

We have seen that the space between anode and cathode represents an electric field. This field is established at the instant when voltage is applied to these two electrodes by charges rushed from the source of voltage to the two electrodes. In other words, one could consider the two electrodes as parts of a capacitor that becomes charged by the application of voltage to it. As shown in electrical engineering texts, an electric field always starts on positive charges and terminates on negative charges. As soon as the electrons, which are negative charges, are in flight between the two electrodes, the space between them is filled with negative charges, and

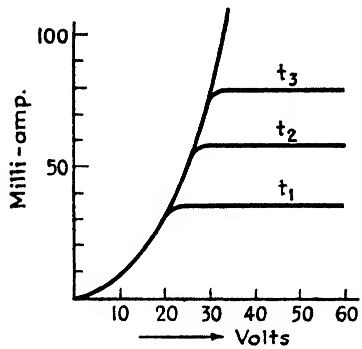

FIG. 6-7.—When the current through a diode is plotted as a function of the applied voltage, something is seen to happen that the analogy of Fig. 6-6 does not explain.

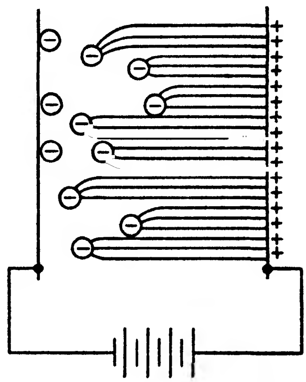

FIG. 6-8.—The electrons in transit between cathode and anode represent the space charge, the presence of which modifies the electric field between the two electrodes.

the electric-field lines that originally started on the positive charges on the anode and terminated on the negative charges on the cathode are now terminating on the electrons themselves. The potential distribution between cathode and anode—if they were parallel plates facing each other—is therefore not linear any more. This situation is pictured in Fig. 6-8, which shows how the field lines emanating from the positive charges on the anode terminate on the electrons just in flight from cathode to anode. This cloud of traveling electrons is also called the "space charge." A little thought will show that the field strength near the cathode must be reduced to just about zero by the presence of the space charge in the interelectrode space. The reasoning for this is as follows. Every electron leaving the cathode is the terminus of a field line emanating from the anode. There will obviously be a point of equilibrium when all the field lines emanating from the anode will terminate on the space charge. When this state of affairs is reached, there will no longer be any field near the cathode, which in our mechanical analogy would mean that the slope near the hopper has been reduced to zero. At the very instant, however, when an electron

crashes into the anode, the field line terminating on it up to this instant will become available, so to speak, again reaching out and terminating on one of the electrons just emitted. Sometimes the action of the space charge is also explained by stating that the electrons in flight repel those just being emitted from the cathode. Whichever method of explanation may be adopted, it is seen that the space charge is then the limiting factor, even if the emission is plentiful.

6–13. Mechanical Analogy of the Space-charge Effect.—Analogies usually have the deplorable habit of breaking down at the most inopportune moment. Sometimes it seems that they are of help only to those who have a clear enough grasp of the subject so as not to need them because only they know when the breakdown point has arrived and when any conclusions drawn from the analogy will no longer apply. Our rubber-sheet model breaks down at the instant when space-charge effects became the dominating factor. But the rubber-sheet model is really of such great help, as will be realized in later chapters, that an attempt to save it and to extend its action to cover the space-charge effect, at least qualitatively, may be worth our while. If the reader wishes to laugh about the Rube Goldberg affair shown in Fig. 6–9, it is his privilege; on the other hand, if he is willing to spend some effort in understanding the action of it, he will find himself repaid with a fairly clear picture of the action of the space charge.

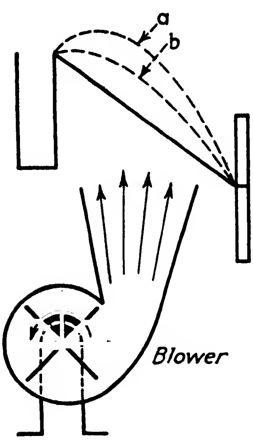

FIG. 6–9.—An attempt to save the mechanical analogy by simulating the effect of the space charge on the electric field by means of a blower deforming the rubber sheet.

Figure 6–9 shows again our familiar rubber sheet stretched in the usual manner between an emitting hopper and a lower board. Underneath the sheet is shown a blower. Let us now imagine that the speed of the motor driving this blower is in some way controlled by the number of marbles on the incline. With no marbles on it, the blower will be standing still; the more marbles on the incline, the harder it will blow. It is left to the reader to scheme out a control for doing this. Here is a suggestion that might work. Use a black rubber sheet and highly polished marbles; then let a *strong light fall on the rubber sheet, catching the light reflected from any marbles there* on a photocell, which in turn controls the motor. No marbles, no reflected light, motor stands still; many marbles, much reflected light, motor runs.

Evidently, with no marbles issuing, the rubber sheet will be stretched, as before, in a straight incline. Now let us assume that many marbles are emitted from the hopper. If they all roll down, the blower will operate so hard as to blow the rubber sheet into a hump, as shown by the dotted line

a, and some of the marbles will therefore roll back into the hopper. But this hump cannot exist very long. All marbles to the right of the hump travel to the lower board where they are absorbed. The blower will consequently slow down and the hump will disappear. As soon as this happens, any marbles ejected by the hopper will begin their journey, thus increasing the speed of the blower again. It is evident that with plentiful emission the blower will adjust itself to such a speed that the slope of the incline near the emitting hopper is just about zero (*i.e.*, the rubber sheet will be level there, as shown by the dotted line *b*). Strictly, it might not even be quite level. If the emitted marbles have an initial speed, the action of the blower might actually produce a slight hump at all times.

If we should lower the board representing the anode, the blower will obviously have to blow harder in order to reduce the slope of the rubber sheet adjacent to the hopper to zero. This means that more marbles will be permitted on the incline, which in turn means a more intense marble stream. If we keep on lowering the anode board, a point will be reached, however, when all the emitted marbles are taken away; from then on, any further lowering (further increase of voltage!) will not increase the current any further.

6–14. Child's Law.—The actual amount of current that is obtained between two flat electrodes with one of them emitting freely and a voltage applied between the two is given by an equation developed by Child. This equation is based on the condition just outlined, that the electric field near the cathode is reduced to zero by the action of the space charge. With a high voltage applied to the electrodes, it is easy to see that more space charge will have to be assumed in the interelectrode space to make all the electric-field lines terminate on it. Furthermore, the speed of the traveling electrons will be higher. The first influence would make the current go up linearly with the applied voltage, while the second one would make it go up with the square root of the voltage, as seen by Eq. (6–1). The current can therefore be expected to go up with the three-halves power of the voltage. This is exactly the conclusion to which Child's equation leads us. For two flat plates, spaced d cm apart, with a voltage E applied, and one of them emitting freely, the current, according to Child's equation, will be

$$I = 2.34 \times 10^{-6} \frac{E^{3/2}}{d^2} \text{ amp per sq cm} \qquad (6\text{–}2)$$

As soon as the applied voltage is sufficiently high so that all emitted electrons are taken away from the cathode, then any further increase in voltage will not lead to an increase in current. The tube is then said to operate under a condition of "temperature saturation" because the only way to increase the current is to increase the filament temperature. If, on the other hand, the space charge is the limiting factor, *i.e.*, when the cath-

ode emits more electrons than are needed to maintain the current, the tube is said to operate under a condition of space-charge saturation. Theoretically, any tube operates under either one or the other condition. If an increase in voltage leads to an increase in current, then an increase in cathode temperature should not lead to an increase of current, and the tube is operating under space-charge saturation. If, on the other hand, an increase in cathode temperature increases the current, then an increase in applied voltage should not increase the current, and the tube operates under temperature saturation. The sharp breakoff point shown in Fig. 6–7 will, however, be found only in tubes using pure metals as filaments; in the case of oxide-coated cathodes, a much more gradual transition from one condition into the other is observed. The reason for this is the difference in surface structure of the two types of cathodes, but it would be beyond the scope of this book to go into further detail.

6–15. Power Loss Caused by Voltage across Tube.—In Sec. 6–14 we saw that in a diode operating under space-charge saturation the voltage required to drive a current through the device is considerable and that it increases with the amount of current passing through it. Diodes are used in rectifying circuits, and it is clear that the voltage across the tube, while it is passing current, multiplied with this current value represents a loss of power. An ideal rectifier evidently passes current in the desired direction without requiring any voltage.

6–16. Effect of Gas in the Tube.—The admission of a small amount of gas or vapor into a diode changes its characteristics radically. The conduction then taking place is conduction not through a vacuum but through a gaseous conductor. The laws governing this type of conduction are much more complicated than those applying to conduction through a vacuum. As already stated, the purpose of this book is not so much to show why various types of tubes have different characteristics as to show how to design circuits making use of these characteristics. We shall consequently again confine ourselves to a most sketchy discussion of the reasons why the admission of a small amount of gas into the tube changes the characteristic so radically. The subject will again be taken up in a little more detail in a later chapter. The reader wishing a more comprehensive treatment will find it in several excellent texts dealing with gaseous conductors only.

Gas molecules are considered as electrically neutral particles. They consist of a nucleus, which is positively charged, carries practically all the mass, and around which a number of electrons equal to the positive charge of the nucleus are spinning in various orbits. The number of gas molecules in every cubic inch, and therefore their distance from each other, depend on the pressure of the gas. There is no such thing as a perfect vacuum. Even in the best obtainable vacuum there are millions of gas molecules in every cubic inch, but their distances from each other are so large compared to their diameter that a particle as small as an electron could travel a long

way before by chance it would collide with one of the molecules. In 1 cc of gas at 0°C and atmospheric pressure, there are 2.7×10^{19} molecules. The average distance between their centers must then be about $\frac{1}{3} \times 10^{-6}$ cm. If we place two electrodes in a vessel filled with gas at atmospheric pressure, one of the electrodes emitting, and apply a voltage between the two, electrons will begin to travel toward the positive plate. Now, it is true that the molecules are very small (about 10^{-8} cm in diameter) and the electron is smaller still (about 10^{-13} cm in diameter); nevertheless, an electron, although it might be passing many molecules, can at best travel only a very small fraction of a centimeter (about 10^{-5} cm on the average) before it collides with a neutral molecule. In such a small distance it has not acquired very much speed and, therefore, more or less bounces off without doing anything to the molecule. If the pressure of the gas inside the container is reduced, say, to 1/1,000 of its original value, the number of molecules will be reduced to 1/1,000, and their average spacing will be increased accordingly. The average distance of travel of the electron before collision, also called the "mean free path," is then greatly increased. Since the electron can therefore move farther, it will acquire higher speed, and its energy at the collision increases rapidly as the pressure of the gas is reduced. Now when an electron hits a gas molecule with sufficient speed, it knocks one or more of the electrons out of the gas molecule. The gas molecule thus deprived of some of its negative charge will show a surplus of positive charge. The gas is said to be "ionized." The knocking of the electron out of its orbit is usually accompanied by the emission of light, and it is this glow that we see in neon or mercury-vapor tubes. We have seen that the speed of an electron can be given in volts, as in Eq. (6–1), and experiments have shown that the speed necessary to disrupt a gas molecule is different for the various gases. The voltage through which an electron must have fallen to acquire enough speed to break up a gas molecule is called the "ionizing voltage" of the particular gas. For nitrogen, this value is about 14½ volts, for neon 21½ volts, for mercury vapor about 10½ volts. Obviously, no ionization can take place if the voltage applied between the two electrodes does not exceed this value. Even if it does, ionization will take place only if the mean free path of an electron is sufficiently long to permit the electron to acquire the necessary energy.

6–17. Cancellation of Space Charge.—Let us assume that the heated cathode and the cold anode of a regular diode are in an enclosure into which a gas or vapor has been admitted. Let the pressure of the gas be low enough so that the mean free path of the electrons emitted by the cathode and speeding toward the anode is sufficiently long to permit the electron to acquire the speed necessary for ionization of the particular gas and high enough so that a sufficient number of electrons will have a chance to collide with the gas molecules. Under this condition it can be assumed that a large number of gas molecules will become ionized. This will have two

results. In the first place, a given electron emitted from the cathode may in this manner produce an additional electron. At first glance this might be a very welcome situation, but, as has been shown, it is the negative space charge that limits the current through a vacuum diode, not the ability of the cathode to emit sufficient electrons. Therefore, the creation of an additional electron in the interelectrode space is not of much help. The second phenomenon *taking place, however, is of the utmost importance.* The ionized gas molecule obviously represents a net positive charge. Thus in the first place it still cancels the electric effect of the liberated electron, and we consequently seem to be right back at the starting point as far as space charge is concerned. This positive gas ion now begins to travel toward the cathode. The force acting on it in this direction is obviously identical with the force acting on the liberated electron because both particles have the same amount of charge and, at the instant of separation, necessarily are also in an electric field of the same strength. But the mass of an ion is much larger than the mass of an electron, even for the lightest element, which is hydrogen; the mass of the hydrogen atom is 1,800 times the mass of the electron. If the gas consists of mercury vapor, for instance, the mass of the ion is again approximately 80 times as much as that of the hydrogen ion. Compared to the speed with which the original electron, as well as the newly liberated electron, is traveling toward the anode, the speed of the positive ion traveling toward the cathode is nothing but a crawl. Long after the two electrons have reached the anode, the positive ion will therefore still be drifting in the interelectrode space. Multiply this with billions of ions drifting in this space and it becomes apparent that this cloud will effectively nullify the action of the negative space charge produced by the electrons in the interelectrode space. This cancellation of space charge causes the gas tube to have a characteristic radically different from that of the vacuum tube. The potential distribution within the tube is also entirely different from that of the vacuum diode. The field near the cathode will be extremely high, while the field within the interelectrode space will be very low. Most of the voltage drop occurs next to the cathode. The mixture of high-speed electrons traveling toward the anode and ions drifting slowly toward the cathode is called the "plasma." The voltage drop across the tube as a whole is usually practically equal to the ionization voltage of the gas; furthermore, and this is of the utmost importance, it is practically independent of the amount of current flowing through the tube up to the limits of emission from the cathode. It is true that the ions and electrons formed by collision with the original electron contribute to the flow of current so that it might at first glance seem that the cathode will be relieved somewhat as the sole source of charge carriers. This contribution is negligibly small, however, and practically the whole current through the tube is still due to electrons originally emitting from the cathode.

6–18. Cathode Construction of Gaseous Tubes.—The cathodes for gaseous tubes can be constructed with considerably higher efficiency than those of a diode of the vacuum type, however. If, for the sake of obtaining a larger emitting surface, we would, for instance, take a ribbon and form it into a zigzag shape, we would find that such a cathode in a vacuum-type diode would be rather ineffective because the electric field could not reach between the corrugations. In a gas tube, on the other hand, the presence of the plasma changes this situation completely, and all the surface of such a cathode will take part in emission. Thus we find the cathodes in gaseous tubes formed in spirals and pleats, not only raising the total obtainable emission but also lowering the power required for heating the cathode compared to what it would take if it were stretched out. These details are again of the utmost importance to the designer of tubes. To the engineer interested in the applications of these devices, they simply mean that, compared to the milliamperes of the average vacuum diode, a gas tube of similar dimensions furnishes amperes without requiring an increase in the power needed for the heating of the cathode.

Gaseous tubes are then simply nonlinear circuit elements, passing current in one direction only, with the voltage across them essentially independent of the current flowing through them. Very little conduction takes place until the applied voltage exceeds a certain value (essentially the ionization voltage of the gas or vapor). From then on any further increase in voltage leads to the destruction of the tube, and we shall have to depend on the circuit elements in series with the tube to prevent the current from reaching destructive values. In other words, the tube cannot protect itself against any current that the source is capable of supplying.

PROBLEMS

6–1. The charge of an electron is 1.6×10^{-19} coulomb. The mass of it is 0.904×10^{-27} gram. (1 watt-sec $= 10^7$ ergs.) Develop the formula $v = 5.93 \times 10^7 \times \sqrt{E}$ cm per sec.

6–2. Two parallel plates, one of them an emitter of electrons, are spaced ½ in. apart in an evacuated vessel. If 20 volts is applied between these plates, with the emitter negative, how long will it take for an electron to fly from the emitter to the other plate? For this problem assume uniform field between the plates.

6–3. A diode with two flat plates, one of them the emitter, spaced 3/16 in. apart shows a current of 120 ma with a voltage of 40 volts applied between the two plates (the emitter negative, the other plate positive). The distance between the plates is now reduced to ⅛ in. and the voltage is raised to 50 volts. What will the current be?

6–4. How do two diodes, one of them operating under temperature saturation, the other under space-charge saturation, react when (*a*) the voltage across the diode is changed, (*b*) the temperature of the cathode is changed?

SUGGESTED ADDITIONAL READING

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CHAPTER VII

FUNDAMENTALS OF RECTIFIER CIRCUITS; THE IDEAL RECTIFIER AND ITS ANALYSIS

7-1. Range of Applications of Diodes as Rectifiers.—Diodes are devices that pass current in one direction only. They therefore offer a convenient method of converting alternating current into direct current. The range of application of these devices is tremendous, reaching from rectified currents of the order of a few microamperes at voltages less than 1 volt, such as encountered in the rectification of signals in a radio set or in sensitive control circuits, to hundreds of amperes at voltages of around 600 volts in mercury-arc rectifiers and to voltages of 50,000 volts and higher encountered in X-ray applications.

7-2. Methods for the Analysis of Rectifier Circuits.—In the discussion of the transformer, it was pointed out that various methods of treating this circuit element are available. The choice of the method of analysis was dictated by the particular type of transformer under consideration. In a similar way, all rectifiers are simply devices whose volt-ampere characteristics depend on the polarity with which the voltage is applied. This was shown in Figs. 5-10 and 5-11. However, depending on the application, certain simplifying assumptions can be made in one case, while not in the other, and these considerations dictate the most suitable type of approach. It is usually the relation between the voltage appearing across the load while the rectifier is permitting current flow to the voltage across the rectifier itself that determines what simplifying assumptions are permissible.

7-3. Behavior of an Ideal Rectifier.—The best insight into the operation of rectifier circuits can probably be obtained by the careful study of an ideal rectifier. When this is once understood, it is usually possible to estimate or predict the effect of the deviation of the characteristic of an actual rectifier from those of an ideal one. It will also be shown how to proceed when these deviations are so significant that a simplified treatment is no longer satisfactory.

An ideal rectifier would obviously be one that would pass current freely in one direction while forbidding it completely in the other. By "passing current freely" is meant that no voltage would be required to drive the current in the desired direction. Its performance can be visualized with the aid of the hypothetical circuit shown in Fig. 7-1. Let the source in this figure furnish an alternating voltage with a very low frequency, say,

one cycle in 10 min. A load of any nature (resistor, motor, capacitor) is connected to the source through a switch. At the switch we have stationed a watchman who keeps his eye on the ammeter measuring the current flowing in the circuit and on the voltmeter connected across the switch (which will, of course, read zero voltage when the switch is closed). We have given him the following instructions. He is to leave the switch closed as long as the ammeter indicates a current from right to left; when the current drops to zero and tends to reverse, he is to open the switch immediately. (Note—and this is very important—that nothing has been said of the voltage of the source at this instant; it may be positive, zero, or negative at this instant, as will become apparent in our discussion of various types of load.) When is he to close the switch again? Well, when the current wants to flow from right to left again, to be sure. But how is our watchman to know when that time has arrived? The voltmeter across the switch will tell him this. At the instant when terminal A becomes positive with respect to C, he can be sure that closure of the switch will result in a current from A to C. From then on he watches the ammeter again. Note again that the voltage of the source has not entered our instructions for the closing instant, except that we might have formulated the closing condition by stating that he is to close the switch when the voltage of the source exceeds that of the load, which, as a glance at Fig. 7–1 shows, will make terminal A positive with respect to C. (How can there be a voltage across the load before the switch is closed? Suppose the load happens to consist of, or at least contain, storage batteries, what then?)

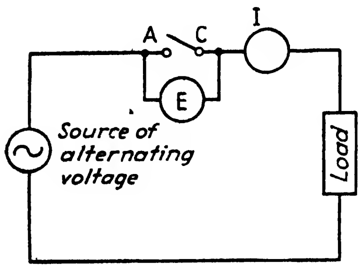

FIG. 7 1.—The performance of an ideal rectifier may be analyzed by means of a switch operated as explained in the text.

An ideal rectifier is the combination of switch and watchman described in the preceding paragraph, and if we understand the latter, we also understand the former. The terminals of the switch were marked A and C; in the rectifier these two letters now mean "anode" and "cathode." If it performs like the watchman and his switch, it means that current is permitted to flow freely from A to C, but (1) it opens the circuit instantly when the current wants to reverse and (2) it closes the circuit instantly when A becomes positive with respect to C. In our "voltage-equal-height" concept, A can be below the level of C as much as it wants, but the moment when A tries to jump above C, current will begin to flow, and C will then follow A (since it is connected to A by the switch) as long as the current flows from A to C.

Sometimes the action of a rectifier in an alternating circuit is explained by likening it to a synchronously driven switch, which closes its contact

for half a cycle and then opens the circuit for the same length of time. The reader is advised to banish this concept from his mind, because, although under certain conditions a half-wave rectifier may permit current flow for half a cycle, in general this is not the case, and the above concept is liable to create much confusion.

7-4. Ideal Rectifier with a Resistance Load.[2]—Let a source of alternating voltage be connected to a series combination of a perfect rectifier and a resistor, as shown in Fig. 7–2*a*. Consider the instant when terminal A is just becoming positive, *i.e.*, is going up. That would make A positive with respect to C (place it above C), and our watchman will close the

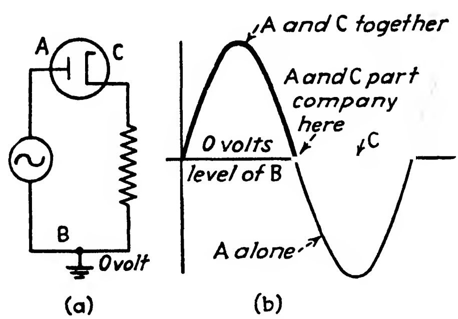

FIG. 7–2.—An ideal rectifier connects a resistive load to the source of alternating voltage during the half cycles when A is positive with respect to B.

switch; he now observes the ammeter, which shows current in the permissible direction. The current flows in the permissible direction as long as C is positive (above B). With the switch closed, points A and C are connected, and the resistance is therefore simply placed across the source. Current consequently flows as long as A is positive with respect to B, *i.e.*, during the half cycle when A is up. Now consider the instant when A comes back from its excursion into the positive (upper) region. When it reaches the ground level, the current will be zero, and after passing through the zero level, the current through the resistor would reverse, if the switch remained closed. The watchman goes into action and opens the switch, and points A and C, which had kept company while A was above the ground level, now part; A goes down into the basement while C remains at the street level. The maximum voltage appearing across the open switch will clearly be the amplitude of the alternating voltage. It will be reached when point A is "way down" while C is at the ground level.

The potential of terminal C with respect to B consists therefore of a series of half waves, as shown in Fig. 7–2*b*. The maximum current flowing will be equal to E_{max}/R. As shown in Eq. (3–3), the average current of one *half cycle* is given by

$$I_{av} = \frac{2}{\pi} I_{max} = 0.638 I_{max} \tag{3-3}$$

For a half-wave rectified current, every alternate half cycle is missing; consequently, the average value of such a current will be only one-half of the value given by Eq. (3–3). The current registered on a dc meter by a half-wave rectified current is therefore given by

$$I_{dc} = \frac{1}{\pi} \times \text{amplitude of half wave} \qquad \text{(half-wave rectified current)} \qquad (7\text{–}1)$$

This is an important relation, and the reader will do well to memorize it.

In the case of a purely resistive load, we see that our watchman closes and opens the switch at the instants when the supply voltage passes through zero. We shall see that this is not true for other types of load.

7–5. Ideal Rectifier with Capacitive Load.[2]—In Fig. 7–3 the load consists of a capacitor. With zero charge on the capacitor, its two terminals

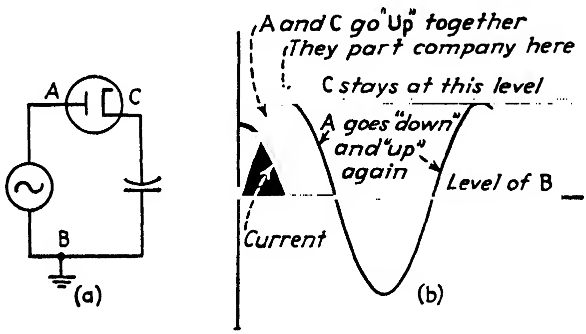

FIG. 7–3.—An ideal rectifier connects a capacitive load to the source of alternating voltage only during the first quarter cycle, when point A is swinging positive with respect to B. After this, current will no longer flow.

are at the same potential, *i.e.*, point C is at the level of point B. Let an alternating voltage appear across the ac generator and consider the instant when A becomes positive with respect to B. The rectifier will close the circuit, as explained before, and will keep it closed as long as current flows from left to right. Ohm's law for the capacitor stated that current flows through a capacitor only as long as the voltage is changing; since the level of A is going up, this condition is fulfilled. The current will have a maximum value when the voltage is changing fastest, which is obviously at the instant when A is just starting to go up. The reader should refer to Sec. 4–6 because, with the rectifier representing a closed switch during the first quarter cycle, the phenomena taking place during this part of the cycle are not any different than if there were a jumper in place of the rectifier. As shown in Chap. IV, the current through the capacitor will lead the voltage by 90 deg. When A has reached the maximum level, there will be an instant when it does not change, which means that the current through the capacitor and rectifier is zero. Point A then starts its journey downward;

its voltage will be decreasing, and if our rectifier keeps the circuit closed, point C would also go down. The current through the capacitor would then reverse. That this is true becomes even clearer when we think of the capacitor as a storage battery. When the voltage of the charging generator becomes less than the voltage to which the battery has already been charged, it begins to discharge into the generator. With the rectifier in the circuit, this reversal cannot take place. The switch will be opened, and the capacitor will remain charged to the peak value of the alternating voltage. Points A and C are parting company, C remaining up while A goes down to the zero level and beyond it. The maximum voltage that a voltmeter connected across the switch will indicate is obviously equal to twice the amplitude of the alternating voltage. This voltage will occur at the instant when point A is at its lowest level (way down in the basement) while C is way up. It is called the "inverse" voltage, to which the rectifier (or switch) is subjected. The foregoing analysis shows that the inverse voltage is twice as high in the case of a capacitive load as in the case of a resistive load.

Our rectifier switch closes the circuit again when A becomes positive with respect to C. But since C has been left at a level equal to the amplitude of the voltage of point A, this will not happen until A has reached its maximum again, and then for only a theoretically infinitely short time interval. Therefore, no current will flow through the rectifier after the capacitor has been initially charged to the peak of the applied alternating voltage.

A dc meter connected across the capacitor—provided that it is an instrument measuring the voltage without drawing any current from it—will show a voltage equal to the peak of the alternating voltage. Note that this is π times the value that a voltmeter connected across the resistive load shown in Fig. 7–2 would show.

7–6. Parallel Combination of Resistor and Capacitor.[2]—Before we pass on to the third circuit element, the inductance, the very important combination of a capacitor and a resistor in parallel should be considered. In practical applications of this combination the resistor is usually the load in which direct current is desired, and the placing of a capacitor parallel to it will not only increase the direct voltage across this load compared to that given by Eq. (7–1) but will also smooth it out considerably. Common sense alone tells us this, because a parallel combination of resistance and capacitance can be expected to perform somewhere between the performance of the two individual elements. In an R circuit, we had half waves of current (and voltage of course) with an average value equal to $1/\pi$ times the amplitude; with a pure C load, we had an absolutely smooth direct voltage equal to the amplitude of the alternating voltage. The performance of the combination will fall somewhere between these two extremes.

Figure 7–4*a* shows a load with the resistance R shunted by the capacitor C. During the first positive quarter of a cycle the alternating voltage will then have to furnish over the rectifier not only the charging current of the capacitor, as in Fig. 7–3, but also a current through the resistor. Consider the instant when the alternating voltage has reached its maximum. At this moment the current through the resistance R will have reached a maximum while the current through the capacitor will be zero because the alternating voltage is not changing at this instant, as shown in Fig. 7–4*b*.

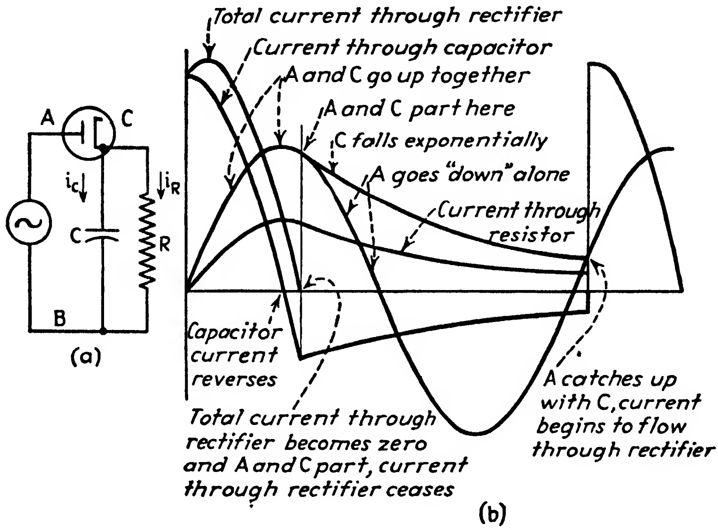

FIG. 7–4.—When the load consists of a parallel combination of resistance and capacitance the capacitor will act like a storage battery being charged once every cycle during a relatively small portion of the half cycle when A is positive with respect to B. It then will maintain the flow through the load resistance R during the time when A is less positive than C.

Now consider the period just following this instant. The alternating voltage begins to fall, at first slowly and then more rapidly. The capacitor current reverses its direction and therefore now helps the source of the alternating voltage to furnish current into the load. A moment will obviously be reached when the rate of change of the alternating voltage is such that the current furnished by the capacitor is equal to the current carried by the load. A little thought shows that this is the case at the instant when the total current taken by a parallel combination of resistance R and capacitance C from an ac source directly connected to the combination, *i.e.*, without rectifier, passes through zero. At this instant anode and cathode of the diode will part company, the anode falling away faster than the cathode; from then on the voltage across the capacitor will have nothing more to do with the alternating voltage. We shall then be dealing simply with a capacitor discharging through a resistor, a process treated in detail in Chap. II. The voltage across the capacitor will drop accord-

ing to an exponential curve until the anode, after having passed through the negative half cycle, has become sufficiently positive to be at the same level as the cathode. From then on the capacitor is charged again to the maximum value. If the current drawn by the load from the capacitor is small, then the variations of the capacitor voltage will be small also, and the average voltage across the load will be only a few volts less than the peak of the alternating voltage. On the other hand, if the load is draining the capacitor quickly, the voltage fluctuation will be very much more severe. The capacitor is acting like a storage device, receiving an electric charge during part of the cycle and keeping up the current flow through the load while the alternating voltage is low or negative. Since the capacitor neither creates nor consumes electrical charges, it is evident that all the direct current flowing in the load must have come through the rectifier. This means that the rectifier, while it is passing current, must carry a much larger current than that indicated by a dc meter in series with the load. Thus, if a dc meter in series with a load indicates an average value of 50 ma, for instance, and if an analysis shows that conduction through the tube is confined to one-quarter of the total time, it is quite obvious that the average current through the tube *while it is conducting* must be 200 ma, and the peak may be even much higher than this value. A more detailed analysis of these relations will be made in connection with filter circuits.

7-7. Pure Inductance in Series with Half-wave Rectifier.[2]—We shall now investigate the performance of a circuit containing a pure inductance in series with a rectifier. It may be well to warn the reader that this combination is considerably harder to understand than the performance of a resistive or capacitive load. Careful attention to the following considerations is therefore urged. An understanding of the underlying principle is absolutely essential for a later understanding of gaseous-tube circuits.

Figure 7–5*a* shows a pure inductance in series with a perfect rectifier connected to a source of alternating voltage. The fundamental law of the inductance states that the application of a voltage to it will cause the current to change (*i.e.*, the inductance "defends" itself against the application of voltage by taking a *changing* current). With a direct voltage applied to it the current will keep on changing in one direction. With an alternating voltage applied to it over a rectifier, one might reason as follows. The rectifier converts the alternating voltage into a pulsating direct voltage; consequently, there must be applied to the inductance a voltage having a dc component, and the current will therefore keep on increasing, not at the same instantaneous rate, to be sure, but always in the same direction. Such reasoning is erroneous in the case of a half-wave rectifier, as the following more detailed analysis will show. Figure 7–5*b* shows the voltage applied to the inductance. When A becomes positive, the rectifier closes the circuit, which applies voltage to the inductance. The

inductance takes a current changing at such a rate that at any instant Ohm's law for the inductance [see Eq. (2-4)] is satisfied. As long as the voltage is positive, the current will change in the same direction, *i.e.*, it will increase. At the instant, for example, when the applied voltage is a maximum, the current will be increasing at the highest rate (note the difference between the behavior of an inductance and that of a resistance!), but the current will *keep on increasing* even while the voltage is *decreasing* again, as long as the voltage is *positive*.

Now consider the instant when point A (and with it point C, since our watchman has kept the switch closed) has reached the zero level again.

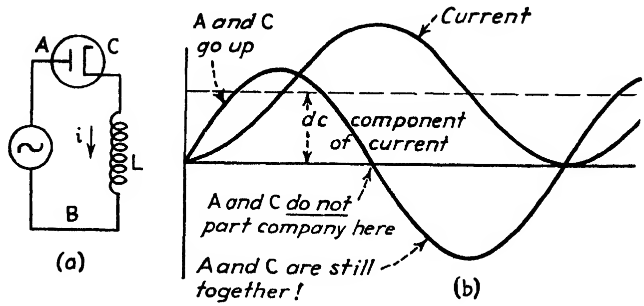

FIG. 7-5.—When an alternating voltage is connected to a pure inductance over an ideal half-wave rectifier, current will be flowing all the time. The dc component of this current will be equal to the amplitude of the alternating current that would flow through the inductance if no rectifier were present.

The current has reached a maximum value at this instant and flows in such a direction that our watchman sees no reason for opening the switch. Point A is now becoming negative with respect to B. Will the watchman open the switch or not? He has orders to keep it closed as long as current is flowing in the right direction, and we have only to ask ourselves whether with the switch closed the current would reverse when the voltage becomes negative. Application of a negative voltage to the inductance means that the current will change in a direction opposite to that when the voltage was positive. Since it was a maximum at the instant when the voltage was zero, it will now begin to decrease, *but it will not reverse its direction.* The watchman has therefore no reason to open the switch, in spite of the fact that the supply voltage has become negative. Since the positive and the negative half waves of the voltage are alike, the current will decrease during the negative half wave in exactly the same manner as it increased during the positive half wave. The startling result of this analysis is therefore that the switch is *never opened,* the rectifier is carrying current during the *full cycle,* and points A and C are always connected. But if the inductance is connected to the source of alternating voltage at all times, then the voltage applied to it has no dc component, which is the reason why our suspicion that the current might increase indefinitely is not correct.

We have something else to explain, however, before everything is in agreement. If our rectifier watchman never opens the switch, then the rectifier could just as well be entirely absent from the circuit. Is the current shown in Fig. 7–5*b* actually the graph of the current that would flow if we connect an alternating voltage directly to an inductance? Did we not learn that the application of an alternating voltage to an inductance simply produces an alternating current? The current shown in Fig. 7–5*b* certainly has a dc component. Where did it come from? A complete answer to this question cannot be given without entering the subject of transients, which would be beyond the scope of this book. Let those who are familiar with them be reminded that the current that flows when an alternating voltage is applied to an inductance consists of a steady-state current, and a transient. This transient depends on the instant of the cycle when the circuit is closed. The transient will be a maximum when the circuit is closed at the instant when the voltage is zero, which is exactly the moment when our watchman closed the switch for the first time. If the inductance has zero resistance, the transient will never die out. In Fig. 7–5*b* *this transient is the dc component.* With this in mind we can now make a final statement. When an alternating voltage is applied to the series combination of an inductance and a half-wave rectifier, an alternating current will flow equal in magnitude to the current that would flow without the rectifier in the circuit, plus a direct current equal to the amplitude of the alternating current.

7–8. Inductance, Resistance, and Ideal Rectifier in Series.[2]—If the inductance has resistance or if it is placed in series with a resistive load, as shown in Fig. 7–6*a*, an exact analysis can be made only with the aid of the

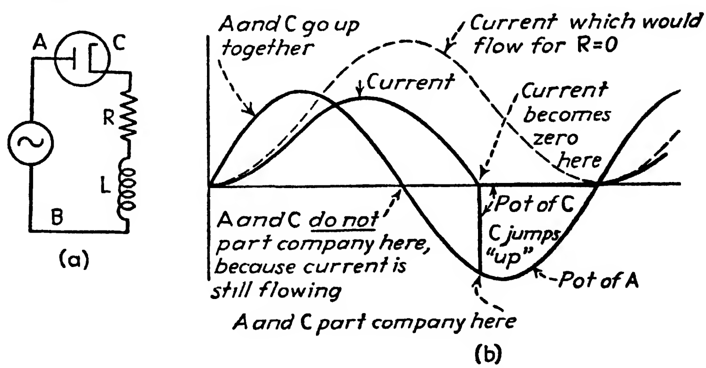

FIG. 7–6.—When the load consists of a series connection of resistance and inductance, the current will consist of disconnected loops, its dc component will, of course, be smaller than it would be if either the inductance or the resistance were alone in the circuit.

methods used in the analysis of transients. The few remarks that follow should give at least some qualitative understanding. In the case of a pure inductance, the full voltage of the source was at all times applied to the

inductance, causing the current to rise as shown in Fig. 7-5*b*. If there is resistance in series with the inductance, the voltage appearing across the resistance due to the current must also be supplied from the source. This leaves less voltage across the inductance, which means that the rate of rise of current must at all times be less than without the resistance. In Fig. 7-5*b* the current reached its maximum when the voltage of the source was just zero; with resistance in series, the current will already be decreasing at the instant of zero voltage because whatever current may be flowing in the resistance at this instant can be supplied only by virtue of a voltage produced by the inductance, *i.e.*, at the expense of the energy stored in the magnetic field of the latter. The current therefore reaches zero before a whole cycle is completed, but reversal will, of course, be impossible with the rectifier included in the circuit. The important conclusion to be drawn from this analysis is that the direct current flowing in a series combination of resistance (load) and inductance in a half-wave rectifier circuit cannot be more than the amplitude of the alternating current that would flow if the inductance alone were connected directly to the source of alternating voltage, nor can it, of course, be more than the resistance alone would draw. Furthermore, far from "smoothing" the current, the latter will consist of a series of disconnected loops, no matter how large the inductance.

The voltage across the series combination of inductance and resistance deserves particular attention. As long as current is flowing, there will be no voltage across the rectifier; consequently, the cathode will have at any instant the same voltage with respect to the reference point as the anode. But at the instant when the current has reached zero the voltage across the resistance is, of course, zero, and since the diode prevents the current from changing beyond the zero point, the voltage across the inductive part also becomes zero at that instant so that the voltage across the combination will suddenly become zero. As considerations of this kind become very important in the discussion of grid-controlled gaseous-tube circuits, the reader will do well to give the last statement careful attention.

7-9. Modifications Due to Deviation from Ideal Performance.—This chapter has dealt so far with the performance of an ideal rectifier. It is now appropriate to investigate the deviations of the actual performance of practical rectifiers. The first imperfection to be expected would be that the rectifier permits current in the reversed direction, in other words, that its volt-ampere characteristic extends into the third quadrant. Copper oxide and other dry-type rectifiers show a current in the reversed direction, which, although small, is not always negligible. Tube-type rectifiers, on the other hand, are for most practical purposes perfect in this respect. As long as rated voltage is not exceeded in the reverse direction, the current in this direction can be considered as negligible.

The other condition laid down for an ideal rectifier in Sec. 7-3 was that it should pass current "freely," *i.e.*, without requiring any voltage, in the desired direction. No known rectifier is perfect in this respect, but in many applications the voltage required to drive the current through it in the conducting direction is of no consequence. This may best be illustrated by considering an ordinary switch. A switch is a circuit element supposed to permit current to flow freely after closure. A small toggle switch may have a resistance of 0.1 ohm; with a current of 3 amp (their usual rating) flowing, a voltage of 0.3 volt will appear across it. If this switch is used in a 110-volt circuit, the load, instead of receiving 110 volts, has only 109.7 volts applied to it. This deviation is evidently so small that the load circuit performs practically in the same way, as with a perfect switch or, expressed differently, the switch gives a performance practically equal to that of a perfect switch. But let this same switch be used in a circuit where it has to operate a 3-amp lamp from a 1.5-volt dry cell. This time the switch will cause a voltage reduction of approximately 20 per cent on the lamp (compared to about 0.27 per cent in the preceding case), which certainly is far from the perfect switch letting current pass "freely." It is still the same switch, however! In the same way a rectifier requiring perhaps 10 or 15 volts to drive rated current through it will give results practically equal to that of an ideal rectifier (as discussed in the preceding sections), if used in a circuit operating from 1,000 or 2,000 volts. Its performance will differ radically from that of an ideal rectifier if the voltage of the source is 20 or even 50 volts. It is the engineer's task to analyze a given application and to decide whether performance predictions based on ideal conditions are sufficiently accurate.

7-10. Approximations Used for Actual Rectifiers.—When a rectifier is used in a circuit of low voltage so that it is not permissible to neglect the voltage required to drive the current through it, it is often possible to take this effect into account by substituting for the actual rectifier an ideal one and another circuit element in series with it. Suppose that the voltage required to drive the current through a rectifier is approximately proportional to the current, in which case the volt-ampere characteristic will be as shown in Fig. 7-7. It is obvious that an ideal rectifier and a resistance in series with the rectifier would have an identical characteristic; therefore we can take account of this characteristic simply by adding such a resistance in series with the load and then proceeding with the analysis as if the rectifier were an ideal one.

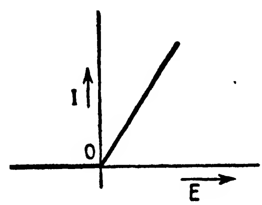

FIG. 7-7.—Volt-ampere characteristic of a rectifier having resistance in the direction of current flow.

If the rectifier happens to be a gaseous tube or a mercury-arc rectifier, its characteristics are not even approximately represented by Fig. 7-7.

As explained in Sec. 6-17, practically no current will flow until the voltage across the tube has reached a certain value; when this point has been reached, the tube breaks down or "fires," and from then on the voltage does not increase any more with an increase in current. The characteristic is therefore of the shape as shown in Fig. 7-8. The voltage which must be reached before conduction begins, and which then remains constant regardless of the amount of current, is usually in the order of 10 to 20 volts. Now a little thought on this matter shows that the current flowing in a load supplied through such a rectifier will be the same as if the rectifier were perfect but with a constant voltage (such as a storage battery) of 10 to 20 volts (whatever the operating voltage of the particular tube happens to be) placed in series with the load. It is left to the reader to convince himself of the truth of this statement.

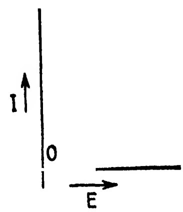

FIG. 7-8 —Volt-ampere characteristics of a rectifier that does not conduct any current until the voltage has reached a certain value, but which from then on will require the same voltage no matter how large the current flowing through it.

We saw that whenever the alternating voltages to be rectified and, consequently, the direct voltages appearing across the load are large compared to the voltage appearing across the rectifying element during the time that it carries current, no appreciable error results either by considering the rectifier as an ideal one or, to be more accurate, by substituting an ideal rectifier plus a resistance or a constant voltage. But if the voltages to be rectified are small, say, of the order of 1 volt, then this method of analysis fails, even with the approximations discussed in this section. The problem can then be solved only by graphical methods, and even these are reasonably simple only in the case of a resistive load. Since similar graphical methods are to be used also for certain problems pertaining to amplifier tubes, the following example of a diode with a resistive load is discussed in detail.

7-11. Graphical Solution of Rectifier Problems.[1]—In Fig. 7-9*a* let a diode of the vacuum type be connected to a source of alternating voltage; the load consists of the meter *M* only, the resistance of which may be neglected. The characteristic of the diode is shown in Fig. 7-9*b*. Before we go on with the discussion, it may be well to call attention to one part of the characteristic. It is seen that with zero volts applied to the diode there is still a certain amount of current flowing, which will not cease until about 1.5 volts are applied to the diode in the opposite direction, *i.e.*, until the anode is made 1.5 volts negative with respect to the cathode. In order to be sure that there is no confusion about "applying zero volts" to the diode, let it be repeated here (see discussion of Figs. 5-4 and 5-5) that the only way to apply zero volts to a device is to short-circuit it and *not* to leave it open. To do the latter would only make sure that the cur-

rent is zero but not that the voltage across the terminals is zero. It may look rather surprising to find that short-circuiting a diode rectifier results in a current. The short circuit of a copper oxide rectifier certainly does not have a result like this. Where does the current come from? Have we found a way to get something for nothing? The explanation of this phenomenon is found in the initial speed of the electrons as they are being emitted from the cathode. To return to our rubber-sheet analogy, it is clear that if the marbles have a horizontal component of speed as they are ejected from the hopper onto the rubber sheet they will keep on traveling even if the sheet is stretched absolutely level (zero volts between

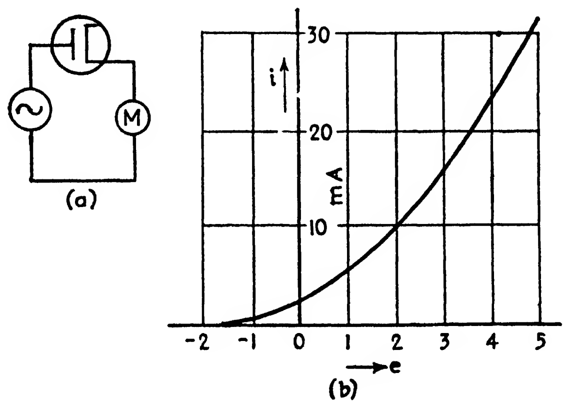

FIG. 7 9.—The volt-ampere characteristic of a typical vacuum-type diode.

anode and cathode). By raising the board representing the anode, however, fewer and fewer marbles will have enough momentum to run uphill, as it were, and to reach the anode. The fact that with the anode made about 1.5 volts negative the current will be zero tells us that none of the electrons emitted from the cathode has a speed in excess of 1.5 volts (see Sec. 6–10).

To come back to our problem shown in Fig. 7–9a, how much current would the meter M indicate if the alternating voltage has an amplitude of 5 volts, which would be equivalent to an rms value of approximately 3.53 volts for a sinusoidal wave shape? Knowing that the amplitude of the alternating voltage is 5 volts would permit us to calculate the instantaneous values at any time, and from the characteristic we can then obtain the current that is flowing at any given instant. A graphical method is indicated in Fig. 7–10. Since the instantaneous values of a sinusoidally varying alternating voltage can be considered as the projections of a vector with a length equal to the amplitude of the voltage, we draw a circle with a radius of 5 units (the same units as used for the voltage axis of the volt-ampere characteristic) with the intersection of the two axes of the volt-ampere characteristic as the center. We then divide the circle into a con-

venient number of parts, such as 24, an equivalent of 15 deg for each part. Choosing the instant when the alternating voltage passes through zero as the time $t = 0$, the circle is marked off as shown in this figure. Vertical

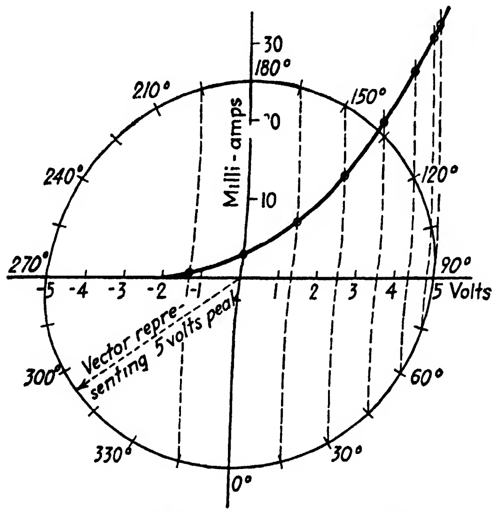

FIG. 7-10 —If the alternating voltage shown in Fig 7 9a has an amplitude of 5 volts, the drawing of a circle with this value as a radius into the volt-ampere characteristic of the diode permits us to determine the current that will flow at any instant

lines drawn through the selected points on the circle give at the intersection with the voltage axis the instantaneous values of the alternating voltage, and their intersection with the volt-ampere characteristic gives the value of current at any given instant. Plotting these current values results in the curve shown in Fig. 7-11. It we wish to find the current indicated by

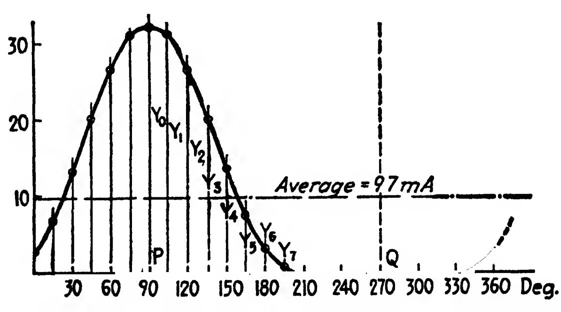

FIG. 7-11 —When the current values obtained from Fig 7-10 are plotted on a linear time scale, the curve shown here results Its average can be determined by the method discussed in connection with Fig 7 12.

a dc meter, we shall now have to average this curve. This means that we have to determine the area under the curve and then convert it into a rectangle of equal area stretching over a length equivalent to a full cycle

There are several methods available for the determination of the area of a curve, some of them more accurate than others. Since the characteristics of vacuum tubes vary from tube to tube as much as 10 per cent, there is obviously no justification in the use of highly accurate methods. The simplest one replaces the actual curve by straight lines drawn between ordinates placed at equal distances. Suppose that in Fig. 7-12 it is desired to obtain the area underneath a given curve between points A and B. Divide this region into n strips of equal width, resulting in the ordinates $Y_0, Y_1, \cdots Y_n$. (Note that the number of ordinates is one more than the number of strips.) By connecting the end points of each ordinate with the one following, the actual curve is replaced by a broken line. Each individual strip is a trapezoid with the width d and two sides formed by two successive ordinates. The area of the first strip, consequently, will be $d(Y_0 + Y_1)/2$; of the second strip, $d(Y_1 + Y_2)/2$; and of the last strip, $d(Y_{n-1} + Y_n)/2$. Adding all strips together results in the total area, given by

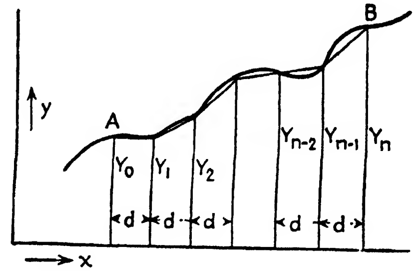

FIG. 7-12.—Determination of the average value of a curve between two ordinates Y_0 and Y_n by replacing the actual curve by a series of straight lines.

$$A = d\left(\frac{Y_0}{2} + Y_1 + Y_2 + \cdots Y_{n-1} + \frac{Y_n}{2}\right) \tag{7-2}$$

The average value of the curve can then be found by dividing this area by nd, which is nothing but the length of the base line. The value d is then seen to cancel out. With this method applied to Fig. 7-11, it becomes clear that it is sufficient to average the curve of one half cycle since the two branches are perfectly symmetrical with respect to each other. The half cycle chosen is located between points P and Q. This region is divided in 12 equal parts (half a cycle is equivalent to 180 deg, and each part was chosen as 15 deg). The first ordinate would be the one located at P, which is seen to be 32 ma. The last one is zero. From Eq. (7-2), the accompanying table results.

$\frac{Y_0}{2}$	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}	$\frac{Y_{12}}{2}$
16	31	26.5	20	13	7	3	0.5	0	0	0	0	0

Summing up these values results in a value of 117, and dividing this by the number of strips (in our case 12) gives us the average value of 9.7 ma.

It will be noted that in arriving at this value no assumption was made with regard to the frequency of the alternating current. We can therefore expect the reading to be independent of the frequency of the ac source. It is consequently possible to calibrate such a device with a 60-cycle voltage, easily obtainable and measurable, and use the arrangement thus calibrated for the measurement at higher frequencies. More will be said about this subject in connection with vacuum-tube voltmeters. The wave shape of the current would be sinusoidal only if the volt-ampere characteristic of the diode were a straight line. This is not the case in this particular problem, and the wave shape is therefore seen to differ from a sine wave.

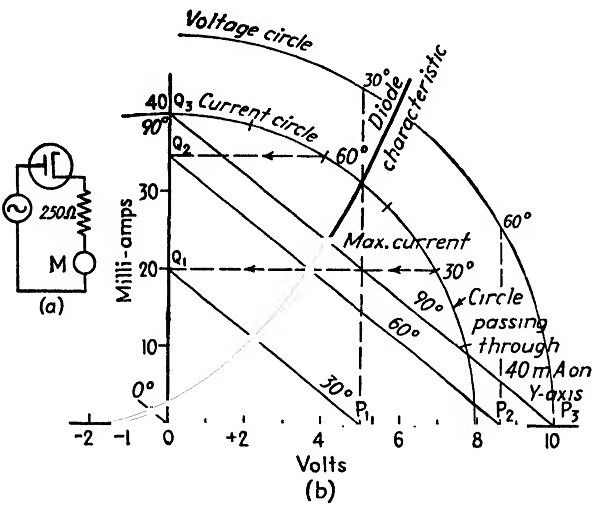

FIG. 7-13.—When there is a load in series with the meter, the instantaneous values of the current may be determined by the method of the load line.

In the example just treated, there was no load in series with the diode except if one wishes to consider the indicating meter as a load. Although the condition described above does arise occasionally when the only purpose of the circuit is to measure or indicate the alternating voltage, more commonly a load is placed in series with the diode and the direct current is supposed to perform work in this load. In Fig. 7-13*a* let a voltage with an amplitude of 10 volts be applied to a resistor of 250 ohms and the same rectifying device discussed in the preceding example. We have now a condition where, at any given instant, the voltage across the diode plus the voltage across the resistor must add up to the value which the alternating voltage has at this instant. In Sec. 5-13 it was shown how to find the current in a series combination of a Mazda lamp and a resistor. This was done by drawing the load line, which was nothing but a fancy name for the characteristic of the resistor, drawn in reverse (*i.e.*, from right to left). The Mazda lamp has now become a diode, and the voltage does not

remain at a fixed value. (In Fig. 5–13, the voltage was 140 volts but this does not change the principle one iota.) It is only necessary to draw the load line (*i.e.*, the characteristic of the 250-ohm resistor) for a number of instantaneous values of the alternating voltage (preferably spaced at equal time intervals). The intersections of these lines give us the instantaneous currents, and the average value can then be found in exactly the same way as shown in Fig. 7–11. Thus, at the instant when the alternating voltage has reached its maximum value of 10 volts, the load line drawn through the 10-volt point on the voltage axis and the 40-ma point on the current axis (convince yourself that this is the characteristic of a 250-ohm resistor, plotted from right to left, with the zero point of the graph located at the 10-volt point of the diode characteristic) intersects the diode characteristic at 23.5 ma, as shown in Fig. 7–13*b*. This procedure is repeated for the 30- and 60-deg points, when the instantaneous voltage would be 5 and 8.66 volts (10 × sine 30 and 10 × sine 60, respectively), but it is left to the reader to carry the problem through to its full solution. The average or direct current is found to be 7.4 ma.

7–12. Merging of Accurate and Approximate Methods for Larger Voltages.—In Fig. 7–13*b* the lengths OP_1, OP_2, and OP_3 represented the instantaneous values of the voltage at 30, 60, and 90 deg, respectively. Since the load lines drawn through these points are all parallel (they represent the same resistor), the lengths OQ_1, OQ_2, and OQ_3, cut off on the current axis, are in the same ratio as the voltages. They could therefore be considered as the instantaneous values of an alternating current with an amplitude of 40 ma. Evidently, they would also represent the value that the current would have if the diode were short-circuited.

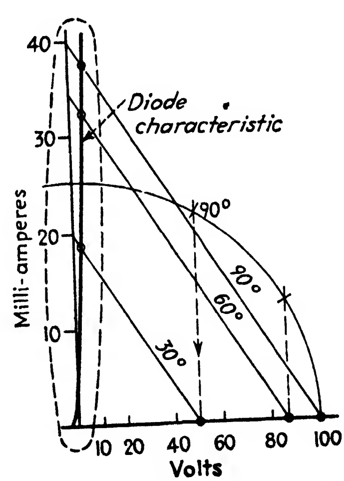

Fig. 7–14.—When the alternating voltage is high compared to the voltage across the diode, the intersection of the load lines with the characteristics is seen to give almost the same current values as the intersection with the current axis.

Suppose now that the alternating voltage is raised to 100 volts amplitude and that the load resistor is increased from 250 to 2,500 ohms. If we now want to apply the graphical method for the determination of the current at every instant, we shall have to draw a circle with a 100-volt radius, which will obviously be inconvenient, if not impossible, if we keep the same scale for the voltage as in Figs. 7–10 and 7–13. There are two ways of solving the problem. We can compress the voltage scale so that 100 volts amplitude can be drawn, as shown in Fig. 7–14. The characteristic of the diode will then be quite close to the Y axis of the

graph, and the intersection of the various load lines will give almost the same values as the intersection with the Y axis itself. Since the Y axis itself can be considered as the characteristic of an ideal rectifier, our contention is corroborated that for this voltage a performance prediction based on the assumption of an ideal rectifier will not cause any serious error. The second method avoids the necessity of redrawing the diode characteristic to a smaller voltage scale but makes use of the fact that the load lines intersect the current axis at points in the same ratio as the voltage values, as explained in the preceding paragraph. In other words, we

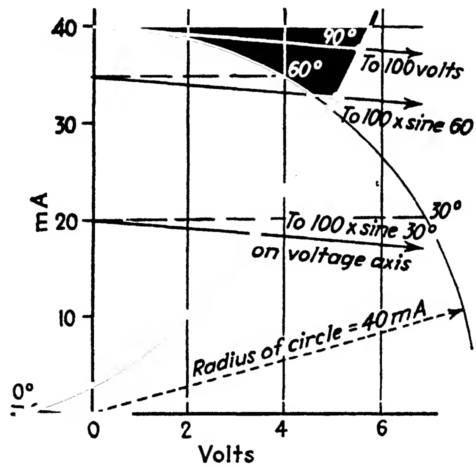

FIG. 7–15 —Another method of obtaining accurate values of current if the alternating voltage is large compared to the voltage remaining across the diode.

draw the load lines from points on the current axis instead of the voltage axis, using a slope equal to that of a 2,500-ohm resistor. This results in Fig. 7–15, which is seen to be simply an enlargement of the small region encircled in Fig. 7–14. We recognize again that the current values do not differ significantly from those of an ideal rectifier.

In most power circuits the rectifier is operating under a condition where the rectified voltage is large and where the assumption of ideal performance does not lead to serious errors. In communication circuits, on the other hand, the alternating voltages may be so small that they cover only a part of the characteristic shown in Fig. 7–9*b*.

PROBLEMS

7–1. A sinusoidally varying alternating voltage with an rms value of 115 volts is connected to a series combination of a perfect rectifier and a 500-ohm resistor.

a. What is the peak value of the current flowing in the resistor?
b. What current would a dc meter connected in series with the resistor show?
c. If the resistor is rated at 10 watts, will it be overloaded?

d. What will a dc voltmeter read when connected across the rectifier? Should its plus or its minus terminal be connected to the anode?

7-2. A capacitor of 5 μf and a perfect rectifier are connected to a sinusoidally varying alternating voltage with an rms value of 200 volts at the instant when the voltage passes through zero. Plot the current flowing through the rectifier. The frequency of the alternating voltage is 60 cps.

7-3. A capacitor of 20 μf already charged to 300 volts is connected in series with a rectifier (cathode of rectifier to plus terminal of capacitor) to a sinusoidal alternating voltage of 350 volts rms, 60 cps. Assume the circuit to be closed at the instant when the alternating voltage passes through zero. When will conduction begin, and what is the maximum current that will flow?

7-4. A type-83 rectifier tube with a tube drop of 15 volts is used for rectification with a transformer having a voltage of 350 volts rms, 60 cps, sinusoidal. The tube furnishes the current for a parallel combination of a resistor and a capacitor of 16 μf. It has been ascertained that conduction begins 18 deg before the transformer voltage reaches its peak. What will the peak charging current be?

7-5. An inductance of 3 henrys is connected in series with a rectifier to an alternating voltage of 115 volts rms, 60 cps. If its resistance is negligible, what direct current will flow?

7-6. Figure 7-16 shows the characteristics of a diode and the circuit in which it is to be used. Determine the direct current that will flow in this circuit.

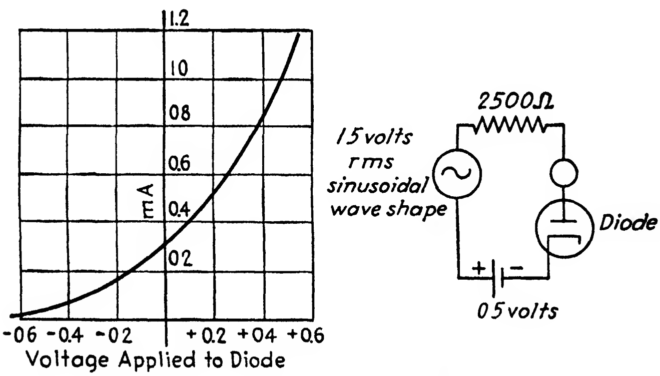

FIG. 7-16.—Characteristics and circuit diagram for Prob. 7-6.

SUGGESTED ADDITIONAL READING

Cobine, J. D.: "Gaseous Conductors," Chap. 12, pp. 472-486, McGraw-Hill Book Company, Inc., New York, 1941.

Eastman, A. V.: "Fundamentals of Vacuum Tubes," Chap. 4, pp. 35-52, McGraw-Hill Book Company, Inc., New York, 1937.

Millman, J., and S. Seely: "Electronics," Chap. 12, pp. 362-378, McGraw-Hill Book Company, Inc., New York, 1941.

M.I.T. Staff: "Applied Electronics," Chap. 6, pp. 247-307, John Wiley & Sons, Inc., New York, 1940.

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1. Panofsky, W. K. H., and C. F. Robinson: Graphical Solution of Rectifier Circuits, *Electronics*, April, 1941, p. 42.
2. Wallis, C. M.: Half Wave Gas Rectifier Circuits, *Electronics*, October, 1938, p. 12.

CHAPTER VIII

FULL-WAVE RECTIFIER CIRCUITS; FILTER CIRCUITS FOR RECTIFIERS

8-1. Practical Application of Rectifier Analysis.—In Chap. VII we investigated the performance of a single rectifier when connected to various circuit elements; in the present chapter we shall apply the results of these investigations to the analysis of practical rectifier circuits. The electronic engineer requires direct voltages for the operation of most of his amplifiers and other electronic devices. Since the use of batteries is not desirable, except, of course, in portable equipment where the question of maintenance and replacement is subordinated to the demand for portability, it is usually necessary to obtain the required direct operating voltages by rectifying the alternating voltage commonly available. Since transformers offer a convenient means to step up or down the available alternating voltage, the combination of transformer and rectifier is very flexible in providing direct voltages over a wide range. Combinations converting the alternating voltage into a direct voltage for the operation of electronic circuits are called "power supplies."

8-2. Components of Power Supplies.—A power supply must furnish direct current to a given load. The circuits discussed in Chap. VII were capable of doing this, but a practical power supply usually consists of many more components than simply a source of alternating current and a rectifier. We saw that a half-wave rectifier connected to a resistance load caused a current consisting of a series of alternate half waves of sinusoidal shape. Such a pulsating current might be perfectly satisfactory for the charging of a battery, for instance, but for the operation of an amplifier it is absolutely unusable. The pulsating voltage must therefore be "smoothed" out before it can be used for the operation of electronic devices. To what degree this smoothing out has to be carried depends entirely on the particular application. For the operation of an oscillator used for induction heating, for instance, practically no smoothing is necessary; if the same oscillator is to be used in connection with broadcasting, the smoothing must be quite thorough. The voltage furnished by a dc generator, one might think, should certainly be good enough for any purpose requiring direct current. Let it be said here that, owing to the commutator segments and slots in the armature of such a generator, the voltage furnished by it is far from satisfactory for the operation of certain electronic equipment, especially high-gain amplifiers. This statement gives an idea as to

the extent to which the direct voltage furnished by a power supply may have to be smoothed. The components in the power supply accomplishing this smoothing—or expressed differently, the components that remove the "ripple" from the direct voltage—are usually referred to as "filter elements" or "filter circuits." How much or how little filtering has to be done depends entirely on the particular application.

As far as the rectifier itself is concerned, any filtering elements are simply a part of the load. In Chap. VII it was shown that a capacitor would charge to the peak value of the alternating voltage and remain at this voltage if no other circuit element was parallel to it. The voltage across it is, therefore, a pure direct voltage without any ripple. The capacitor may be considered as a filter element instead of a load, and this will be the view taken in the present chapter. For the analysis of the circuit operation, however, it should be clear that the filter elements are simply part of the load, as far as the rectifier is concerned.

8-3. Full-wave Rectifier Circuit.—The first important step toward a smoother direct voltage is the use of a full-wave arrangement in place of

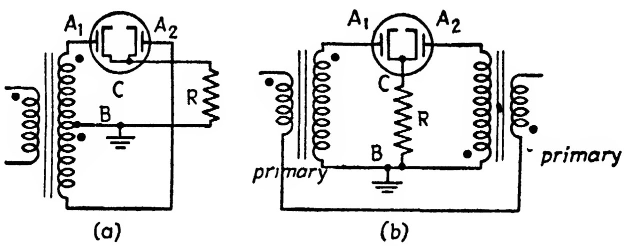


FIG. 8-1.—A full-wave rectifier may be considered as two half-wave rectifiers supplied from two sources of alternating voltage 180 deg out of phase with each other.

the half-wave circuits analyzed in Chap. VII. The fundamental full-wave circuit supplying a purely resistive load is shown in Fig. 8-1*a*. It is seen that this circuit could be considered as two independent half-wave rectifiers feeding into the same load. The cathodes of the two rectifiers are connected to each other; as a matter of fact, the arrangement most commonly found is one where both rectifiers are in the same glass envelope, with either one cathode serving both anodes, or at least only one connection brought out for both cathodes. The center-tapped secondary winding of the transformer could be considered as two windings furnishing two voltages 180 deg out of phase. If we consider point B as the reference point, it is clear that at the instant when A_1 is positive, A_2 will be negative by the same amount (equal number of turns assumed on each side of the center tap).

8-4. Two-phase and Biphase System.—Recently the term "biphase" has been applied to this arrangement and seems to be a very good choice. We speak of a six-phase system when we have six alternating voltages

reaching their positive maximum voltage one-sixth of a cycle, or 60 deg apart; of a four-phase system when dealing with four alternating voltages reaching their maximum one-fourth of a cycle, or 90 deg apart; of a three-phase system when the voltages are one-third of a cycle, or 120 deg apart. Logically, a two-phase system should be one where the two voltages are one-half of a cycle, or 180 deg apart. Unfortunately, the term "two-phase" is applied to a system having two voltages 90 deg apart, which is really a four-phase system with two phases left out. The best compromise seems therefore to accept the term "biphase" for two voltages 180 deg apart, although the most satisfactory solution is probably to apply the term "two-phase" to the 180-deg displaced system and the term "biphase" for the pruned four-phase system.

8-5. Indication of Transformer Polarity.—Since a center-tapped transformer could be considered as two separate sources of alternating voltages, with one terminal from each of the two sources connected, the circuit shown in Fig. 8-1*a* could also be represented as in Fig. 8-1*b*. This representation will not be used in this book, but the reader should realize that every center-tapped transformer could be considered in this manner. If these two separate sources are connected so that the voltage of point A_1 with respect to the common terminal B is 180 deg phase-displaced against the voltage of point A_2, then at an instant when A_1 is up or above the level of B, A_2 will be down or below B. The two diagrams show for the first time in this book a method (recently introduced) of letting the reader know about the polarity of transformer windings. The method consists of placing a dot on one end of each winding, or each section of a winding. The dots indicate that the terminals next to which they are placed all have the same instantaneous polarity with respect to the unmarked terminal. At this point, the reader may not realize the importance of properly marking terminals. Let it be said that time spent on trying to understand this scheme will be amply repaid when we shall have to analyze grid-controlled gaseous-tube circuits. The transformer shown in Fig. 8-1*a* has a total of three windings: the primary, a secondary between terminals A_1 and B, and a secondary between terminals A_2 and B. The three dots indicate that at an instant when the dotted terminal of the primary is, for instance, positive with respect to the undotted terminal, A_1 will be positive with respect to B (above B) and B will be positive with respect to (above) A_2. The last statement may, of course, be expressed by saying that A_2 will be below B.

8-6. Average and Rms Value of Direct Current; Peak Inverse Voltage of Full-wave Rectifier with Resistance Load.—If the two rectifiers are ideal, the current will consist of consecutive half waves of sinusoidal wave shape. It will be remembered that, in the half-wave circuit, anode and cathode parted company when they had reached the zero level and when the anode was beginning its "downward" half cycle. The same holds true

for the full-wave circuit as far as that particular anode is concerned. But owing to the 180-deg phase displacement, the second anode is just coming up when the first is going down, and this second anode then takes the common cathode on another upward half cycle. Each of the two transformer sections carries a half wave of current, entirely unaware of the fact that the cathode is not waiting faithfully at the zero level (as it did in the case of the half-wave circuit), until the anode connected to that particular transformer section is coming up again, and that it takes an upward excursion with the anode connected to the other section. The current flowing in the load has therefore the shape shown in Fig. 8–2. A dc meter

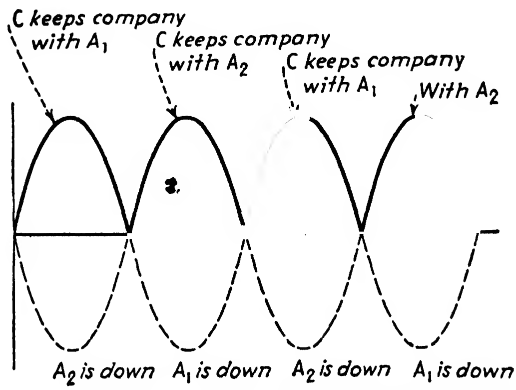


FIG. 8–2.—The common cathode of a full-wave rectifier follows the two anodes alternately during their positive swing.

would read $2/\pi$ times the amplitude of the half waves. The amplitude of the half waves of current is evidently equal simply to the amplitude of the alternating voltage divided by the load resistance (for an ideal rectifier). The rms value of the current, *i.e.*, its heating value, on the other hand, is exactly the same as that of an alternating current of the same amplitude, *i.e.*, it will be 0.707 times the amplitude. Using the indications of a dc meter for calculating the wattage that such a rectified current would produce in a resistor would therefore lead to an error in excess of 20 per cent.

During the time that one section of the full-wave rectifier is inactive, the anode of that section is negative with respect to the reference point (center tap of the transformer), but its cathode, instead of being at the potential of the reference point, as was the case in half-wave rectification, is now positive with respect to the reference point. The "spread" between the cathode and anode of the inactive section is seen to reach a value equal to twice the amplitude of the voltage of one of the transformer sections. Consequently, in full-wave rectifier circuits with resistance load, each rectifying section will be subjected to a peak inverse voltage twice as high as will be the case if the same diode is used in a half-wave circuit with resistance load.

The peak current of the full-wave and the half-wave arrangement will be the same for the same value of load resistance, but the average value will, of course, be twice as high in the full-wave arrangement as in the half-wave circuit.

8–7. Full-wave Rectifier with Capacitor Load.—If the resistor in Fig. 8–1 is replaced by a capacitor, the circuit shown in Fig. 8–3 results. The reader should have no difficulty in proving to himself that this circuit is in no way superior to the half-wave circuit shown in Fig. 7–3. We saw that in the half-wave circuit the capacitor was charged to the peak value of the alternating voltage and remained at this value after the first quarter cycle. It is clearly superfluous to have a second source of alternating voltage with the same peak value trying to accomplish the same thing. This indicates the possible field of application for half-wave circuits. Wherever a capacitor must be charged to the desired direct voltage and the current drain from it is very small, a half-wave circuit is just as satisfactory and a great deal simpler than a full-wave circuit.

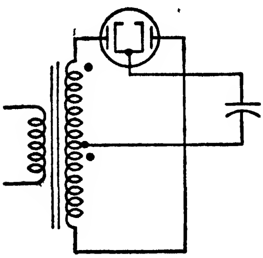


FIG. 8–3.—If the load consists of a pure capacity, the full-wave circuit has no advantage over the half-wave circuit; in both cases the capacitor simply becomes charged to the peak voltage of the alternating voltage.

8–8. Full-wave Circuit with Capacitor and Resistor as Load.—When the rectifier is supplying a parallel combination of a resistor and a capacitor, the situation is somewhat different. Figure 7–4*b* showed how the voltage across the combination decreased according to an exponential curve until the anode had caught up again with the cathode. If there should be another anode coming up while the anode shown in Fig. 7–4*b* was down, cathode *C* in this figure would not fall so far as shown there but would be yanked up twice as often as with the half-wave rectifier. This is indicated in Fig. 8–4, where *a* shows the diagram and *b* the potentials and currents in the circuit. For two ideal rectifiers, the peak of the direct voltage is equal to the peak of the alternating voltage, just as in the case of the half-wave rectifier but, owing to the fact that recharging to this value occurs twice every cycle instead of only once, the average value of the direct voltage is higher in the case of the full-wave circuit. The less the current in the load resistor, however, the nearer will the circuit perform to the condition outlined in Sec. 8–7.

As was stated at the beginning of this chapter, it is the usual practice in the analysis of rectifier circuits to consider as "load" only the resistor or whatever is consuming energy (such as amplifier tubes) and to refer to the elements causing a smoothing of the load current as the "filter elements." It is clear that in the parallel combination of resistor and capacitor the latter is causing a considerable smoothing, and it is therefore often

referred to as the "filter capacitor." As will be seen later, smoothing can also be achieved by means of an inductance. Systems where the first element after the rectifier is a capacitor are said to have a "capacitor-input filter," while in the case where the current flowing through the rectifier must pass first through an inductance, they are referred to as "choke-input filters." The circuit shown in Fig. 8–4 is therefore the simplest case of a capacitor-input filter.

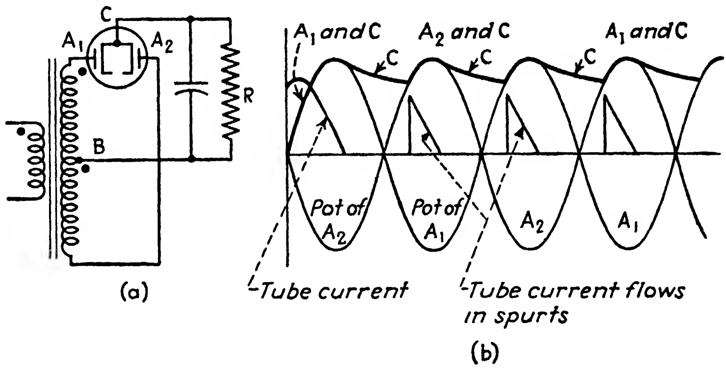


FIG 8 4—When the load consists of a parallel combination of resistance and capacity, the capacitor receives a charge twice during each cycle, and the voltage fluctuation across the load will therefore be less in this case than in the half-wave circuit.

8–9. Ripple Voltage on First Capacitor.[2]—The designer of a dc power supply naturally wants to know at least approximately the amount of voltage variation encountered across the filter capacitor. An examination of Figs. 7–4*b* and 8–4*b* makes it clear that the capacitor receives a charge every cycle or half cycle and then supplies the current taken by the load during that portion of the cycle (or half cycle), during which the alternating voltage supplied by the source has a lower instantaneous value than the capacitor voltage. It is therefore clear that the capacitor must supply the load current for a period less than a full cycle in the case of a half-wave circuit and for a period less than a half cycle in the case of full-wave circuit. When a charged capacitor, or a charged storage battery (there is no difference), supplies a load, its voltage decreases. If we can calculate how much the voltage across the capacitor decreases when it supplies the load current for a full cycle, or a full half cycle, we shall obtain a figure larger than the actual voltage variation since the ac source itself furnishes current to the load during part of the cycle. The wave shape of the actual voltage variation appearing across the filter capacitor of a full-wave rectifier circuit is shown in Fig. 8–5*a*. If, for the sake of simplified calculations, we are going to assume that the capacitor has to furnish the load current for a full half cycle, we are replacing the actual curve by the one shown in Fig. 8–5*b*. The capacitor is charged in an in-

finitely short time and is then supplying the load current for a full half cycle. As a further simplification, the exponential dropoff is replaced by a straight line. Physically this means that we consider the load current as constant. Before we develop the general formulas, an example will show how fundamentally simple the calculations really are. Let us assume that a radio receiver, or an amplifier, requires a direct current of 90 ma for its operation. Suppose that a full-wave rectifier circuit with capacitor-input filter is employed and that the first filter capacitor has a capacity of 30 μf. What voltage fluctuation will occur across the capacitor? If we word the problem as follows: "How much will the voltage across a charged 30-μf capacitor drop in $\frac{1}{120}$ sec, when it has to furnish a current of 90 ma?" we can give the answer immediately with the aid of nothing more than the fundamental law applying to the capacitor. Equation (2–9) told us the rate at which the voltage across a capacitor changes when current flows through it. In our case, the current is 90 ma, or 90×10^{-3} amp, while the capacitance is 30×10^{-6} farad; roc e, as given by the above equation, is therefore 3,000 volts per sec. Expressed in words, when a charged capacitor of 30 μf furnishes a current of 90 ma, the voltage will collapse at the rate of 3,000 volts per sec!

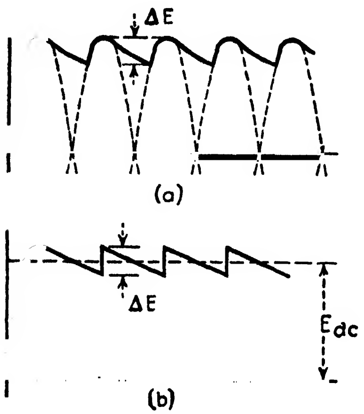


FIG. 8–5.—For the calculation of the voltage fluctuation, the actual wave as shown in (*a*) may be replaced by the saw-tooth wave shown in (*b*).

Fortunately, we do not intend to discharge the capacitor for more than $\frac{1}{120}$ sec, in which time the voltage will decrease by $3{,}000 \times \frac{1}{120} = 25$ volts. Note that it will not make any difference whether the first filter capacitor is followed by additional elements or whether it feeds directly in the load. Furthermore, this voltage variation occurs regardless of the operating voltage of the load, whether it is 1,000 volts or 250. The relative, or percentage, variation is, of course, less when 1,000 volts fluctuate by 25 volts than when 250 volts fluctuate by the same amount.

8–10. Calculation of Ripple Voltage.—We shall now turn to the development of the general formulas for the ripple voltage encountered in full-wave and half-wave capacitor-input circuits. Let the average direct voltage supplied to the load be E and the resistance of the load R. The direct current flowing in the load is then given by the equation:

$$I_{dc} = \frac{E}{R} \tag{8-1}$$

As shown in Chap. II, the rate of change of voltage across a capacitor with the current i flowing through the capacitor is

$$\text{roc}\ e = \frac{i}{C} \tag{2-9}$$

Introducing for i the value from Eq. (8-1), we find

$$\text{roc}\ E = \frac{E}{RC} \tag{8-2}$$

For a half-wave circuit the discharge lasts $1/f$ sec; consequently, the voltage variation ΔE across the capacitor (see Figs. 8-5*a* and 8-5*b*) is

$$\Delta E = \frac{E}{fRC} \qquad \text{(for half-wave circuit)} \tag{8-3}$$

For a full-wave circuit the period of discharge is $1/2f$, and the voltage variation is

$$\Delta E = \frac{E}{2fRC} \qquad \text{(for full-wave circuit)} \tag{8-4}$$

The relative voltage variation, *i.e.*, the ratio $\Delta E/E$, is given by the two equations:

$$\frac{\Delta E}{E} = \frac{1}{fRC} \qquad \text{(for half-wave rectifier)} \tag{8-5}$$

and

$$\frac{\Delta E}{E} = \frac{1}{2fRC} \qquad \text{(for full-wave rectifier)} \tag{8-6}$$

It will be noted that RC is the time constant of the load circuit and, in order to keep the variations low, obviously the time constant of the load circuit must necessarily be much larger than one cycle or one half cycle of the supply frequency.

It should be noted that ΔE is the *total* voltage variation, *i.e.*, the difference between the maximum and minimum voltages appearing across the capacitor.

The voltage across the capacitor can now be considered as consisting of a steady direct voltage on which is superimposed a saw-tooth wave with an amplitude equal to one-half of the total variation ΔE. Thus, if the voltage across the capacitor varies between a minimum value of 400 volts and a maximum value of 440 volts, the total variation will be 40 volts. The same minimum and maximum values would be obtained if, on a direct voltage of 420 volts, we would superimpose an alternating voltage with an amplitude of 20 volts. During one half cycle the two voltages would have the same polarity and give a maximum of 440 volts, while during the other half cycle the result would be 400 volts.

It has been agreed to express the waviness, or lack of smoothness, of a periodic current or voltage by the "ripple factor." This quantity is defined as

$$\gamma = \frac{\text{effective value of the alternating components of voltage (or current)}}{\text{dc or average value of the voltage (or current)}}$$

For a triangular wave, the rms value is equal to approximately 0.58 times the amplitude, and the ripple factor is consequently 0.29 times the relative variation given by Eqs. (8–5) and (8–6). It should be remembered that these considerations were carried through under the assumption that the capacitor would have to carry the load for a full or a half cycle, respectively. The actual values of the ripple factor will therefore be less than found by these equations. However, with so many factors, such as inductance of the transformer winding and voltage across the diode, influencing this value, figures calculated on the basis of Eqs. (8–5) and (8–6) are accurate enough for design purposes.

8–11. Definition of Load Resistance.—It may be desirable to say a word about the value of the load resistance R. Only on very rare occasions does the load on the dc side consist of a true resistance. In most electronic circuits the direct current is required for the operation of amplifier tubes. These tubes do not behave like a resistance. Depending on the type of tube, the current may reduce at a faster rate than the voltage does or it may remain constant until the voltage has dropped to a very low value. Under these circumstances, the drop of capacitor voltage will certainly not follow an exponential curve. This is another reason why too high an accuracy in these calaculations is a waste of effort and time. In order to make use of Eqs. (8–5) and (8–6), however, we must have a value for R. It will generally be found satisfactory to use simply a value of resistance which at the operating direct voltage will pass just as much current as the actual circuit. The error due to this assumption is obviously the smaller, the smaller the ripple itself is. If, on the other hand, the load current is so large that it would reduce the capacitor charge to a fraction of the maximum charge, then a graphical analysis would have to be made, taking into account the actual volt-ampere characteristics of the connected load.

8–12. Additional Filter Elements.[2]—Suppose, now, that the ripple as calculated from Eqs. (8–3) to (8–6) was found to exceed a permissible value for a given application. Enlarging the capacitor will, of course, be of help, but it will also make the peak current, which the tube has to carry, higher. Is there any other way of reducing the voltage fluctuation on this capacitor? As already stated, for a given value of capacitor and load current, nothing that we may do beyond the first capacitor will have any influence on the ripple appearing across it. The voltage existing across this capacitor was seen to consist of a direct voltage and a superimposed

alternating voltage of essentially triangular shape with an amplitude equal to one-half the voltage variation as calculated with the aid of Eqs. (8–3) and (8–4). If we can now interpose between this capacitor, which, after all, is the voltage source for the load, and the load, a circuit element with low resistance to direct current but high impedance to alternating current, we shall obviously improve the performance. Such an element is an inductance or choke. As far as direct current is concerned, it produces a loss of voltage determined only by the resistance of the winding, while the alternating

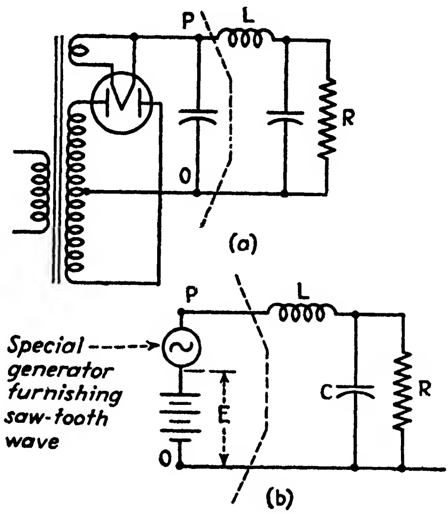


FIG. 8–6.—The ripple voltage across the load R may be reduced by the addition of an inductance and another capacitance as shown in (*a*). The calculation of the remaining ripple voltage may be carried out with the aid of the equivalent circuit shown in (*b*).

ripple voltage now feeds into a series combination of this choke and the resistance of the load. A further improvement may be obtained by placing an element across the load offering a low impedance to alternating current but a high resistance to direct current. Such an element is evidently another capacitor, and the circuit incorporating the changes just discussed is shown in Fig. 8–6*a*. In order to analyze this circuit, we replace it by the one shown in Fig. 8–6*b*. The first capacitor can be considered as the source of voltage for the following filter elements and the load. These should therefore perform in exactly the same way if we were to apply a voltage with the same wave shape as the voltage appearing across the first capacitor. This substitution is made in Fig. 8–6*b*. The first capacitor and the voltage existing across it are replaced by a dc source with a voltage equal to the average voltage existing across the capacitor, while the saw-tooth ripple of Fig. 8–5*b* is imagined as being produced by a special ac generator. These two voltages are now seen to act on the combination of the filter elements and load resistance. According to Helmholtz's

theorem, the current that will flow can be considered as the superposition of the individual currents flowing under the influence of each voltage with all others short-circuited. As far as the direct voltage is concerned, the current produced by it in the circuit with the alternating voltage short-circuited will simply be given by $E_{dc}/(R + R_L)$, where R_L is the dc resistance of the choke. The capacitor parallel to the load resistance will, of course, affect the time of build-up of the current but, as far as a steady state is concerned, it can be considered as absent. To the alternating voltage, on the other hand, the inductance presents a high impedance; the capacitor, a low one. In most filter circuits the capacitor across the load resistance is made so large that its capacitive reactance $1/2\pi fC$ is very much smaller than the load resistance. (This is usually indicated by the mathematical symbol $x_c \ll R$.) Under this condition, the alternating current produced by our fictitious saw-tooth generator, after flowing through the inductance, will flow mostly through the capacitor, with only a small part flowing through the load. Therefore, we shall not commit any serious error by considering the load resistance R absent as far as the alternating current is concerned. It is then evident that the saw-tooth generator simply feeds into a series combination of an inductance and a capacity, and if we know the alternating voltage furnished by the saw-tooth generator, we should be able to calculate the alternating current flowing in this combination. Multiplying the current with the capacitive reactance of the second capacitor will give us finally the alternating voltage that we may expect across this capacitor and therefore also across the load.

8–13. Calculation of Ripple Reduction.—As was shown in connection with Fig. 4–4, the voltages appearing across an inductance and a capacitance with alternating current passing through the two in series will be 180 deg out of phase. This means that the two voltages are of opposite polarity at every instant. The voltage appearing across the inductance will have an rms value of $x_L I$, where I is the rms value of the alternating current, while the voltage across the capacitor will be equal to $x_C I$. Owing to the 180-deg phase difference of these two voltages, the total voltage appearing across the two connected in series will then be $I(x_L - x_C)$. If the voltage is given instead of the current and we wish to find the latter, it is only necessary to transpose the equation and solve for I.

The terms "inductive reactance" and "capacitive reactance" have a meaning only if the alternating voltage is of sinusoidal wave shape and if we know its frequency. We know that the frequency of the alternating voltage appearing across the first filter capacitor is equal to the line frequency in the case of a half-wave rectifier and equal to twice the line frequency in the case of a full-wave rectifier, but we also know that this voltage is not of sinusoidal wave shape but looks rather like a saw-tooth wave. The questions must, therefore, be raised whether we can use for our calcu-

lation the rms value of the saw-tooth wave and, furthermore, whether the quantities x_L and x_C calculated with the appropriate frequency may be used in this calculation. As mentioned once before, an alternating voltage of any wave shape can be considered as consisting of sine waves of different frequencies. The lowest frequency, usually called the "fundamental," is equal to the frequency of the particular alternating voltage or current, while the other frequencies, called "harmonics," are multiples of the fundamental frequency. By means of an analysis known as "Fourier" analysis, it can be shown that the amplitude of the fundamental sine wave of a saw-tooth wave is equal to the amplitude of the saw-tooth wave multiplied by $2/\pi$. The amplitude of the saw-tooth wave appearing across the first filter capacitor was equal to one-half the *total* voltage variation appearing across this capacitor. The amount of this total voltage variation was given by Eqs. (8–3) and (8–4) for the half-wave and the full-wave circuits, respectively. Combining the relations just outlined, we obtain the amplitude of the fundamental sine wave of the ripple voltage for the full- and half-wave rectifier circuits when used on an alternating system with the frequency f.

$$e_{\max\ \mathrm{fr}} = \frac{E_{\mathrm{dc}}}{\pi fRC} \quad \text{(for half-wave circuit)} \qquad (8\text{–}7)$$

and

$$e_{\max\ \mathrm{fr}} = \frac{E_{\mathrm{dc}}}{2\pi fRC} \quad \text{(for full-wave circuit)} \qquad (8\text{–}8)$$

where $e_{\max\ \mathrm{fr}}$ is the amplitude of the fundamental sine wave of the ripple voltage, in this case the saw-tooth voltage appearing across the first filter capacitor, and E_{dc} the direct voltage appearing across the load. Since the reactance of the choke coil increases with frequency, while the reactance of the capacitor decreases with frequency, it is usually permissible to neglect the higher harmonics and design the filter circuits so that they will give a satisfactory reduction of ripple at the fundamental ripple frequency. The values x_L and x_C are therefore calculated with a frequency equal to the ripple frequency. The alternating current that flows through the circuit under the influence of the fundamental of the ripple voltage will then be given by

$$I_{\mathrm{fr}} = \frac{E_{\mathrm{fr}}}{x_L - x_C} \qquad (8\text{–}9)$$

where I_{fr} is the rms value of the fundamental sine wave of the ripple current, while E_{fr} is the fundamental sine wave of the ripple voltage. The latter is equal to 0.707 times the amplitude as found with the aid of Eqs. (8–7) and (8–8) for the half-wave and the full-wave circuit, respectively. The voltage across the capacitor is then equal to the current, as given by Eq. (8–9), multiplied by the capacitive reactance of the capacitor calcu-

lated with a frequency equal to or twice the line frequency, depending on whether we are dealing with a half-wave or a full-wave rectifier circuit. If we call the alternating voltage applied to the filter the input voltage and the voltage appearing across the load and capacitor the output voltage, it is seen that the ratio of input voltage to output voltage is given by

$$\alpha = \frac{x_L - x_C}{x_C} = \omega^2 LC - 1 \tag{8-10}$$

α is called the "smoothing factor."

If the ripple voltage thus reduced by a combination of inductance and capacitance is still too large for a particular application, a second section consisting of inductance and capacitance can be connected across the second capacitor C. Although the derivation of the exact equation for this case is rather complicated, an approximation, made under the assumption that each section thus formed reduces the voltage by a factor $1/\alpha$, does not lead to any appreciable error provided the inductive reactance is considerably larger (ten to twenty times) than the capacitive reactance. This is usually the prevailing condition in typical filter circuits.

Suppose that we have, for example, two sections, each consisting of a choke of 3 henrys and a capacitance of 10 μf and that these sections are to follow the first capacitor in a full-wave rectifier circuit operating from the 60-cycle line. The ripple frequency is then 120 cycles and the inductive reactance is $x_L = 2\pi \times 120 \times 3 = 2{,}260$ ohms. The capacitive reactance x_C is found to be 132.5 ohms. The smoothing factor is 2,127/132.5 = 16.1. Therefore, the reduction of the fundamental of the ripple voltage by one section would be approximately 16. An additional section would reduce this value once more in the ratio 16:1 so that the ratio of the input to the final output voltage would be 16^2 or approximately 260:1. Consequently, if the fundamental of the ripple voltage appearing across the first filter capacitor is, for instance, 13 volts, there will then appear across the filter capacitor of the second section only a voltage of 13/260, which is 50 mv.

8–14. Choke-input Filter.[1,3,4]—All the preceding considerations refer to the condition where the first element after the rectifier is a capacitor. As already stated, this type of filter circuit, regardless of whether it is followed by additional inductances and capacitances, is referred to as "capacitor-input filter." We shall now discuss the second possibility, *i.e.*, where the diode is followed by an inductance before going to the load or to any additional filter elements. The behavior of this type of filter is radically different from that of the capacitor-input filter.

In Sec. 7–7 we analyzed the performance of a pure inductance when connected in a half-wave rectifier circuit. We found—more or less to our surprise—that the current would not rise indefinitely even if the induct-

ance had no resistance whatsoever. The reason for this unexpected behavior was that the rectifier really never ceased conducting or, expressed differently, the inductance was at all times connected to the source of alternating current. We shall now proceed to investigate what will happen if a pure inductance is connected to a full-wave rectifier circuit as shown in Fig. 8–7*a*. While point A_1 is positive, current will flow through the anode connected to it, through the diode, and through the inductance. The current will build up during the first half cycle in exactly the same way as described in the half-wave rectifier circuit; but at the instant when

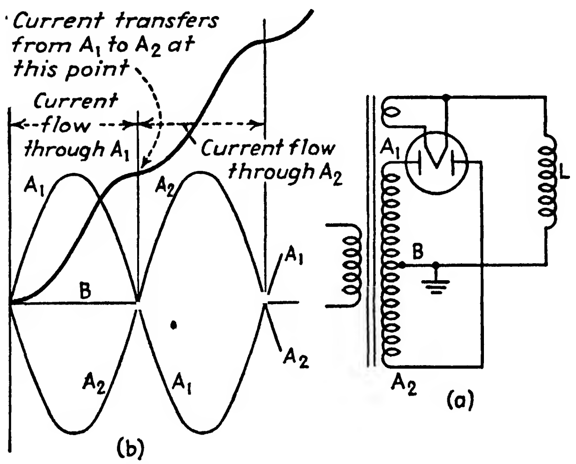


FIG. 8–7.—If the load of a full-wave rectifier were a pure inductance, the current would keep on rising until destruction of one component of the circuit took place.

A_1 becomes negative with respect to the reference point B, point A_2 will, of course, become positive; the current flowing in the circuit at this instant will therefore transfer from the anode A_1 to A_2. Up to this moment, the current was rising exactly in the same manner as was shown in Fig. 7–5*b* for the half-wave circuit but, although in the latter the current decreased from here on (because a *negative* voltage was applied to the inductance), another half wave of *positive* voltage is now applied to the inductance since the rectifier, acting as a high-speed switch, disconnects the inductance from the anode going negative and reconnects it to the anode just going positive. The current therefore increases during the next half cycle in the same manner as it did during the first half cycle. (Remember that a voltage applied to an inductance does not have anything to say about the actual amount of current but only about the *rate* at which it changes.) Obviously, during every additional half cycle the current will increase in exactly the same way, and we would therefore get a condition as shown in Fig. 8–7*b*. We have here, then, a condition similar to the one that we would have if a steady direct voltage were applied to an inductance.

In that case we know that the current would keep on rising forever until some other circuit element acted as a limiting device. The important part to recognize in this circuit is that current is flowing at all times, which is strictly different from the condition of capacitor input.

8-15. Fictitious Circuit for the Analysis of Choke-input Filter.—If there is resistance in series with the inductance, as shown in Fig. 8-8*a*, then quite obviously the current cannot rise indefinitely, but a steady state will be reached. Let us now attempt to find the current that flows under this condition. If the inductance is very large, it will take a long time to reach this steady state, but the variation from half cycle to half cycle will also be small since a large inductance prevents any significant change of current in as small an interval as a half cycle. This means that current will flow through the inductance and the resistance at all times, transferring suddenly from the anode connected to A_1 to the anode connected to A_2. But if this is true, then the inductance and the load are connected to one or the other section of the center-tapped transformer winding at all times. It is easy to see that circuit conditions are then exactly the same as if we had a single source of voltage furnishing nothing but positive half waves of voltage to the load and inductance, with the diode removed completely from the circuit. This is shown in Fig. 8-8*b*.

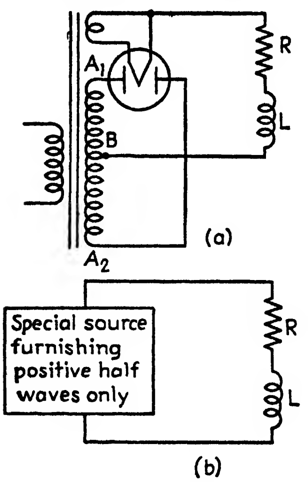


FIG. 8-8.—When the load consists of a series combination of resistance and inductance, current will be flowing at all times; circuit performance can be analyzed with the aid of the equivalent circuit shown in (*b*).

Such a condition is susceptible to mathematical treatment, with nothing more than the principle of superposition, ac theory, and Fourier analysis. The direct current that flows in the circuit will be determined only by the dc component of the voltage, which is $2/\pi$ times amplitude or simply the average of the alternating voltage of one section of the center-tapped transformer. This will also be the direct voltage that would be measured across the load resistance. This discussion brings out two fundamental differences between this type of filter circuit and the capacitor-input circuit. With the capacitor-input circuit, the direct voltage across the load reached values near the peak value of the alternating voltage, while with the choke-input circuit, we realize only approximately 63 per cent of this value. But on the credit side of the circuit, we find that the diodes, instead of having to pass high spurts of current for a short time, pass an almost steady current during their conduction period, with the current transferring from one to the other anode abruptly at the instant when the one anode becomes negative, the other one positive.

8–16. Voltage Regulation of Choke-input Filter.—An additional item on the credit side of the circuit is found in considering the voltage regulation, *i.e.*, the drop in voltage across the load with an increase in load current. With a capacitor-input filter, even under the assumption of a perfect rectifier and no loss in the transformer windings, we still observe a fall in voltage with an increase in load due to the drop of capacitor voltage when it has to furnish the load current. With a choke input, on the other hand, the characteristic is completely flat under the assumed ideal conditions. With actual tubes in place of ideal rectifiers, a choke-input filter will, of course, also show a certain amount of voltage regulation, but it is considerably less than that shown by capacitor input.

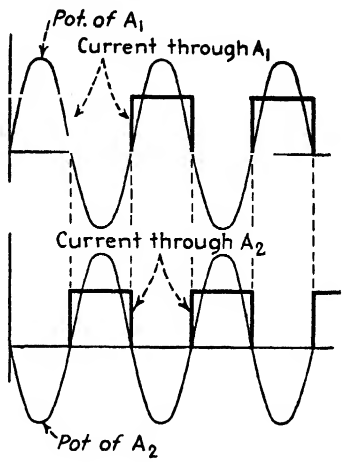


Fig. 8–9.—If the inductance of the circuit in Fig. 8–8 is very large, the current flowing in the load will be very nearly a true direct current, the currents through the individual rectifiers will be square waves.

8–17. Criterion for Choke- and Capacitor-input System.—In the foregoing analysis we assumed an inductance so large that it would keep the current at an almost steady value. Remember that a rectifier is nothing but a high-speed switch. If in the circuit shown in Fig. 8–8*a* the current is an almost steady direct current (where the inductance is large), then the current flowing in the two sections of the secondary transformer winding necessarily must be half waves of rectangular wave shape, as shown in Fig. 8–9. It would be impractical as well as uneconomical to make the inductance so large in actual circuits that the resulting current would be a steady direct current. In actual circuits there remains therefore usually an alternating current, and in order to minimize its effect, it is customary to place a capacitor across the load. If the reduction of ripple obtained by this means is not sufficient, it is, of course, again permissible to follow the capacitor by additional filter elements consisting of inductance and capacitance.

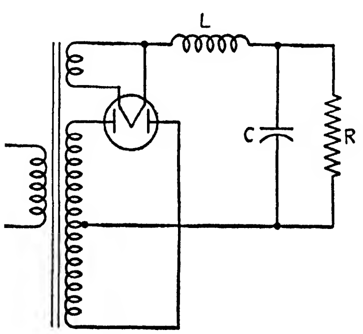


Fig. 8–10.—Further reduction of the ripple voltage may be obtained by placing a capacitor across the load

Figure 8–10 shows the fundamental choke-input filter, with an additional capacitor placed across the load, as explained in the preceding paragraph. Suppose now we were to decrease the inductance from the large

value assumed in the foregoing section. Obviously, when the inductance becomes zero, the circuit becomes a capacitor-input circuit and the current does not flow continuously but in spurts. It is therefore of interest to ask how large the inductance would have to be so that the current would never cease to flow in the circuit. This condition is really the criterion between capacitor and choke input.

8-18. Size of Choke Required for Continuous Current Flow.—So long as we insist on current flowing at all times, the substitution of the center-tapped transformer and the two rectifying elements by a single special source furnishing only positive half waves of voltage is permissible, as shown in the fictitious circuit in Fig. 8-8*b*. The current in this case will consist of a dc component given by the average or dc component of the voltage divided by the load resistance. The ac component will be due to an alternating voltage of a wave shape given by a succession of half waves of sinusoidal wave shape all facing the same way, as indicated in Fig. 8-11. If the negative peak of the ac component of the current in this fictitious circuit at any time reaches a value in excess of the dc component, then there will be a reversal of current in the fictitious circuit. (Remember there is no rectifier element in the fictitious circuit.) In the actual circuit the presence of the rectifying element prevents such a reversal of current, and the performances of actual and fictitious circuits are therefore no longer identical. Only if the inductance in the fictitious circuit shown in Fig. 8-11 is large enough to prevent the ac component from having an amplitude in excess of the direct current, can this circuit represent a permissible substitution for the actual rectifier circuit. Therefore, if we can calculate the size of the inductance in the fictitious circuit shown in Fig. 8-11 so that the above condition is fulfilled, we also have found the value of the inductance in the actual rectifier circuit if we wish to be sure that current is flowing through the rectifying elements at all times.

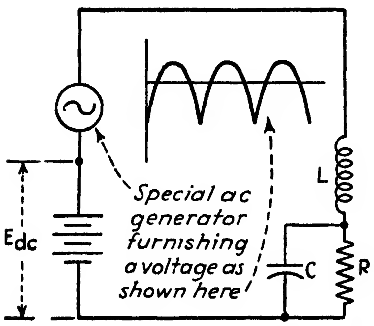


FIG. 8-11.—If the inductance is large enough to keep the current flowing at all times, the circuit shown in Fig. 8-10 may be analyzed with the aid of the equivalent circuit shown here.

The direct current that flows in the fictitious circuit shown in Fig. 8-11 will be, according to the superposition principle, equal to the direct voltage divided by the resistance. This direct voltage is evidently nothing but the average or dc value of a rectified alternating voltage of sinusoidal wave shape, given by Eq. (3-3). Therefore, if the amplitude of the voltage of one section of the center-tapped secondary winding of the

transformer is E_{max}, the direct current flowing in the load is given by

$$I_{dc} = \frac{2}{\pi}\frac{E_{max}}{R} \tag{8-11}$$

The alternating voltage acting in the fictitious circuit (Fig. 8-11) is not of truly sinusoidal wave shape but consists of positive half waves only. If a Fourier analysis is applied to a rectified sinusoidal wave, it is found that the amplitude of the fundamental sine wave of a rectified sine wave is equal to two-thirds the average or dc value. The frequency of this voltage is, of course, double the frequency of the source of alternating voltage. The amplitude of the fundamental sine wave of our fictitious voltage in Fig. 8-11 is, therefore,

$$e_{fr\ max} = \frac{2}{3}\frac{2}{\pi}E_{max} = \frac{4}{3\pi}E_{max} \tag{8-12}$$

The inductive reactance of the choke at the ripple frequency f is

$$x_L = 2\pi fL \tag{8-13}$$

Under the assumption that the capacitor across the load has a reactance very much smaller than the inductive reactance of the choke coil, the current will have an amplitude of

$$I_{max} = \frac{e_{fr\ max}}{x_L} = \frac{4E_{max}}{3\pi 2\pi fL} \tag{8-14}$$

If the current is never to have negative values, the maximum value as given by Eq. (8-14) must be smaller than the dc component as given by Eq. (8-11). By equating these values, we arrive at

$$\frac{2}{\pi}\frac{E_{max}}{R} = \frac{4E_{max}}{6\pi^2 fL}, \quad \text{or} \quad R = 3\pi fL \tag{8-15}$$

With a ripple frequency at 120 cycles, as would be encountered in 60-cycle full-wave rectifier circuits, the relation will therefore be

$$R = 1{,}132L \tag{8-16}$$

This value of L would just let the current drop to zero and therefore represents the minimum value. It is desirable to make the inductance somewhat larger and, for design purposes, the minimum value is usually taken as

$$L_{min} = \frac{R}{1{,}000} \quad \text{(in full-wave 60-cps circuits)} \tag{8-17}$$

where R is the total load resistance including the resistance of the choke.

8-19. Swinging Choke.—Equation (8-17) indicates that the minimum inductance of the choke is directly proportional to the load resistance.

This poses a problem if the load resistance of the circuit changes from one value to the other. When the load resistance is the highest, *i.e.*, when only a small amount of direct current is demanded from the rectifier, the choke should have a very high value; with an increase in load current, the value of the inductance could be reduced. It is a fortunate coincidence that inductances used as filter elements in rectifier circuits usually show a reduced inductance with an increase in direct current, owing to the fact that the core saturates when the current becomes large. Chokes that are expressly designed with the thought of reducing the inductance with an increase in direct current are called "swinging" chokes. Ordinarily one would think it satisfactory to design the choke with such a value of inductance that Eq. (8–17) would be fulfilled even under the condition of highest load resistance. It must not be forgotten, however, that a high value of inductance at this place affects the speed with which the whole circuit adjusts itself from one load condition to another. For this reason the use of a swinging choke is often dictated by circuit requirements more than by economy of material used for the construction of the choke. A further improvement can be obtained here by adding to the original section more stages, each consisting of an inductance and a capacitance. The same relations as those derived for capacitor input will hold true.

8–20. Comparison of Capacitor- and Choke-input Filters.—To summarize the distinguishing features and the advantages and disadvantages of the two systems, we can state that a capacitor-input filter circuit will deliver a higher voltage from the same center-tapped transformer than will a choke-input circuit. Where the current requirement is not severe, the economy in the transformer winding obtained with this type of filter circuit is the determining factor. But the voltage regulation of a filter with capacitor input is much greater than for choke input. Furthermore, the tubes are subjected to much more severe service in the case of a capacitor-input circuit since the current flows in very short spurts of high value. This also means that the heating of the transformer winding will be higher for the same amount of direct current than in the case of choke input. Choke-input systems are therefore usually more desirable, but economy sometimes dictates the use of a capacitor input.

8–21. Voltage-doubler Circuits.[5–7]—It is possible to obtain direct voltages exceeding the peak value of the available ac source by the use of so-called voltage-doubler, -tripler, and -quadrupler circuits. The simplest circuit accomplishing this result is shown in Fig. 8–12. This circuit, as well as complicated modifications of it, was first proposed by Greinacher in 1914. Examination of the circuit shows that it is nothing but the combination of two separate half-wave rectifier circuits, charging two capacitors during alternate half cycles. Consider terminal B of the transformer in Fig. 8–12 as the reference point. The anode A_1 of the one rectifier and the cathode C_2 of the other rectifier are both connected to the other ter-

minal of the transformer and are therefore swinging way up and way down together. During the excursion in the up direction, anode A_1 takes its cathode C_1 along, which charges the capacitor connected to it. If there is no load connected across the capacitor, it will retain this voltage, as explained in Sec. 7–5. During the excursion into the negative region, C_2 becomes negative with respect to A_2, current begins to flow (remember the watchman closing the switch!), and the capacitor connected to A_2 is charged. The capacitors are therefore both charged to the peak value of the alternating voltage. If a load is connected across the two, points C_1 and A_2 will, of course, not remain at the levels just discussed; Fig. 8–13 shows the voltage variations of the two points. The total output voltage is the vertical distance between the two curves. The ripple frequency of the voltage across each capacitor is seen to be equal to the line frequency since each capacitor receives a charge once during each cycle, but the ripple frequency of the total output voltage is equal to twice the line frequency because the series combination of the two capacitors considered as a whole receives a charge every half cycle.

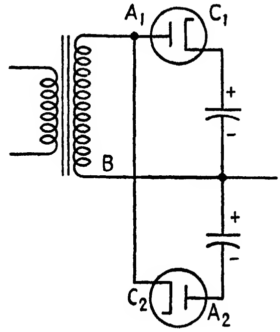


FIG. 8–12.—In a voltage-doubler circuit, two capacitors are charged during alternate half cycles of the alternating voltage.

With the system just described, two direct voltages are obtained, one positive and one negative with respect to one terminal of the ac source.

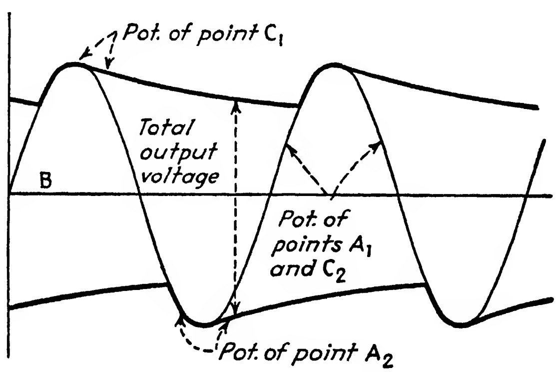


FIG. 8–13.—Potential variations of the various points of the voltage-doubler circuit shown in Fig. 8–12.

If the latter has a grounded terminal, it is therefore possible to obtain a positive and a negative voltage with respect to ground. As will be seen later, this is quite often a very desirable situation, but there are also cases where it is more desirable to have the full output voltage with respect to

ground. A voltage-doubler circuit giving such a voltage is shown in Fig. 8–14. The anode of the diode D_1 is connected to B, the reference or ground level. Its cathode, and therefore point G, can consequently never become negative with respect to ground. During the first excursion of terminal F of the transformer into the negative, or down region, capacitor C_1 will be charged to the peak value of the alternating voltage by the current flowing through D_1. This capacitor now represents a charged storage battery, and the potential of G with respect to B consists of the alternating voltage across the transformer plus the direct voltage across the charged capacitor. When F reaches its positive maximum, G will then be at a level twice this maximum. This voltage is now used to charge capacitor C_2 over the diode D_2; with no load, or very light load, there appears across C_2 a voltage equal to twice the amplitude of the alternating voltage. The action of the circuit can also be described by stating that, during the negative half cycle of the transformer voltage, C_1 is charged; during the positive half cycle, the voltage across the capacitor in series with the then positive voltage of the transformer is used to charge the second capacitor.

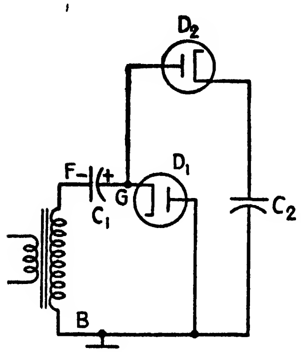
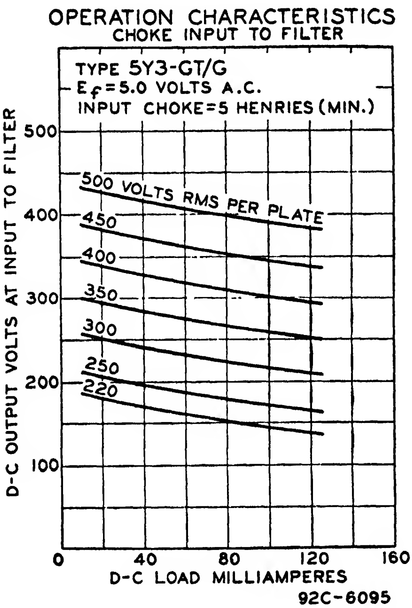


FIG. 8–14.—Voltage-doubler circuit having one terminal of the total output voltage common with one terminal of the alternating voltage.

There are many other rectifier circuits, especially the multiphase circuits, which space does not permit to discuss. It is hoped, however, that the methods of analysis presented in this chapter will aid the reader in analyzing such circuits himself.

8–22. Manufacturer's Data for the Design of Rectifier Circuits.—Before closing this chapter, it may be well to emphasize again that, in the analysis of rectifier circuits presented here, we assumed in most cases that the rectifying element was perfect. For a choke-input filter, for instance, such an assumption would mean that the alternating voltage of each section of the center-tapped winding of the transformer would have to furnish a voltage, the average value of which is equal to the desired direct voltage. In order to make up for the voltage lost across the rectifier, the choke, the resistance of the transformer winding, etc., the transformer must furnish an appreciably higher voltage. As the exact calculation is a rather difficult task, the manufacturer of rectifier tubes usually furnishes the desired information in the shape of a set of curves. Figure 8–15 shows, for instance, what can be expected from a 5Y3 rectifier tube, one of the most popular rectifier tubes used for radio receivers and amplifiers. If we wanted to design a rectifier circuit furnishing 300 volts dc at a load current of 80 ma, for instance, and decided to use choke input, the curves tell

us that we would have to use a transformer furnishing an rms voltage of approximately 390 volts to each side of the center tap (compared to a value of 333 volts if everything were perfect). With capacitor input on the other hand, the transformer would have to furnish only 290 volts each side of the center tap. The curves also indicate that the voltage regulation (*i.e.*, the change of voltage with a change of load) is much poorer in the case of



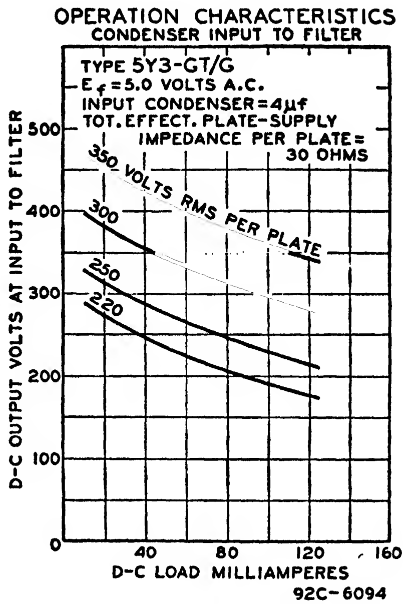


FIG. 8-15.—Operation characteristics of 5Y3 rectifier. (*Courtesy Radio Corporation of America.*)

the capacitor-input circuit. In order to limit the peak current encountered in the case of capacitor input, the manufacturer usually also specifies a minimum effective plate-supply impedance, a precaution not required in the case of choke input.

PROBLEMS

8-1. A 5Y3 rectifier tube (for data refer to Fig. 8-15) is to be used in a full-wave rectifier circuit, to furnish 60 ma at 250 volts dc. How high must the alternating voltage per plate be for

a. Choke input (choke of 5 henrys or larger)?
b. Capacitor input (capacitor 4 μf or larger)?

8-2. In the circuit shown in Fig. 8-16, assume a perfect rectifier and neglect transformer resistance and reactance. The load takes 50 ma. With the capacitor C_1 of 10 μf as shown,

a. What is the approximate ripple across C and the load?

b How much will the average voltage drop *at least* from the open-circuit value?
c What is the amplitude of the fundamental wave of the ripple voltage?

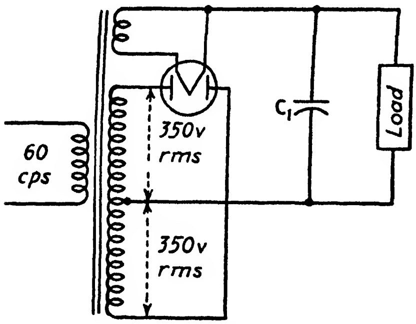


FIG 8-16 —Circuit diagram for Prob 8-2

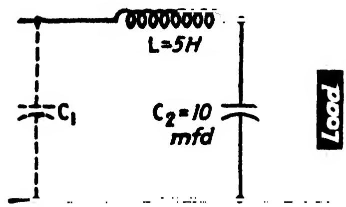


FIG 8-17 —Filter to be added to circuit shown in Fig 8 16 (see Prob 8-3)

8-3. To the circuit analyzed in Prob 8-2 is added another filter section between C_1 and the load, as indicated in Fig 8-17 Taking into consideration the fundamental wave of the ripple voltage only, what will the ripple voltage (amplitude or rms) now be across the load?

8-4. A dc power supply is to be built, using a transformer with a center-tapped secondary The total voltage of the secondary is 700 volts (or 350 volts each side of center tap) The load is equivalent to a resistance of 4,000 ohms, and a choke-input filter

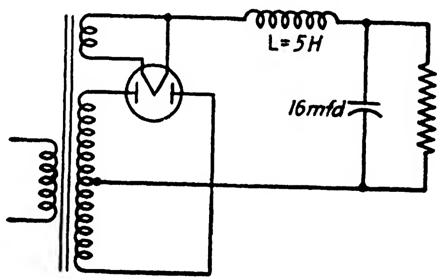


FIG 8-18 —Circuit diagram for Prob 8-4

system is to be used, with values as shown in Fig 8-18 Neglecting the transformer resistance as well as the tube drop, determine

a The direct output voltage across the load
b The ripple factor, expressed in per cent

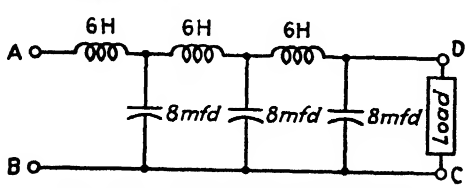


FIG. 8-19.—Filter circuit of Prob. 8-5.

8-5. If at the input terminals *AB* of the filter system shown in Fig. 8-19 a 120-cps voltage with an amplitude of 200 volts is applied, to what value will this be reduced at the load terminals *CD*?

8-6. Design a dc supply employing a voltage-doubler tube 25Z6, the characteristics of which are shown in Fig. 8-20, to supply 60 ma dc at 180 volts from an ac system of 117 volts rms.

a. What capacitor value must be used?
b. What is the ripple voltage, or the maximum variation of the output voltage?

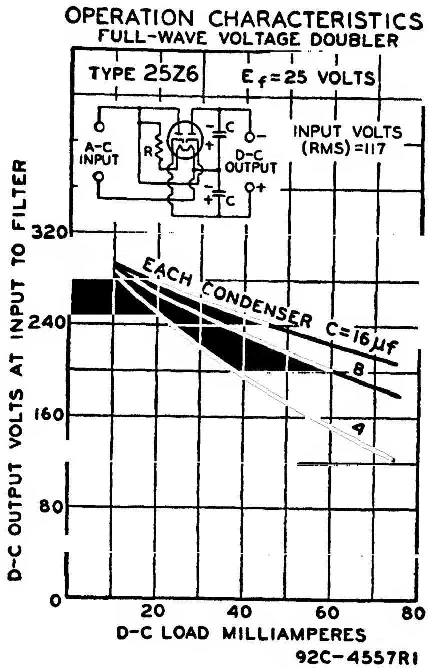


FIG. 8-20.—Operation characteristics of 25Z6 rectifier doubler. (*Courtesy Radio Corporation of America.*)

8-7. Applying the methods of analysis explained in this chapter to the circuit shown in Fig. 8-21, what will the dc meter read when 600 volts (rms) of sinusoidal wave shape is applied to the terminals *AB*?

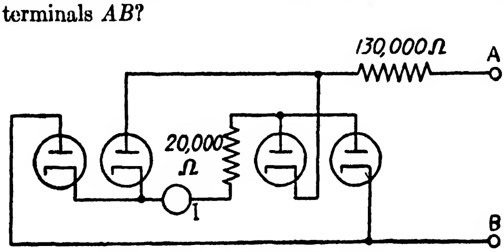


FIG. 8-21.—Circuit diagram for Prob. 8-7.

Can you give any good reason why the 20,000-ohm resistor should not be removed from the circuit, and the 130,000-ohm resistor increased to 150,000 ohms, without changing the circuit performance to the slightest degree?

SUGGESTED ADDITIONAL READING

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CHAPTER IX

THE TRIODE

9-1. Effect of Adding a Third Electrode to a Diode.—A diode, or rectifier tube, is unquestionably a very useful device because it provides a convenient means for the conversion of alternating current into direct current. The many amazing feats achieved by electronic means could not have been accomplished, however, without an additional invention of the greatest importance. In 1907, Lee De Forest placed a third electrode, called the "grid," between the cathode and anode of an ordinary diode. He found that by varying the voltage applied to this third electrode he could control the current flowing from anode to cathode. This control, as we shall see, can be obtained with the expenditure of an extremely small amount of electric power on the third electrode.

A tube with the two elements, anode and cathode, was called a "diode"; it is therefore quite logical to call a tube having a third electrode a "triode."

9-2. Sizes of Triodes; Cathode Construction.—Small triodes, such as are used in radio receiving sets, usually have indirectly heated cathodes; for larger tubes, operating with high voltages, the cathode is of the filamentary type and consists of pure tungsten or tantalum. The reason for this was given in Sec. 6-6. Triodes are built for the control of power as high as 200 kw but, although the construction of tubes designed to handle large amounts of power differs considerably from that of the smaller tubes, the same fundamental principles apply to both.

9-3. Reference Point for the Electrode Voltages of a Triode.—All voltages in a triode are given with respect to the cathode. This does not lead to any ambiguity in the case of indirectly heated or unipotential cathodes, as shown in Fig. 9-1, because every point of the cathode surface is at the

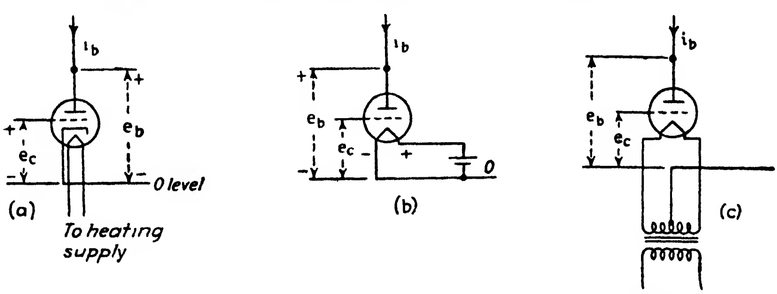

FIG. 9-1.—The level of the cathode is the reference point for all voltages in a triode, in the case of a filamentary type of cathode, the negative end of a filament serves as reference point.

same potential. In the case of a filamentary type of cathode, however, the heating current flowing through the filament produces a voltage, which means that every point along the cathode—which is at the same time the filament—is at a different level. For this case it has been agreed to consider the negative terminal of the filament as the reference point, as shown in Fig. 9–1*b*.

If alternating current is used for the heating of the filament, the transformer furnishing the heating current should have a center tap, or a center-tapped resistor should be placed across the filament. The center tap is then considered as the reference point to which all voltages are referred. This is shown in Fig. 9–1*c*.

9–4. Mechanical Analogy of the Triode.—Let us now see whether we can predict, at least qualitatively, what the introduction of the third electrode and the application of voltage to it may do to the current flowing between anode and cathode. A quick insight may be gained by going back again to a mechanical analogy. We saw that the mechanical equivalent of a diode was a rubber sheet stretched between a box from which marbles were ejected and a board lower than this emitting box. Can we extend this analogy to cover the case of the triode? Let us assume that cathode and anode are flat plates. With no third electrode present, the application of voltage to the two elements produces a uniform field between them, at least as long as no current is flowing (with the cathode cold, for instance), so that there would be no space-charge effects. A uniform field is equivalent to an incline of constant slope. Suppose that we now introduce between anode and cathode a grid of wires running in the vertical direction. If we impress on these wires a certain voltage with respect to the cathode, then the surfaces of these wires will be at this potential, regardless of the potential at which the other points of the interelectrode space might be. It is clear, however, that the wires will modify the whole potential distribution in this space. If they are spaced relatively far apart, the field distribution along a line between them might be relatively undisturbed from the condition when no grid is present. If they are very closely spaced, on the other hand, the voltage distribution, even along a line passing between the wires, will be affected by the voltage applied to the wires. In searching for an arrangement that would behave in a similar way on a rubber sheet, the solution indicated in Fig. 9–2 is found to give a very satisfactory picture. A rack with a number of pins is pushed against the rubber sheet from underneath. At the exact place of contact between the tops of the pins and the rubber sheet, the level of the sheet is obviously determined exactly by the amount the tops of the pins are pushed up with respect to the marble-emitting box. Furthermore, the level of every point on the rubber sheet is more or less affected by the deformation caused by the row of pins. Along a line between two pins, the rubber sheet will clearly have a saddle; if the pins are spaced

relatively far apart, the line through the saddle obviously deviates to only a small extent from the position it has with the pins absent (*i.e.*, an incline of constant slope). Now suppose we wanted to picture the region around the pins representing the grid wires in a manner similar to the way in

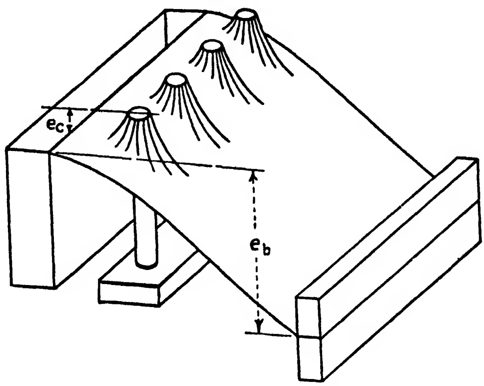

FIG. 9-2.—The modification of the electric field existing between anode and cathode in a triode can be visualized with the aid of this rubber-sheet model.

which a map is drawn. In order to do this, we draw lines of equal height or elevation on our rubber sheet, as shown in Fig. 9-3*a*, where the region around the pins is shown in perspective. These lines can then be drawn in a top view, as shown in (*b*), and marked with their respective levels. From such a picture it is easily seen whether a marble jumping from the emitting box onto the rubber sheet will have a chance to run down through the saddles. In the particular case shown in Fig. 9-3*b* this is obviously impossible because the line marked 2.5 indicates that a marble emitting

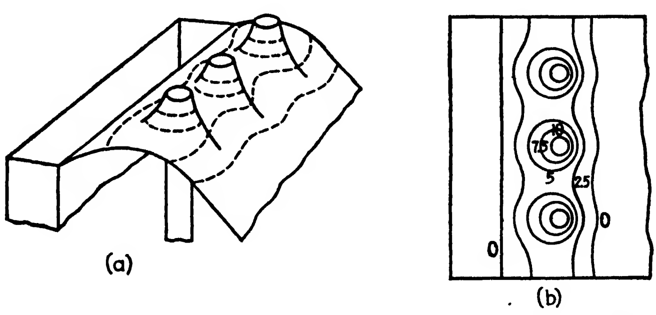

FIG. 9-3.—Equipotential lines in the electric field are equivalent to lines of equal height in the rubber-sheet model.

at the level zero finds no downward slope on any point along the cathode; or, expressed differently, the region between the saddles is everywhere higher than the rim of the emitting box. It is clear that the row of pins

representing the grid wires in this mechanical analogy can be lowered until a downward slope exists, at least at some points of the emitter. This means that marbles will begin to roll down between the saddles. This, then, is the picture we should form about the potential distribution between cathode and anode. In many books we find field pictures of various arrangements of electrodes within tubes. These pictures show the field lines and the equipotential lines in an electrical sense, but it is of great help if one considers these lines as the top view of a topographical map depicting mountains and valleys in a gravitational field.[2–5]

9-5. Weakness of Analogies.—As stated once before, analogies always have their weak points. In our circuit diagrams we decided to let positive be "up" and negative "down." Electrons in an electric field travel from the regions of negative potential to the region of positive potential, but marbles unfortunately fall from the upper region to the lower one. Our rubber model is not a circuit diagram, and in order to make it conform to the behavior of electrons, we have no choice other than to consider "up" as negative and "down" as positive. An attempt[1] has been made to rectify this weakness of the rubber-sheet model by turning it upside down, placing the whole model in a tank of water, and replacing the marbles by air bubbles issuing from a pipe with holes, to which compressed air is fed. The air bubbles will then travel along the underside of the rubber sheet in an upward direction. At first glance this seems to be a very satisfactory solution; its weakness is that an air bubble does not acquire kinetic energy or momentum, as an electron or marble does. As will be seen later, an analogy that is not capable of reproducing this condition is all but useless when it is applied to multigrid tubes.

9-6. Effect of Space Charge in the Triode.—As just shown, a rubber sheet stretched between the emitting box and the anode, deformed by the row of pins pressed against the sheet from below, represents the potential distribution in the interelectrode space. It should be kept in mind, however, that this distribution is valid only as long as there are no electrons in the interelectrode space, *i.e.*, as long as we do not have to consider space-charge effects. Just as in the diode, where the effect of the space charge caused the field near the cathode to be reduced to zero, so in the triode the space charge reduces the field near the cathode to zero. It is obvious, however, that it will take a larger space charge—*i.e.*, a larger number of electrons in flight—to reduce this field near the cathode to zero if it was of high value before emission started. It is therefore easy to see that the amount of current flowing between anode and cathode can be controlled by means of the voltage applied to the grid because that voltage (the amount by which the rack of pins is pushed up) decides the electric field (the incline) that would exist without space charge (the blower).

9-7. Grid Current and Power Required for Control.—There is another important phase about which we can learn from our rubber model. Strictly

speaking, we have to consider the pins pressed against the rubber sheet as tubes with the sheet glued to the edge of the tube and then cut away so as to leave an opening in the center. It is then evident that a marble might fall into the tube if it happened to reach the edge, but it is also clear that as long as the rack is pressed against the rubber sheet so far that the top edge of these tubes is at a higher level than the box from which the marbles emit there will be practically no chance of this happening. The electrical equivalent of marbles falling into the pins would be a current flowing to the control grid; the equivalent of the top edge of the pins being higher than the emitting box is the grid being negative with respect to the cathode. It is therefore seen that the amount of current between anode and cathode is subject to control by the voltage placed on the grid but that this control voltage applied to the grid does not have to furnish any current so long as the grid is negative with respect to the cathode. This statement will be modified by certain factors to be discussed later. At present we can assume that the control over the current is exercised with no expenditure of energy at the grid.

9–8. Relative Influence of Plate and Grid Voltages.—The field existing near the cathode before space-charge effects reduce it to zero (in other words, the field existing while the cathode is cold) is seen to be the determining factor as far as the current between anode and cathode is concerned. This field is equivalent to the incline existing next to the marble-emitting box in our analogy. It is quite clear that this slope will be determined by the difference of level existing between anode and cathode as well as by the amount that the grid pins have been pushed upward. Furthermore, the geometry of the whole arrangement is of the greatest influence. With the grid pins spaced close to each other and the whole row close to the cathode, it is evident that the field (or slope) will be determined mainly by the level of the row of grid pins; on the other hand, if the grid is spaced farther away from the cathode and if the individual grid wires themselves are spaced farther apart, the level of the anode will have a large influence. Thus, there are tubes which, with an anode voltage of 250 volts, may require the application of -90 or -100 volts to the grid in order to interrupt the anode current completely; with other tubes, it may require only -6 to -8 volts to obtain this result. In the former tube, the grid wires are spaced farther apart, while the latter has a closely spaced grid structure. Both types serve useful purposes, as we shall see later, but the actual electrode spacing and geometry are matters of interest to the tube designer only.

9–9. Transfer and Plate Characteristics of a Triode.—From the point of view of the application engineer, the triode is a device in which the current i_b flowing from anode to cathode depends not only on the voltage e_b applied to these two electrodes but also on the voltage e_c applied between cathode and grid. A mathematician would describe this state of affairs

by saying that the anode current is a function of the anode voltage and the grid voltage, and he would write it as follows:

$$i_b = f(e_c, e_b) \tag{9-1}$$

When there exists a relationship between *two* values, it can be shown most conveniently by means of a graph; thus we have shown the relation between the current and the voltage of a Mazda lamp in this manner. To portray the relationship between three variables, which is the problem we face in the case of a triode, it is necessary to use a three-dimensional graph. As this is a solution too cumbersome for practical use, it is therefore common practice to show the relation between three variables by so-called "families" of curves. Suppose that on a type 6C5 tube we had applied various plate and grid voltages and had obtained current values as given in the accompanying table, which permits us to determine the current for any

PLATE CURRENT OF A 6C5 TUBE IN MILLIAMPERES FOR VARIOUS PLATE AND GRID VOLTAGES

e_b	e_c						
	0	−2	−4	−6	−8	−10	−12
50	4	1					
100	9.2	5	1.9	0 3			
150	15	10.2	6	2.7	0 8	0 2	
200			11.4	7	3.6	1.5	0.5
250				12 6	8	4.7	2.3

value of grid voltage and plate voltage given. Thus for −8 volts on the grid and 200 volts on the plate, the tube passes a current of 3.6 ma. How can the information given in this table be converted into a graph? This can be done in two ways: we can convert either every horizontal row or every vertical column in a separate curve. Suppose we try the first. In this case we shall use the grid voltages as the abscissas and the plate currents as the ordinates and obtain a curve for every plate voltage given in the table. The result of plotting the values given in the table in this manner is shown in Fig. 9–4, and the curves so obtained are called the "transfer" or "mutual" characteristics of the tube. Each transfer characteristic is marked with the plate voltage with which it was taken. The reason for the term "mutual" is found in the fact that the current in one circuit, namely, the anode or plate circuit, is plotted as a function of the voltage applied to another circuit, namely, the grid circuit. (It will be remembered that we spoke of "mutual inductance" when we dealt with

the voltage produced in one circuit by a rate of change of current in another circuit.)

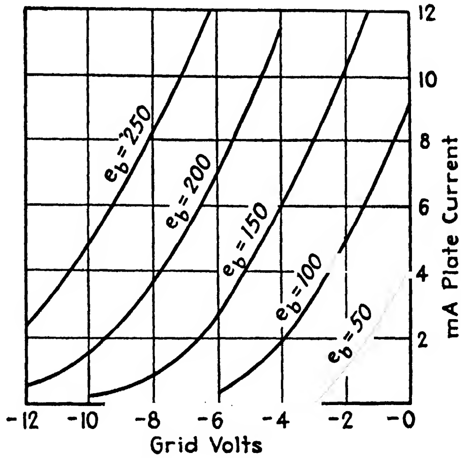

FIG. 9-4.—The relation between plate current, grid voltage, and plate voltage may be plotted in the manner shown here; these curves are called "mutual" or "transfer" characteristics.

The second way of plotting the figures given in the table consists of making a curve out of every vertical column. This means that the plate current is plotted as a function of the plate voltage. The results are

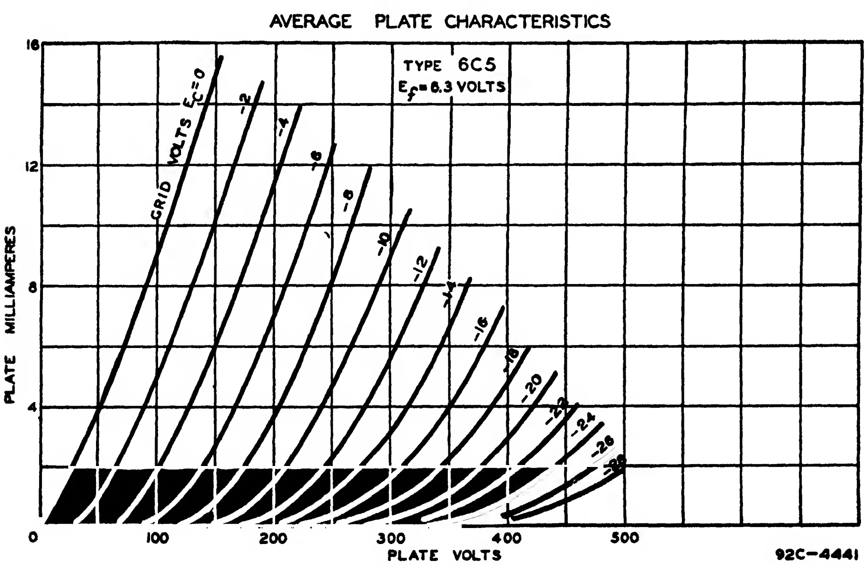

FIG. 9-5.—The information contained in Fig. 9-4 may also be presented in the shape of so-called "plate characteristics." (*Courtesy Radio Corporation of America.*)

shown in Fig. 9-5. Here the plate voltages are the abscissas and the currents are the ordinates. Each of the curves is marked with the grid volt-

age applied while the curve was taken. The family of curves thus obtained is called the "plate characteristics" of the tube.

It should be clear to the reader that both representations convey exactly identical information. If one set of curves is given, there is no difficulty in converting it into the other method of representation. If one asks, however, whether the transfer characteristics or the plate characteristics are the more desirable method of presenting the results of the experiment carried out on the tube, it is quite likely that most people would consider the transfer characteristic as the more important. For is it not true that the main function of a tube is to change the plate current when the grid voltage is changed? Under this condition, it would seem that a set of curves with grid voltages as the abscissas and the plate currents as ordinates would be the more desirable. For reasons to be seen later, however, plate characteristics are far more important, and it is only occasionally that transfer characteristics are still used.

9–10. Amplification Factor of a Tube.—Let us now consider the plate characteristics of a type 6C5 tube, as shown in Fig. 9–5. The graph shows that with 150 volts on the plate and -4 volts on the grid the current will be 6 ma. If we increase the negative voltage applied to the grid to -6 but keep the plate voltage unchanged, the current drops to 2.7 ma. If we wish to bring the current back to the original value of 6 ma, we do not necessarily have to change the grid voltage back to -4 volts, but we can achieve this result by increasing the plate voltage to 190 volts. It took, therefore, a change of 40 volts (from 150 to 190) to wipe out the effect of a grid-voltage change of only 2 volts (from -4 volts to -6 volts). When this experiment is repeated with smaller voltage changes—as a matter of fact, the changes should be infinitely small—then the ratio of these two voltage changes is called the "amplification factor" of the tube. It carries the Greek symbol μ. In this particular case the amplification factor of the tube at the plate and grid voltages as chosen would be $40/2 = 20$. Sometimes the amplification factor is also defined as the ratio of the effectiveness of a plate-voltage change to a grid-voltage change producing the same current change. Thus, if it takes at a particular operating point a plate-voltage change of 5 volts to produce the same current change that is produced by a grid voltage change of $\frac{1}{10}$ volt, then the amplification factor of the tube at this particular operating point would be 50. Why this ratio is called the "amplification factor" will become apparent later. With these explanations, the mathematical definition of the amplification factor as given in the following equation:

$$\mu = \frac{\Delta e_b}{\Delta e_c} \quad \text{(with } i_b \text{ held constant)} \qquad (9\text{–}2)$$

becomes clear. It simply states that in order to find μ for a given tube, we must determine the ratio of a voltage change on the plate to the voltage change required on the grid to keep the plate current constant.

Can we visualize the amplification factor on the rubber-sheet model? Suppose that, with the rack representing the grid and the board representing the anode at given levels, a certain number of marbles arrives per second at the lower board. Now if we lower this board some more, we shall stretch the rubber sheet downward an additional amount with a consequent increase of slope near the emitting box. The stream of marbles will therefore increase. In order to bring this value back to the original, we have to push up the rack representing the grid a certain amount. If we find, for instance, that the effect of lowering the board representing the anode by 10 in. can be canceled by raising the rack with the pins by 1 in., then the amplification factor of the model would be 10.

9–11. Plate Resistance of a Tube.—In Chap. V we saw that the question of the resistance of a nonlinear device is a tricky one. It was shown that there are two values that can lay claim to the title resistance. One was the static resistance and was defined simply as the ratio of the actual voltage to the actual current. The other was the dynamic resistance and was defined as the ratio of a small voltage change to the small current change produced by the former. It was also shown that the behavior of a nonlinear device over a narrow range of voltages could never be described satisfactorily by means of the static resistance, but that the device could be replaced by a battery and a resistance equal to the dynamic resistance in series with this battery. The static resistance of a tube is obviously quite meaningless because it can be made to assume almost any value by adjusting the grid voltage properly. The dynamic resistance of a tube, on the other hand, is determined by the slope of the tangent to the plate characteristics of the tube, and a glance at these curves indicates that this value, although not constant, nevertheless does not vary greatly. Suppose that we were asked to determine the dynamic resistance of the 6C5 tube operating at 200 volts on the plate and -6 volts on the grid. Under this condition the tube is carrying a current of 7 ma (see Fig. 9–5). The slope of the characteristic can be determined by drawing a tangent to the operating point and determining the resistance that would have the same slope, or we may determine the ratio of small current and voltage changes around this operating point. To change the current from 6 to 8 ma with -6 volts on the grid, the anode voltage will have to be changed from 190 to 210 volts, or a change of 20 volts, to produce a current change of 2 ma. The dynamic resistance, according to the principles discussed in Chap. V, is therefore $20/0.002 = 10{,}000$ ohms. In the case of tubes, the dynamic resistance is called the "plate resistance" of the tube, and the manufacturer usually gives this value for one or several operating points. Mathematically, the plate resistance of a tube is defined by the following:

$$r_p = \frac{\Delta e_b}{\Delta i_b} \quad \text{(with } e_c \text{ held constant)} \qquad (9\text{–}3)$$

In the mechanical rubber analogy, the plate resistance is equal to the ratio of the amount of displacement of the anode board to the change in the amount of the marble stream, as the reader may easily reason out for himself.

9-12. Transconductance of a Tube.—There remains one additional characteristic value usually given by the manufacturer. We saw that the amplification factor represented the ratio of plate- and grid-voltage changes for constant plate current and that the plate resistance represented the ratio of plate-voltage change to plate-current change for constant grid voltage; it seems that the third value should have to do with the ratio of plate-current change and grid-voltage change for a constant plate voltage. The ratio of the voltage and current of a device is called the "resistance," while the reciprocal, *i.e.*, the ratio of current to voltage, is called the "conductance." In the present case we observe the ratio of the change of current in one circuit, namely, the anode circuit, produced by a voltage change applied to another circuit, namely, the grid circuit. For reasons similar to those discussed in connection with the term "mutual characteristic," the last-mentioned ratio is called the "mutual" conductance or "transconductance" of the tube. This value is usually given in micromhos by the manufacturer. Referring to the transfer characteristics shown in Fig. 9-4, we observe a current of 7 ma with a plate voltage of 200 volts and a grid voltage of -6 volts. Reducing the grid voltage by 1 volt, namely, to -5 volts, is seen to cause the current to increase to a value of 9 ma. The ratio of plate-current change to grid-voltage change, or the transconductance g_m, is therefore seen to be $g_m = 0.002/1$ mho. Since 1 mho is equal to 1,000,000 micromhos, the above value expressed in micromhos would be 2,000. We can also state this in another way. At the particular operating point just mentioned, the plate current will change 2,000 μa for every volt of grid-voltage change, *provided that the plate voltage remains constant during this current change.* The mathematical definition of the transconductance is given by

$$g_m = \frac{\Delta i_b}{\Delta e_c} \quad \text{(with } e_b \text{ held constant)} \tag{9-4}$$

It is seen to be equal to the slope of the mutual or transfer characteristic.

In the rubber model the transconductance is represented by the ratio of the increase of the marble stream to the movement of the rack of pins representing the grid or, expressed differently, the increase of the marble stream per inch of movement of the rack.

9-13. Relation between μ, r_p, and g_m.—Strictly on the basis of the mathematical definitions of the three characteristic values the reader will see that the three are related to each other; when two of them are given, the third one has a definite value satisfying the following relation:

$$\mu = g_m r_p \tag{9-5}$$

The reader will have no difficulty in proving this for himself by substituting the values for g_m and r_p, as given by Eqs. (9–3) and (9–4) for the right-hand side of Eq. (9–5).

One does not have to arrive at Eq. (9–5) by purely mathematical methods, however. The truth of it can be easily seen in the following manner. The plate resistance tells us the relation between a plate-voltage change and the plate-current change produced by it. Thus, if the plate resistance of a tube is given as 8,000 ohms, it means that a plate-voltage change of 1 volt will produce a plate-current change of 1/8,000 amp or ⅛ ma. If the manufacturer tells us that the amplification factor of the tube is 10, he means that the grid voltage is ten times more effective in changing the plate current than the plate voltage is. Consequently it takes only 0.1-volt grid-voltage change to produce the same plate-current change as the above 1-volt plate-voltage change. We can therefore state that 0.1-volt grid-voltage change will produce a plate-current change of 0.125 ma. A 1-volt grid-voltage change will consequently produce 1.25-ma plate-current change, or 1,250 μa. The transconductance of this particular tube at the operating point for which the plate resistance and the amplification factor were given is therefore 1,250 micromhos.

9–14. Load of a Tube.—The vacuum tube has gained the tremendously important position it holds now by its ability to control the current flowing in the anode circuit by changing the control voltage applied to the grid. It is evident that this ability is of use only if the tube is used to control the current in some *other device*, or load. In a radio receiving set the load is a loud-speaker, and the current flow in this load is controlled by the tube so that its variations represent as nearly as possible the variations in air pressure occurring at the microphone in the broadcasting studio. (That the results are not always pleasing is, of course, due to the fact that whatever issues from the loud-speaker cannot be better than what the microphone receives.) In a photoelectric relay the final load is usually a mechanical relay, the coil of which is operated by the plate current of a tube. In other cases, the load may be simply a resistor. By controlling the flow of current through this resistor the voltage across it will, of course, vary, and these voltage variations may be used for the control of additional tubes.

9–15. Use of Tube Data for Predicting Circuit Performance.—In the preceding sections we studied the tube constants and characteristic curves of the tube itself. This study could be compared to the investigation of the characteristic of a starting resistor for a motor. But just as a motor starter is never used by itself just for the satisfaction of having current flow through it but becomes useful only when combined with the motor it is to control, so a tube becomes useful only when it controls the current in a load. When we know the resistance values for the various steps of a motor starter, we can predict how it will perform when combined with any motor

or load. In a similar way, the plate characteristics of a tube and the tube constants discussed in Secs. 9–6 to 9–8 are of value only if they give us some way to predict the performance, *not of the tube alone,* but of the combination of tube with a load of any kind. In the following sections it will be shown how to use the information given by the manufacturer in the shape of curves or by characteristic values to predict circuit performance of a tube in connection with a load. In order to avoid confusion on the part of the reader, it may be well to state right here that several methods are available for the solution of this problem. This would evidently be rather welcome if only the various methods all gave the same answer to a given problem. This is unfortunately not so, and the choice of the appropriate method depends on the size of the voltage variation applied to the grid as well as on the nature of the load operated by the tube. For small voltage variations applied to the grid (where "small" is one of those vague terms meaning ¼ volt for one type of tube and 5 to 10 volts for another type), a mathematical treatment based on what is known as the "equivalent-plate-circuit theorem" gives very satisfactory results; its outstanding advantage is that it will give correct results for any type of load. When the grid-voltage variation becomes large, a graphical solution making use of the plate characteristics must be used. If the load is resistive, the graphical solution is nothing more than an application of the principles of the load line; when the load is inductive, as is often the case, the graphical solution becomes anything but simple.

9–16. Equivalent-plate-circuit Theorem.[6]—Let Fig. 9-6 represent the plate characteristics of a certain triode for grid voltages of -2, -4, and -6 volts. These characteristics tell us the relation between plate current, plate voltage, and grid voltage. With each grid voltage shown in Fig. 9–6, the tube behaves like a definite nonlinear conductor with the characteristics as given for the particular grid voltage in this graph. In Chap. V it was shown that, for any given operating point on the characteristic of a nonlinear conductor and for a small range of voltages above and below this point, the nonlinear conductor could be replaced by a resistance equal to the dynamic resistance at the operating point in question, and a battery in series with this resistance. With -2 volts on the grid and 80 volts on the plate, the tube is seen to pass 8 ma (point A_1). There its static resistance is 10,000 ohms, while the dynamic or plate resistance is found by drawing a tangent at the point in question. This is done in Fig. 9–6, and the tangent is seen to intersect the voltage axis at 40 volts (point B_1). The dynamic resistance is only 5,000 ohms (a value arrived at by drawing a parallel to the tangent at A_1 through the origin as per the dotted line). Suppose now that we make the grid -4 volts negative. This would drop the current to less than 3 ma if we kept the plate voltage constant, or would require a plate voltage of 110 volts if we intended to keep the current at its value of 8 ma. The latter will bring us to point A_2 on the -4-volt

characteristic. In the region around A_2 the tube is now acting again as a nonlinear conductor, and we can therefore substitute again a combination of resistance and a battery. If the characteristics are reasonably parallel, a tangent drawn at A_2 will have the same slope as a tangent drawn at A_1. With -2 volts on the grid, the tube was seen to act like a battery of 40 volts with a resistance of 5,000 ohms in series; with -4 volts on the grid, the battery has changed to 70 volts, but the resistance of the substitution has remained at 5,000 ohms. Suppose now that this tube is used in combination with any other circuit elements and that we have made

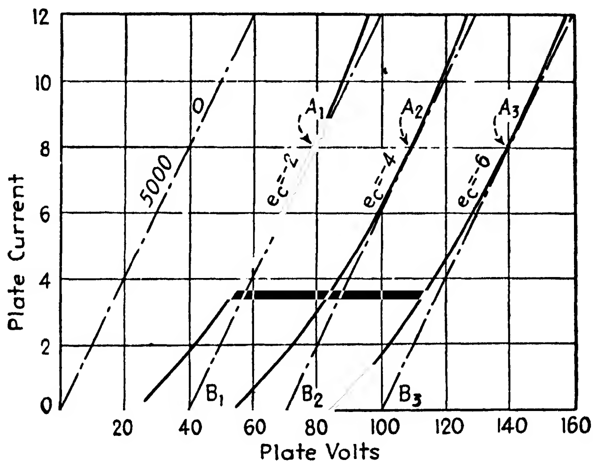

FIG. 9-6.—Over a small operating range each plate characteristic can be considered as the characteristic of a resistance with a battery in series with it.

sure that with -2 volts on the grid of the tube the anode voltage is actually 80 volts; we would now like to find some method by which we could predict or calculate what happens in the circuit containing the tube if we change the grid voltage from its -2 value to another value. The theorem by which this can be accomplished is known as the "equivalent-plate-circuit theorem." In the case just discussed, the circuit containing the tube with -2 volts applied to the grid obviously performs in exactly the same manner as if the tube were removed and a resistance of 5,000 ohms and a battery of 40 volts were substituted for the tube. If we change the grid voltage in the actual circuit from -2 to -4 volts, it is quite certain that changes are going to take place in the circuit containing the tube; these changes will evidently be exactly the same as those that would take place in the circuit where the tube was replaced by a 5,000-ohm resistor and a 40-volt battery, if this battery was changed from 40 to 70 volts.

9-17. Use of the Tube Constants for the Equivalent-plate-circuit Theorem.—In the above example, the substitution of a resistance and a fixed voltage in place of the tube was made on the basis of information obtained

from the plate characteristics of the tube. The slope of the tangent gave us the 5,000-ohm value for the resistor, the intersection of the tangent with the voltage axis gave us the 40 volts, and the position of the plate characteristic for -4-volt grid voltage told us that a 2-volt change on the grid of the actual tube would produce the same changes in the circuit as the change of the 40-volt battery to 70 volts. It seems, therefore, that we must have the plate characteristics if we wish to make this substitution for any given tube. It will be shown now that the substitution can be made with the aid of the characteristic values discussed in Secs. 9–6 and 9–7 with only one piece of additional information. This will be shown for a type 6C5 tube for which the manufacturer gives us the following values: plate resistance, 10,000 ohms; amplification factor, 20; transconductance, 2,000 micromhos. [The last value would not have to be given since it can be obtained from the other two with the aid of Eq. (9–5).] The manufacturer states further that with 250 volts on the plate and -8 volts on the grid the plate current has a value of 8 ma. Suppose now that this tube has been incorporated with other circuit elements and that we have chosen or adjusted all voltages and values of circuit elements so that there will be 250 volts across the tube itself. We now wish to find the resistance and the fixed voltage to replace the tube. If we had the plate characteristics of the tube, we would draw a tangent at the operating point of 250 volts and 8 ma, and let it intersect with the X axis. The dynamic resistance of a nonlinear conductor, *i.e.*, the slope of a tangent drawn at the operating point, was the ratio of a small voltage change to the small current change produced by it as explained in Chap. V. This is exactly the value defined as the plate resistance in the case of a tube, as examination of Eq. (9–3) shows. Consequently, the resistance to be used in our substitution for the 6C5 tube must be 10,000 ohms. It now remains to find the value of the battery that we must place in series with the 10,000-ohm resistor so that this combination with 250 volts applied to it will take exactly as much current as the actual 6C5 tube. It is clear that this battery must have a value of 170 volts because, with 250 volts applied to a series combination of 10,000 ohms and 170 volts, the current will be 8 ma, as required. The substitution is shown in Fig. 9–7.

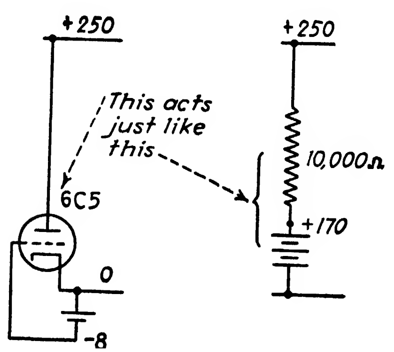

FIG. 9–7.—With -8 volts on the grid and plate voltages in the range around 250 volts, a 6C5 tube will act just like the substitution shown in this figure.

Suppose now that in the circuit containing the actual 6C5 we were to change the grid voltage from -8 to -8.5 volts. What shall we have to

do to our substitution of 10,000 ohms and 170 volts in order to cause the same changes in the circuit as the grid-voltage change would produce in the actual circuit? In the case discussed in connection with Fig. 9–6 we saw that a grid-voltage change from -2 to -4 volts shifted the plate characteristics by an amount equal to the distance between A_1 and A_2. Owing to the fact that the characteristics are reasonably parallel, the distance B_1B_2 between the intersection of the two tangents drawn at points A_1 and A_2 was the same as the distance A_1A_2. With -2 volts on the grid, the tube was equivalent to a resistance of 5,000 ohms and 40 volts in series with it; with -4 volts on the grid, it was equivalent to 5,000 ohms and 70 volts in series with it. Coming back to the 6C5, we do not have the plate characteristics at our disposal, but if we could only tell how far the characteristics would move to the right when the grid voltage is changed from -8 to -8.5, our problem would be solved. The manufacturer tells us that the amplification factor of a 6C5 is 20. With 250 volts on the plate and -8 volts on the grid, the tube takes 8 ma. Making the grid $\frac{1}{2}$ volt more negative will decrease the current some unknown amount, but by stating the amplification factor as 20, the manufacturer tells us that we can bring the current back to 8 ma if the plate voltage is raised $20 \times \frac{1}{2} = 10$ volts. The plate characteristics for -8.5 volts must therefore pass through the point determined by 260 volts plate voltage and 8 ma plate current. This is just what we have been looking for. The characteristic has moved 10 volts to the right. Since it is reasonably parallel to the original one, a tangent drawn at the new point will pass through 180 volts on the voltage axis. The change taking place in the circuit containing the actual tube when its grid voltage is changed from -8 to -8.5 volts is therefore the same as if we were to change the battery of 170 volts in our black-box substitution for the tube by 10 volts, which is equal to the grid voltage multiplied by the amplification factor. This substitution is the equivalent-plate-circuit theorem and, since it is most important in the solution of problems involving vacuum tubes, it may be desirable to restate it in its entirety as follows: *When a triode is included in a circuit with other circuit elements, we can replace it as far as the circuit is concerned by a resistance and a battery in series with it. The resistance is equal to the plate resistance of the tube. The battery is found by drawing a line with a slope given by the plate resistance through the operating point of the tube and extending it to the intersection with the voltage axis. Then the effect that a grid-voltage change at the actual tube has on the circuit is the same as if this fictitious battery is changed by an amount equal to μ times the grid-voltage change.* To return to Fig. 9–7 then, we saw that the substitution for a 6C5 at the particular operating point was a battery of 170 volts and a resistance of 10,000 ohms in series with it. The amplification factor of this tube is given as 20. This means that the current changes taking place in the circuit containing the actual tube, when the grid voltage is changed

by, say, 1.5 volts, are the same as if the 170-volt battery of the fictitious substitution is changed $20 \times 1.5 = 30$ volts.

9–18. Application of the Equivalent-plate-circuit Theorem.—In Fig. 9–8*a* the 6C5 tube is shown with a resistance of 20,000 ohms in series with the tube. If we wished to operate the tube at the values recommended by the manufacturer (which is by no means a necessity), then the plate voltage should be 250 volts, the grid voltage −8, and the plate current 8 ma. But since the 8 ma must flow also through the 20,000 ohms, causing a voltage of 160 volts to appear across this resistance, the total voltage applied to the combination must be equal to $250 + 160 = 410$ volts. Suppose now that we place in series with the battery furnishing −8 volts for

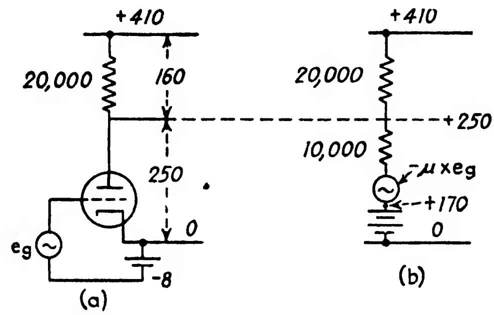

FIG. 9–8.—The introduction of an ac signal to the grid of the 6C5 will have the same effect as the introduction of a voltage μ times the alternating voltage in series with the fixed battery.

the grid a source of alternating voltage, such as the secondary of a transformer, and let the amplitude of this voltage be 2 volts. This evidently means that the actual grid voltage will fluctuate between −6 and −10 volts. What will take place in the plate circuit of the tube?

According to the principles discussed in the preceding paragraphs, we replace the tube by a battery of 170 volts and a resistance of 10,000 ohms, as shown in Fig. 9–8*b*. The reader may convince himself easily that the current taken by the circuit is exactly 8 ma, *i.e.*, exactly the same as taken by the actual tube and load resistance. The 2-volt grid variation up and down from the steady value of −8 volts will now, according to the equivalent-plate-circuit theorem, have the same effect as a variation twenty times as large down and up of the 170-volt steady value of the battery in the plate circuit. Observe the sequence of the words "up-down" for the grid circuit and "down-up" for the fictitious battery. The correctness of this statement can be ascertained as follows. When the grid voltage swings in the up direction, *i.e.*, during the time that the superimposed alternating voltage is positive, thus reducing the steady value of −8 volts to −6 volts, it is clear that the current in the plate circuit will increase; but in order to make the fictitious circuit with its battery act in the same way, it is evidently necessary to reduce the value of the fictitious voltage from 170 to

some lower value, in this case to $170 - 40 = 130$ volts. Similar reasoning will, of course, indicate that, while the grid is swinging negative to its extreme value of -10 volts, the fictitious battery will rise to a voltage of $170 + 40 = 210$ volts. If the alternating voltage supplied to the grid voltage is of any particular wave shape, the variations of the 170-volt fictitious battery will be of the same wave shape, which could be considered simply as the result of placing in series with the 170-volt battery a source of alternating voltage of exactly the same wave shape as that applied to the grid but μ times (in this case, twenty times) as large and 180 deg out of phase. Generally speaking, whatever voltage is placed on the grid in series with the -8 volts steady value (also called the "bias" voltage applied to the grid) must also be placed in series with the fictitious battery in the load circuit, but multiplied μ times and 180 deg out of phase. Communication engineers in their circuits are usually interested only in the fluctuating part of the plate current. Since the principle of superposition is applicable to our substitution circuit, it is clear that as far as the ac component flowing in the circuit is concerned, the fictitious battery as well as the steady voltage supplying the circuit can be considered as short-circuited. Therefore, for the calculation of the fluctuating or ac component the actual circuit shown in Fig. 9–9*a* can be replaced by the equivalent circuit shown in *b*. Most amplifier problems are solved by making use of this equivalent circuit.

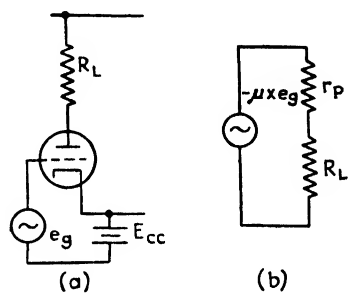

FIG. 9–9.—If we are interested only in the ac component of the plate current, all dc voltages of Fig. 9–8 may be considered as short-circuited.

9–19. Mathematical Derivation of the Equivalent-plate-circuit Theorem.—In Sec. 9–18 the equivalent-plate-circuit theorem was derived strictly on the basis of our ability to substitute for any given nonlinear conductor a fixed voltage and a resistance in series with it. The equivalent-plate-circuit theorem is of such importance that it may be desirable to derive it on a mathematical basis as is usually done in most books on electronics. This method has the disadvantage, however, that it covers only voltage *changes* or current *changes*. For communication engineers a method that furnishes the correct answer as far as the alternating component is concerned may be entirely satisfactory, but the industrial electronic engineer quite often deals with dc amplifiers, and in this case the fictitious substitution presented in the preceding paragraph furnishes the correct answer not only for the alternating but also for the steady component.

Suppose that, with a particular voltage e_c applied to the grid and a particular voltage e_b applied to the plate, the current observed in the plate

circuit is i_b. If the *plate voltage* e_b is now *increased* a small amount Δe_b *while the grid voltage is left unchanged,* the plate current will change a small amount Δi_b. This change can be calculated with the aid of the plate resistance at the particular operating point. It is given by

$$\Delta i_b' = \frac{\Delta e_b}{r_p} \tag{9-6}$$

If we *increase the grid voltage* by an amount Δe_c—by "increase" is meant a change in the positive direction—the plate current also increases. Since

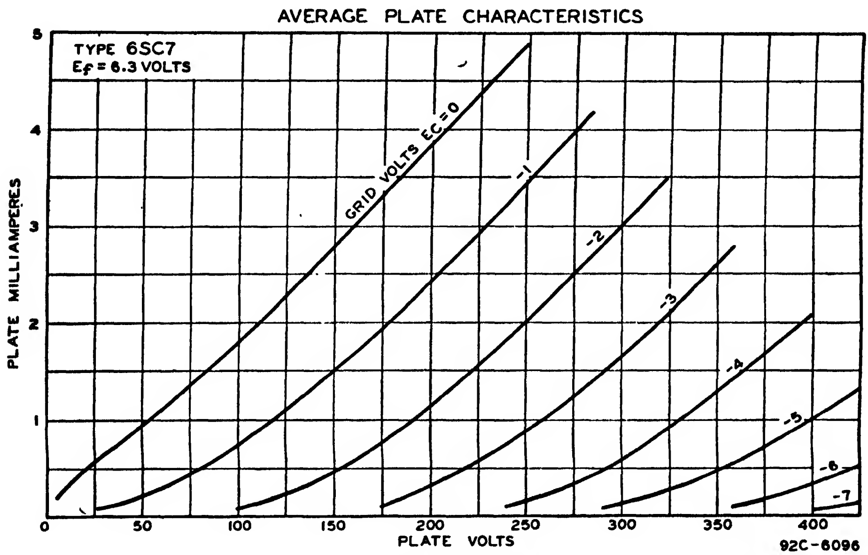

FIG. 9-10.—The plate characteristics of a type 6SC7 tube. (*Courtesy Radio Corporation of America.*)

a grid-voltage change has μ times as much effect on the plate current as a plate-voltage change, the current change $\Delta i_b''$ caused by this grid-voltage change is given by

$$\Delta i_b'' = \frac{\mu \, \Delta e_c}{r_p} \tag{9-7}$$

Since the principle of superposition is valid for small changes, the *total* plate-current change that results when the grid voltage and the plate voltage are changed *simultaneously* is given by

$$\Delta i_b = \Delta i_b' + \Delta i_b'' = \frac{\mu \, \Delta e_c + \Delta e_b}{r_p} \tag{9-8}$$

Equation (9-8) is one of the most important relations pertaining to vacuum tubes. It permits us to calculate the plate-current change taking place

with the simultaneous changes of grid and plate voltage. Suppose, for instance, that the manufacturer tells us that a type 6SC7 with 250 volts on the plate and -2 volts on the grid passes a current of 2 ma, and that the plate resistance is 53,000 ohms and the amplification factor 70. What will the current be if the plate voltage is increased to 300 volts while the grid voltage is decreased (take careful notice of the word) to -3 volts? Δe_b is consequently 50 while Δe_c is -1. Equation (9–8) will give us

$$\Delta i_b = \frac{-70 \times 1 + 50}{53{,}000} = -0.378 \text{ ma} \qquad (9\text{–}8a)$$

The plate current therefore decreases from 2 ma to $2 - 0.378 = 1.622$. A glance at the actual characteristics shown in Fig. 9–10 shows that the value thus calculated coincides with the point that we would locate on the characteristic for a grid voltage of -3 volts at a plate voltage of 300 volts.

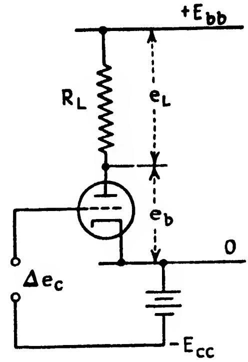

FIG. 9–11.—Designation of various voltages appearing in a vacuum-tube circuit.

Now, in Fig. 9–11, let a tube be placed in series with a load having a resistance R_L across a supply voltage E_{bb}. The voltage across the tube will be e_b, while the voltage across the load will be e_L; the latter is, of course, equal to i_bR_L. If we now increase—*i.e.*, make more positive—the grid voltage by an amount Δe_c, the plate current will increase a certain amount Δi_b, not known to us yet. This increase of current will increase the voltage across the load and consequently decrease the voltage across the tube itself. The plate-voltage increase is therefore negative and is equal to

$$\Delta e_b = -\Delta i_b R_L \qquad (9\text{–}9)$$

Substituting this value in the fundamental Eq. (9–8), which must be satisfied under all conditions, we obtain

$$\Delta i_b = \frac{\mu\,\Delta e_c - \Delta i_b R_L}{r_p} \qquad (9\text{–}10)$$

In this equation Δi_b appears on both sides. By solving for Δi_b, we obtain

$$\Delta i_b r_p = \mu\,\Delta e_c - \Delta i_b R_L$$

$$\Delta i_b(r_p + R_L) = \mu\,\Delta e_c$$

$$\Delta i_b = \frac{\mu\,\Delta e_c}{r_p + R_L} \qquad (9\text{–}11)$$

The plate-current change taking place is therefore, as Eq. (9–11) shows, exactly the same as if a voltage $\mu\,\Delta e_c$ had been made to act on a circuit

consisting of a series combination of $r_p + R_L$. This is the same relation that we arrived at on the basis of replacing the tube by a resistance and a battery.

In deriving Eq. (9–11) a resistive load was assumed. It is clear, however, that no matter what the type of load, the instantaneous voltage across the tube must be equal to the voltage of the supply minus the instantaneous voltage across the load. Equation (9–11) is therefore valid for any type of load.

The reader who has followed the two derivations of the equivalent-plate-circuit theorem may at this time be concerned about the fact that the polarity of the fictitious voltage $\mu\,\Delta e_c$ as given by Eq. (9–11) does not seem to agree with the polarity shown in Fig. 9–9. Since a clear understanding of the phase relationship between grid voltage and voltage appearing across the load is often of great importance in the correct analysis of a given problem, it may be well to take the necessary time to show that both derivations give the same result. Consider the circuit shown in Fig. 9–12; with the voltages as given in this figure a current of 10 amp is evidently going to flow in the resistor. If we wish to increase the current to, say, 12 amp, we can either *increase* the 30-volt battery by 4 volts or *decrease* the 10-volt battery by the same amount. Changing the grid voltage on an actual tube in the positive direction demanded that the fictitious battery in the substitution should be decreased, *i.e.*, changed in a negative direction; but a change of the fictitious voltage in such a direction evidently *increases* the plate current and is therefore entirely in accordance with Eq. (9–11), which tells us that a positive grid-voltage change produces a positive plate-current change.

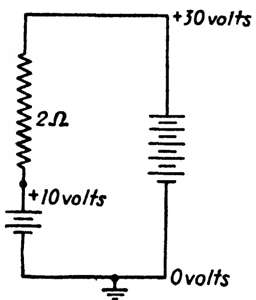

Fig. 9-12.—The current in the circuit shown here will increase with either a positive change of the 30-volt battery or with a negative change of the 10-volt battery.

9–20. Limitations of the Equivalent-plate-circuit Theorem.—The equivalent-plate-circuit theorem is seen to provide a powerful tool for the analysis of circuits combining vacuum tubes with other circuit elements. It must be recalled, however, that this theorem was derived not only on the basis of representing the characteristics of the tube as straight lines (the tangent through the operating point) but also by assuming that all these straight lines were parallel. Circuit performance calculated with the aid of the equivalent-plate-circuit theorem will therefore coincide with actual performance only if the voltage and current variations occurring at the tube fall within an area of the plate characteristics of the tube where the curves can be considered as reasonably parallel and as equally spaced straight lines. In many amplifier circuits every effort is made to fulfill this condition because it represents linear operation, but the industrial

electronic engineer wishes quite often to operate the tubes beyond these limits. Under these conditions he cannot use the equivalent-plate-circuit theorem in the solution of his problems.

9–21. Use of the Load Line for the Analysis of Tube Circuits.—Let it be desired, for instance, to find the current variation taking place when a 6J5 tube is placed in series with a 5,000-ohm resistor across a supply voltage of 250 volts and when the voltage applied to the grid of the tube is varied from -4 to -10 volts. If we wish to apply the equivalent-plate-circuit theorem to this case, we run into the following difficulty: in order to make our famous substitution we must know the plate resistance of the tube at the operating point. A grid-voltage variation from -4 to -10 volts is a rather large variation and, since we do not know at what point of the -4- or the -10-volt characteristic the tube is going to operate, it is impossible to assign a definite value to it. In the example treated in Fig. 9–8 we got around this difficulty by increasing the supply voltage a sufficient amount so that we were sure that the tube was operating with 250 volts on the plate, for which value the manufacturer obligingly gave us the plate resistance as 10,000 ohms. In the problem just presented, on the other hand, the plate-supply voltage is given and the only thing we can say definitely is that the voltage across the tube itself will be lower since the voltage across the resistor and the voltage across the tube must at any instant add up to the supply voltage. The statement just made reminds us that once before we faced the problem of determining the current that would flow when a nonlinear conductor was placed in series with a resistor across a given voltage. It was discussed in connection with a Mazda lamp in Secs. 5–12 and 5–13; the procedure was called "drawing in the load line."

Applying this method to our present case, we draw a straight line representing the characteristic of a 5,000-ohm resistor into the family of plate characteristics shown in Fig. 9–13. As explained in connection with Fig. 5–13, this straight line is plotted from right to left, with the 250-volt point on the X axis considered as the zero point. Since the load line must pass through this point, only one additional point is required. The most convenient additional point is the one where the load line intersects the Y axis. The current value at which it intersects this axis is evidently equal to the supply voltage divided by the value of the resistance, or in our case $250/5{,}000 = 0.05$ amp, or 50 ma. This point is beyond the limits of our graph, and we therefore have to choose another point. With 50 volts applied to a 5,000-ohm resistor, for instance, a current of 10 ma would flow. By going 50 volts to the left of the 250-volt point (remember that this is the zero point for the resistor characteristic!) and by going up 50 ma we can locate the point A, through which the characteristic must evidently pass. The load line is seen to intersect the plate characteristic for a grid voltage of -4 volts at point B, indicating that the current will be 12 ma

for this grid voltage, while it intersects the characteristics for a grid voltage of -10 volts at point C, indicating a current of 3.2 ma. The two intersections tell us all that we could possibly wish to know: not only the current, as just explained, but also the voltages across the tube and across the resistor. In contrast to the method making use of the equivalent-plate-circuit theorem, the load line gives us answers that are not approximations but are absolutely correct within the limits of accuracy to which we can read the characteristics. Whether the characteristics are straight or curved, whether they are parallel or not, will have no influence on the

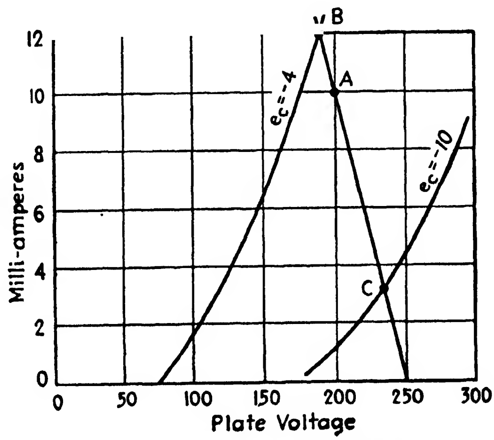

FIG. 9-13.—The load line for a 5,000-ohm load in series with a 6J5 tube.

accuracy of the result. The graphical solution is based on nothing else but the fact that the voltage across the tube—no matter with what grid voltage and in what part of the characteristics it may be working—plus the voltage across the resistor must add up to the value of the supply voltage.

In the example given, the load line served the purpose of finding the value of currents that flow in the load when given voltages are applied to the grid. Of course, it is evident that the load line also gives an answer to the reverse problem: to find the grid voltage required to produce a given current in the load.

9-22. Comparison of Methods.—As just shown, two methods are available to determine the current that will flow in a circuit containing a triode in combination with other elements. The first is the application of the equivalent-plate-circuit theorem, according to which we may substitute a fixed voltage and a resistance equal to the plate resistance for the tube; a change of the voltage applied to the grid in the actual circuit produces the same current change as a change μ times as large of the fixed voltage in the substitute circuit. The theorem may be applied for any kind of load,

but its results are reasonably correct only if the voltages across the tube and the currents flowing through it, when plotted into the plate characteristics, fall within an area where these characteristics can be considered as reasonably straight, equally spaced parallel lines.

The second method, that of the load line, is not subject to the limitations as far as linearity of the characteristics is concerned. It suffers, however, from another type of limitation. The load line, it will be remembered, was the characteristic of a resistance. We cannot plot the load line of an inductance, for instance, or let ourselves fall in the very tempting trap of drawing a load line with the reactance $2\pi fL$ of an inductance. Although it is true, in the case of alternating voltages appearing across the tube and an inductive load placed in series with it, that the *instantaneous* values must at any given instant add up to the total direct supply voltage, this no longer holds for the amplitudes or effective values. If the tube is operating outside the linear region and the load is inductive, an experimental setup with oscillographic observation is the only practical way of solving the problem.

PROBLEMS

9-1. Determine the plate resistance and the amplification factor of one triode unit of a type 6SC7 tube with a plate voltage of 275 volts and a grid voltage of -2 volts. The plate characteristics of this tube are shown in Fig. 9-10.

9-2. The plate characteristics of a type 2A3 tube are shown in Fig. 9-14. What could be substituted for this tube when the grid voltage is adjusted to -30 volts and the plate voltage does not vary beyond the limits of about 220 to 250 volts?

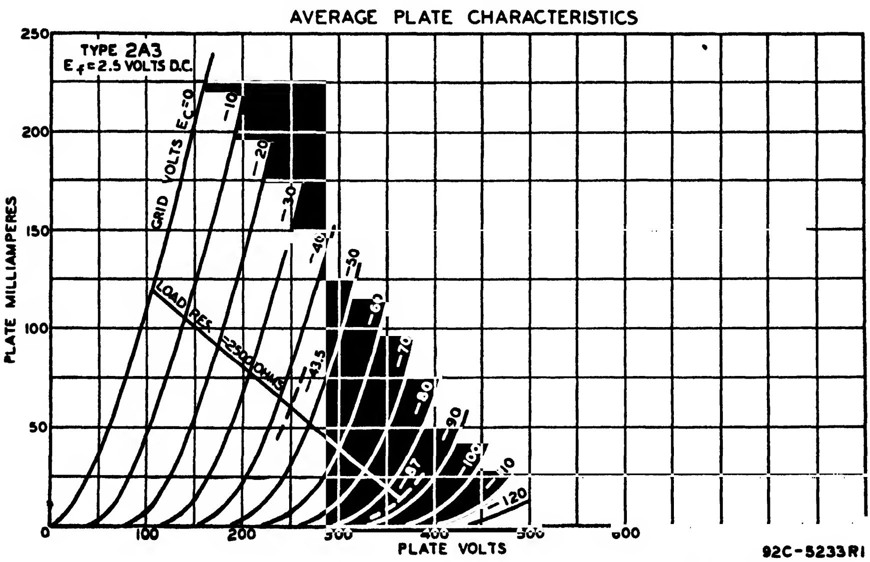

Fig. 9-14.—Plate characteristics of a type 2A3 tube. (*Courtesy Radio Corporation of America.*)

9-3. A 6C5 tube (see Fig. 9-5) is operated with 200 volts on the plate. The grid voltage changes from -6 to -10 volts. Give the corresponding plate currents.

9-4. With 150 volts on the plate of the same tube, the plate current is supposed to change from a value of 1 ma to a value of 8 ma. What are the two grid voltages necessary to accomplish this?

9-5. On the same tube 100 volts is applied to the plate and -3 volts on the grid. The plate voltage is now increased to 200 volts. In order to bring back the current through the tube to the original value, to what value must the grid voltage be changed?

9-6. From the plate characteristics of a type 6J5 tube, shown in Fig. 9-15, plot the transfer characteristics, *i.e.*, curves showing the plate current as a function of the grid voltage. Plot three curves for the three plate voltages of 100, 200, and 300 volts.

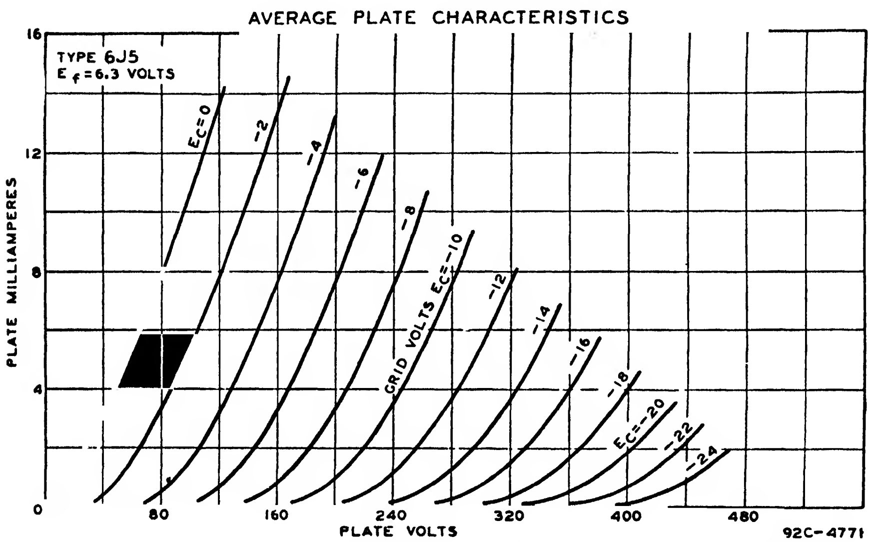

FIG. 9-15.—Plate characteristics of a type 6J5 tube. (*Courtesy Radio Corporation of America.*)

9-7. From the plate characteristics shown in Fig. 9-15, or the transfer characteristics prepared in Prob. 9-6, whichever is more suitable, determine

a. The plate resistance of the tube with 200 volts on the plate and -8 volts on the grid.

b. The amplification factor at these values.

c. The transconductance at the same operating point.

9-8. See whether the relation $g_m = \mu/r_p$ holds true approximately for the values determined in Prob. 9-7. (In order to satisfy this relation, the transconductance must be expressed in mhos!)

9-9. Using the figures obtained in solving Prob. 9-5, what value will be found for the amplification factor of a 6C5?

9-10. In series with a type 6L6 tube in triode connection (characteristics shown in Fig. 9-16) is a constant-current device, keeping the current through the tube constant at a value of 50 ma, regardless of the voltage across the tube. The voltage applied to

the grid is varied from −7.5 to −30 volts. What happens (where, to what, and how much)?

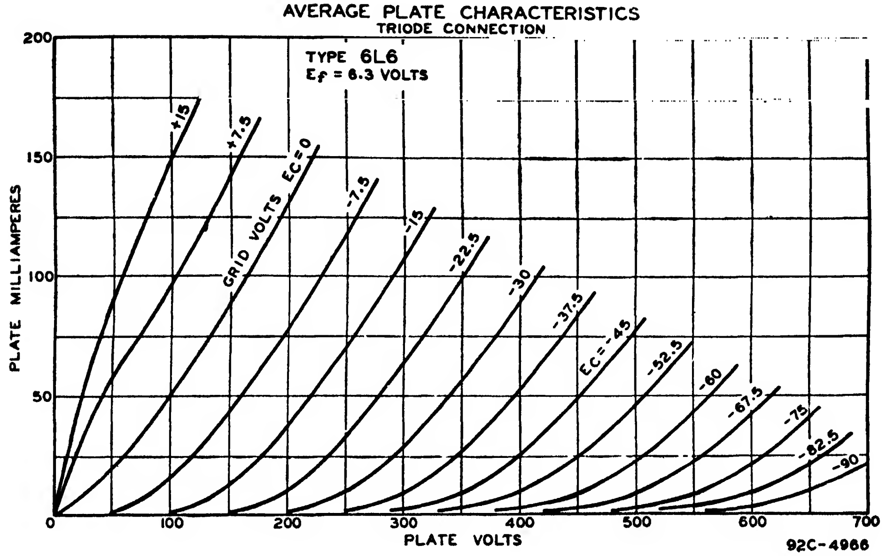

FIG. 9-16.—Plate characteristics of a type 6L6 tube, triode connected. (*Courtesy Radio Corporation of America.*)

9–11. A relay with 3,000 ohms coil resistance is placed in series with a 2A3 tube (see Fig. 9–14); the voltage applied to the combination is 350 volts.

a. If the grid voltage is raised from −80 to −10 volts, what are the two corresponding current values through the relay?

b. If the relay requires for safe operation a current change from 5 to 50 ma, what grid voltages are necessary to produce these two currents?

9–12. A relay with 8,000 ohms coil resistance requires 10 ma to close its contacts safely, and a reduction of the current to 2 ma to open them. It is to be operated in the plate circuit of a 6C5 tube, but for certain reasons it is desired that the grid voltage on this tube never becomes less negative than −4 volts. What supply voltage is needed for the combination and what grid-voltage change will be necessary for the operation of the relay with the two current values given?

9–13. A type 6L6 tube in triode connection is placed across 300 volts. Plot the plate current as a function of the grid voltage with (*a*) 0, (*b*) 500, (*c*) 1,000, (*d*) 2,000, (*e*) 4,000, and (*f*) 8,000 ohms in series with the tube. Sometimes the curve obtained under (*a*) is called the "static mutual characteristic," while the other curves are called the "dynamic mutual characteristics." Don't try to figure out why.

SUGGESTED ADDITIONAL READING

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CHAPTER X

SINGLE-STAGE AMPLIFIER PERFORMANCE; TUBE RELAYS; TIMING RELAYS

10-1. Importance of Load.—In Chap. IX the methods used for analyzing the performance of a triode in combination with an electrical circuit were discussed; the present chapter will deal with the application of these methods to investigating the performance of a tube as an amplifier with various types of load in the plate circuit. An amplifier is defined as "a device whose output is an enlarged reproduction of the essential features of the input and which draws power therefor from a source other than the input signal."

As stated once before, a vacuum tube becomes a useful device only by its ability to control the current in a load, such as the antenna of a broadcasting station, the loud-speaker of a radio receiver, or the relay coil in the case of photoelectric device.[2]

10-2. Symbols and Designations Used in the Analysis of Amplifier Circuits.—The original, and at present still most important, application of the tube is as an amplifier of an alternating voltage applied to its grid. The load that it is desired to operate is placed in the plate circuit, and the essential feature of the circuit is that the power (or wattage) required on the grid is much smaller than the power obtained in the load. In this book no attempt will be made to treat the subject of amplifiers exhaustively but, in order that the reader who may wish to enlarge his knowledge on the subject by the study of the many available good texts will not become confused as to the designations and symbols used in the accurate treatment, the accepted and approved symbols will be used in this text even if they are not all needed in the simple treatment presented here. The reader has already met some of the designations used in the analysis of triode circuits in Chap. IX. We shall now discuss briefly additional quantities used in the analysis of amplifier circuits.

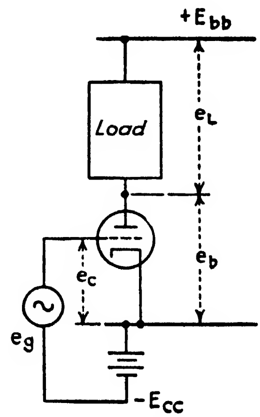

FIG. 10-1.—With an alternating voltage applied to the grid, an alternating voltage will, of course, also appear across the load in the plate circuit.

A summary of the symbols is found in the accompanying table, to which the reader may refer until he has become familiar with them.

	Level of grid with respect to cathode	Level of plate with respect to cathode	Current toward the plate	Voltage across load
Instantaneous total value..	e_c	e_b	i_b	e_L
Quiescent value (when varying component of grid voltage is zero)............	E_{c0}	E_{b0}	I_{b0}	E_{L0}
Average value of the total value..................	E_c	E_b	I_b	E_L
Instantaneous maximum of the total quantity.......	E_{cm}	E_{bm}	I_{bm}	E_{Lm}
Instantaneous value of the varying component.....	e_g	e_p	i_p	e_z
Effective value of the varying component...............	E_g	E_p	I_p	E_z
Amplitude of the varying component...............	E_{gm}	E_{pm}	I_{pm}	E_{zm}
Average value of the varying component........... .	E_{g0}	E_{p0}	I_{p0}	E_{z0}
Supply....................	E_{cc}	E_{bb}		

Figure 10–1 shows the connection of a tube and a load of any kind across a direct supply voltage. If the tube is used as an amplifier in the narrow sense as defined in Sec. 10–1, the voltage applied to the grid will consist of a steady direct voltage, also called the "bias" voltage, and a source of alternating voltage in series with this direct voltage.

Figure 10–1 shows the alternating voltage as being produced by a generator, but it may, of course, also be the voltage produced across a resistor when an alternating current flows through it. The bias voltage is designated as E_{cc}, and the instantaneous value of the alternating voltage is designated as e_g. With this in mind, it is seen that the instantaneous value e_c of the grid voltage—*i.e.*, of the voltage level of the grid with respect to the cathode—is equal to the sum of E_{cc} and the instantaneous value e_g of the alternating voltage furnished by the generator.

10–3. "Up" and "Down" Concept Applied to the Grid.—Let the bias voltage E_{cc} be, for instance, equal to -8 volts and let the alternating voltage have an amplitude of 3 volts. At the instant when the alternating voltage reaches its positive maximum, the total grid voltage will obviously be $-8 + 3 = -5$ volts; at the time when the alternating voltage has reached its negative maximum, the actual grid voltage will be $-8-3 = -11$ volts.

In our voltage-equal-height concept, we could visualize that the grid is held at a level of -8 volts, *i.e.*, below the cathode, by the action of the

bias voltage, but that the action of the alternating voltage makes it "bob" up and down 3 volts from the bias level. If we wish to be really consistent, we would indicate this situation in the manner shown in Fig. 10–2. The cathode is shown at zero level, point F at a level of -8 volts, and the fact that the grid is bobbing up and down is indicated by arrows placed at the terminal of the ac generator connected to the grid. Why one of the arrows is shown in a full line, the other in a dotted line, will become apparent presently.

10–4. Voltages Appearing at the Plate.—Examining now the plate circuit, let us assume for a moment that the alternating voltage applied to the grid is still zero so that the grid is at a steady level E_{cc}. If the load has a dc resistance, a voltage will appear across it, and the voltage across the tube will be equal to the supply voltage diminished by the voltage appearing across the load. If the supply voltage and the dc resistance of the load are given, the current flowing in the circuit as well as the voltages across tube and load can be determined with the aid of the load line as explained in Chap. IX. The values of the two voltages appearing under this condition are called the "quiescent" values and are designated by E_{b0} and E_{L0} for the plate voltage and load voltage, respectively. If the alternating voltage now becomes active on the grid, variations of the plate current from the quiescent value will, of course, take place. These current variations produce a voltage across the load so that point A in Fig. 10–2 also begins to bob up and down. Only if the tube is operating in the linear part of the characteristic, however, will the excursion of A from the quiescent level be of equal magnitude in both the up and the down direction (provided, of course, that the grid made also excursions of equal magnitude in the two directions). If the excursions are not of equal magnitude, the average level of A, *i.e.*, the voltage that a dc voltmeter would record, will change from the value observed in the quiescent state. The average value of the plate voltage under such a condition is designated as E_b and, as just explained, for linear operation this will be equal to E_{b0}. Additional quantities will be discussed as they appear in the text.

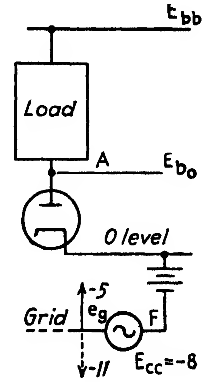

FIG. 10–2.—This is a truly correct way of showing the grid where it belongs "bobbing" up and down *below* the level of the cathode.

10–5. Pure Resistive Load in the Plate Circuit.—Let us now consider the simple case where the load is purely resistive. In a general way this case was disposed of in Chap. IX, but there are a few concepts used in the particular field of the amplification of an alternating voltage that have not yet been discussed. In Fig. 10–3, let a tube be connected in series with a resistor across a direct supply voltage. The quiescent current I_{b0}, which will flow before an alternating voltage is applied to the grid, can be found with the aid of the load line if the supply voltage is given, or we

may raise the supply voltage to such a value that the tube will operate with the voltage and current recommended by the manufacturer. This was explained in Secs. 9–14 and 9–17. When the alternating voltage is applied to the grid, the current will increase and decrease from the quiescent value. While the grid is *above* the level of the bias potential, the plate current will be larger than the quiescent value; the voltage across the resistor will, therefore, also be larger, which means that the anode will be *below* the quiescent level. Therefore, as the grid *bobs up,* the anode *bobs down.* The anode voltage can also be considered as consisting of a steady direct voltage with an alternating voltage superimposed on it. The analysis just presented shows that the alternating component of the anode voltage in the case of a resistive load is 180 deg out of phase with the alternating component of the grid voltage. (When the grid goes *positive* from its steady level, the plate goes *negative.*)

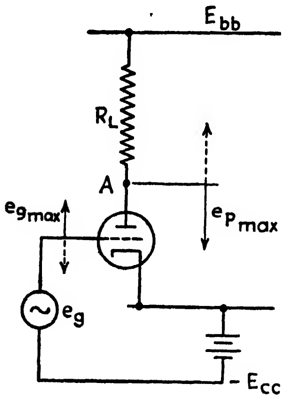

FIG. 10–3.—In the case of a resistive load, the anode swings negative when the grid swings positive.

In Fig. 10–3 the full arrows belong together and so do the dotted ones. We could, of course, also omit the dotted arrows, and—if the alternating voltage applied to the grid is of sinusoidal wave shape—consider the two full arrows as vectors representing the alternating components of the grid voltage and the plate voltage, respectively.

10–6. Voltage Gain of a Tube.—With the application of an alternating voltage to the grid we see that an alternating voltage of essentially the same wave shape, but 180 deg out of phase, will appear at the plate. The ratio of these two voltages is called the "voltage gain" of the tube and will be designated as v_g. The calculation of the voltage gain of a tube can most easily be accomplished by the application of the equivalent-plate-circuit theorem. We have seen that a small voltage variation Δe_c applied to the grid of the actual tube would cause a current change Δi_b exactly equal to the change that would be caused by a voltage change μ times as large taking place in the plate circuit of the tube. This was derived in Eq. (9–11), which read

$$\Delta i_b = \frac{\mu \, \Delta e_c}{R_L + r_p} \tag{9–11}$$

The current change calculated with the aid of Eq. (9–11) will cause a voltage change across the load resistance R_L which will amount to

$$\Delta e_L = \frac{\mu \, \Delta e_c}{R_L + r_p} R_L \tag{10–1}$$

The voltage gain will therefore be given by

$$v_g = \frac{\Delta e_L}{\Delta e_c} = \mu \frac{R_L}{R_L + r_p} = \mu \frac{1}{1 + (r_p/R_L)} \tag{10-2}$$

Equation (10–2) is of the utmost importance in the design of amplifier circuits. It shows that the voltage gain is equal to the amplification factor of the tube multiplied by the value $R_L/(R_L + r_p)$. As the denominator of this fraction is always larger than the numerator, the value of the fraction can never exceed unity. It also becomes clear why the value μ is called the "amplification factor" of a tube. It represents the limit of voltage gain obtainable from a given tube, a limit, however, that cannot be reached in an actual circuit except if it is possible to make the load resistance infinitely large compared to the plate resistance of the tube. For a load resistance equal to the plate resistance of the tube, for instance, the fraction assumes the value one-half, so that the voltage gain obtainable with a given tube under this condition is exactly one-half of the amplification factor.

The case of a purely resistive load is usually encountered when it is desired to use the alternating voltage appearing across this resistor as the signal voltage to be applied to an additional tube. It is then evident that as high a voltage gain as possible will probably be desirable. Equation (10–2) shows that in order to obtain this result the load resistor should have as high a value as possible. It must be remembered, however, that the quiescent plate current must flow through this resistor and that a high value of resistance will mean that the plate-supply voltage must also be high. In actual tube circuits the choice of the load resistance is therefore usually a compromise between a reasonable voltage gain and a supply voltage easily obtainable from the conventional power supply.

10–7. Use of the Load Line for the Determination of v_g.—Since the case of a resistive load can also be treated with the aid of the load line, it may be profitable to show how the results obtained with this method compare with those obtained with the equivalent-plate-circuit theorem. Let us assume that a type 6C5 tube is placed in series with a resistor of 25,000 ohms across a supply voltage of 300 volts. The load line to be drawn will pass through the 300-volt point on the voltage axis and 300/25,000 = 0.012 amp = 12 ma on the current axis, as shown in Fig. 10–4. Suppose that the grid voltage is -6 volts. The intersection of the load line with this characteristic indicates a current a trifle less than 5 ma. The voltage across the tube is indicated as slightly more than 175 volts, and the voltage across the resistance slightly less than 125 volts. (This latter value checks since a current of 5 ma through a resistance of 25,000 ohms would provide a voltage of exactly 125 volts.)

Let us now determine what current changes take place when the grid voltage is varied up and down 2 volts from the steady value of -6 volts.

A glance at the load line indicates that the voltage across the tube will vary from 150 volts to approximately 205 volts, or a voltage variation of 55 volts. This value is accurate within the limits of our ability to read the characteristic curves, but it is not influenced by any nonlinearity of the tube characteristics.

Suppose now that we were to solve this problem by the application of the equivalent-plate-circuit theorem. In order to apply this method, we must have the amplification factor and the plate resistance of the tube at the operating point. But the operating point cannot be found without the aid of the load line and the plate characteristics, so that it would seem that we might just as well solve our problem with the load line only. Overlooking for the moment this perfectly valid argument, let us determine the plate resistance and the amplification factor at the operating point. This we do by drawing a tangent at the point of intersection of the load line with the characteristic for -6 volts. If this is done, the plate resistance is found to be 11,600 ohms at this point. The amplification factor at the operating point is found by drawing a horizontal line through the operating point; to obtain 5 ma current with -4 volts would require a voltage of 140 volts. To obtain the same current with -8 volts would require a voltage of 220 volts. The effect of a grid-voltage change of 4 volts is the same as a plate-voltage change of 80 volts, and the amplification factor of the tube at that point would be $80/4 = 20$. The voltage gain of this combination would, according to Eq. (10–2), be calculated to

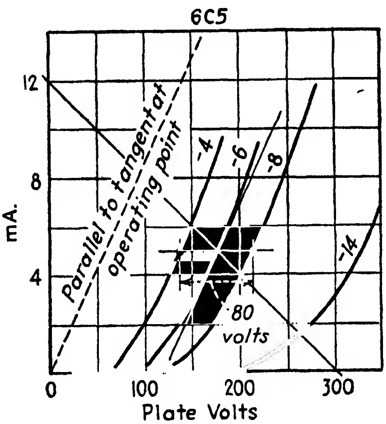

FIG. 10-4.—Determination of the voltage variation taking place across a 25,000-ohm load in series with a 6C5 tube by means of the load line.

$$v_g = \mu \frac{R_L}{R_L + r_p} = 20 \frac{25{,}000}{25{,}000 + 11{,}600} = 13.65 \qquad (10\text{–}2a)$$

A voltage variation of 4 volts, centered around the -6-volt operating point, should therefore give a voltage change across the load of $4v_g = 4 \times 13.65 = 54.6$. This value certainly checks surprisingly well with the value of 55 volts that we read from the load line.

Assume now that the plate characteristics are not available and that we are given only the values of the plate resistance and the amplification factor at the operating point recommended by the manufacturer, which is 250 volts on the plate and -8 volts on the grid, resulting in a plate cur-

rent of 8 ma. Since we have no way of knowing the value that the plate resistance will have at the actual operating point, we simply have to use the values as given by the manufacturer and trust that we shall not be too far off. If the voltage gain is calculated with $r_p = 10{,}000$ (as given by the manufacturer), Eq. (10–2) will yield a value of 14.3, and the voltage variation that could be expected across the load with a 4-volt variation on the grid would come out as 57.2 volts. Since the manufacturer states that characteristics may vary from tube to tube as much as 10 per cent, it is evident that even the value of 57.2 volts would be entirely satisfactory for predicting the tube performance. As long as we are interested only in the variation taking place, the equivalent-plate-circuit theorem would offer a satisfactory solution. But how about the quiescent point? If this tube were to be used in a problem involving direct voltages, we would not be satisfied with predicting the plate-voltage *change* taking place with a given grid-voltage *change* but should also know the quiescent point. The only way to arrive at some approximate value, if the plate characteristics are not given, is to assume again that the values for plate resistance and amplification factor at the operating point given by the manufacturer will hold true over a reasonable range and permit the substitution of the tube by a fictitious battery and resistance. Applying the principles outlined in Chap. IX, we find that the 6C5 tube operating at the values recommended by the manufacturer is equivalent to a battery of 170 volts and a resistance of 10,000 ohms in series with it. (Convince yourself that this combination takes 8 ma just as the actual tube does, and that this current changes with a change of plate voltage exactly as the actual tube current would.) If the grid voltage is raised from −8 to −6 volts, the fictitious battery will be lowered from 170 volts by an amount equal to the amplification factor times the grid-voltage change, or 40 volts in this case, which makes it 130 volts. If a combination of a battery of 130 volts and a resistance of 10,000 ohms is placed in series with another resistance of 25,000 ohms across 300 volts, as shown in Fig. 10–5, a current of 4.85 ma will result.

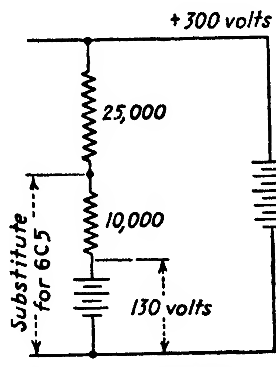

FIG. 10–5.—The same problem as shown in Fig. 10–4 may also be solved with the aid of the equivalent circuit.

This is again in sufficiently close agreement with the value as read on the load line.

From the example just outlined, the reader may gain the impression that the equivalent-plate-circuit theorem is entirely sufficient for the solution of all tube problems. The example was indeed chosen to increase his faith in the agreement of the results obtained by both methods. It must be remembered, however, that the disagreement is very pronounced as the

deviations from the operating point become larger. Suppose that we wished to know what current flows if the grid voltage is changed to -14 volts. The load line gives the correct answer, namely, 1.5 ma. If the characteristics were not available so that we could not use the load-line method, we would go back to our substitution. Decreasing the grid voltage from -8 volts, for which the tube is equivalent to 170 volts in series with 10,000 ohms, to -14 volts will increase the fictitious battery of 170 volts by 6×20 volts. This would make the battery equal to 290 volts. If the battery of 130 volts shown in Fig. 10–5 is increased to 290 volts, the current taken by the combination would be 10/35,000 amp, or approximately $\frac{1}{3}$ ma. This is seen to differ very materially from the actual value of 1.5 ma, as obtained from the load line. These considerations show that the equivalent-plate-circuit theorem gives increasingly erroneous results as the tube begins to operate in the curved parts of the characteristic.

10–8. Inductive Load in the Plate Circuit.—Equation (10-2) was derived under the assumption that the load was purely resistive. The equivalent-plate-circuit theorem, which after all is nothing but the application of the principle of superposition, is valid for any kind of load, however. The only condition that must be fulfilled is that the load must provide a conducting path for the operating or quiescent current of the tube; this condition rules out the use of a pure capacitance. In this case it would be necessary to by-pass the capacitance with a circuit element capable of passing the dc component of the plate current, in other words, resistance or inductance. In Fig. 10–6*a* is shown the diagram applying

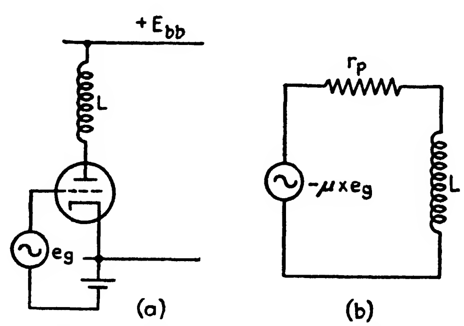

FIG. 10–6.—The load line cannot be used for an inductive load; in this case we must use the equivalent-plate-circuit theorem.

where the load of the tube consists of an inductance. If the voltage applied to the grid is a sinusoidal wave of the frequency f, then the fictitious voltage μe_g acting in the plate circuit of the tube is of the same wave shape and frequency. The performance of the circuit can therefore be calculated simply by applying the laws governing alternating currents to the

equivalent circuit shown in Fig. 10–6*b*. The rms value of the alternating current i_p that will flow in the circuit is given by

$$I_p = \frac{\mu E_g}{z} = \frac{\mu E_g}{\sqrt{r_p^2 + x_L^2}} \tag{10–3}$$

The alternating voltage that will appear across the inductance L is given by

$$E_z = \mu E_g \frac{r_L}{\sqrt{r_p^2 + x_L^2}} \tag{10–4}$$

The voltage gain obtained from the stage under this condition is then given by

$$v_g = \mu \frac{x_L}{\sqrt{r_p^2 + x_L^2}} = \mu \frac{1}{\sqrt{1 + (r_p/x_L)^2}} \tag{10–5}$$

A comparison of Eqs. (10–5) and (10–2) shows that in this case too the voltage gain can never exceed the value of the amplification factor because the numerator of the fraction following μ will always be smaller than the denominator. It will be noted, however, that for numerically equal values of R_L and x_L Eq. (10–5) will give a higher value for the voltage gain than Eq. (10–2). If we make, for example, the load resistance R_L in Eq. (10–2), or the load reactance x_L in Eq (10–5) equal to twice the plate resistance of the tube, the voltage gain will be 66 per cent of the amplification factor in the case of the resistive load, while it will be over 89.5 per cent in the case of the inductive load.

An inductive load in the plate circuit of a tube has another advantage over a resistive load. We have seen that with a resistive load it was necessary to have a high supply voltage since the voltage caused by the plate current through the load had to be compensated for. This is obviously not the case with an inductive load. The dc component of the plate current will cause only a small loss of voltage, determined by the dc resistance of the inductance only. Since for a well-designed inductance the resistance is always small compared to the reactance for the range of frequencies in which it is to operate, it is clear that the loss caused by the dc component is small compared to the values of the alternating voltage. Under these circumstances, it would always seem desirable to use an inductance as the load of a tube. Against the advantages just outlined, the following disadvantages must be taken into consideration, however.

10–9. Frequency Response of an Inductive Load.—For frequencies from a few hundred cycles per second to about 20,000 cps—this range is usually called "audio frequencies"—efficient inductances must usually be wound on an iron core, which makes them bulkier as well as more expensive than resistors. Furthermore, and this is of greater importance, Eqs. (10–4) and

(10–5) indicate that the voltage gain obtained with an inductive load in the plate circuit depends on the frequency of the alternating voltage applied to the grid. At a frequency for which the reactance $x_L = 2\pi fL$ is equal to the plate resistance of the tube, the voltage gain will be 0.707 times the amplification factor of the tube. It is therefore quite evident that the voltage gain will fall off as the frequency decreases. Examination of Eqs. (10–4) and (10–5) would make one feel, however, that with *increasing* frequency the voltage gain would approach the value of the amplification factor closer and closer. But even this is not true. In the construction of an inductance, especially one with a great number of turns, it is impossible to avoid capacitance effects between the turns and the layers of the winding. With increasing frequency these capacitance effects become so pronounced that they begin to act as a shunt parallel to the inductance. This shunting of the load obviously leads to a decrease in amplification; for this reason inductances are rarely used any more in the construction of amplifiers for audio frequencies.

Another feature—usually undesirable—accompanying the use of an inductance as a plate load should be pointed out. From the equivalent circuit of this combination, it is obvious that the voltage appearing across the inductance will be displaced in phase with respect to the voltage applied to the grid of the tube, and that this phase displacement will depend on the frequency of the applied voltage. In many types of amplifiers this is not a serious drawback, but in others it may be a very undesirable feature.

Figures 10–7 and 10–8 show graphs of the factor with which the amplification factor μ must be multiplied to obtain the voltage gain for various

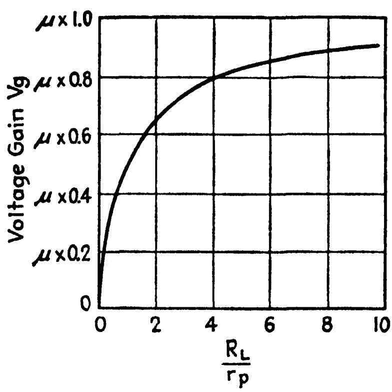

FIG. 10–7.—The voltage gain of a tube operating with a resistor as plate load.

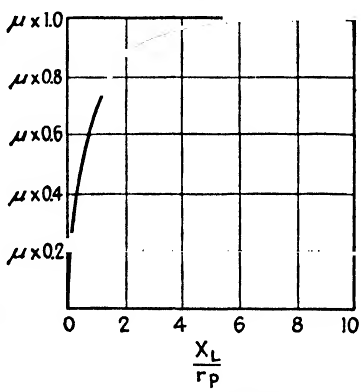

FIG. 10–8.—The voltage gain of a tube operating with an inductance as plate load.

values of R_L/r_p and x_L/r_p, respectively. In connection with Fig. 10–8 it should be pointed out that x_L is a function of the frequency, so that this

graph can also be considered as showing in a general way the relation between the voltage gain and the frequency for a given inductance.

10–10. Load Resistance for Maximum Power.—In order to obtain as high a voltage gain as possible, we saw that it was necessary to make the impedance of the load as high as possible in comparison with the plate resistance of the tube. In Sec. 4–13, we investigated the case of a generator furnishing electric power to a load over a resistance that could not be removed from the circuit. It was shown there that for maximum *voltage* the load resistance would have to be as high as possible, but if it is desired to obtain the maximum *power* in the load, it should have a resistance equal to the resistance in series with it. The equivalent-plate-circuit theorem states that within the tube is hiding a generator with a voltage equal to μ times the voltage applied to the grid, as well as a resistance equal to the plate resistance in series with this voltage. The plate resistance of the tube is consequently a resistance that cannot be removed from the equivalent circuit, and if we wish to develop the maximum ac power in the load connected to the tube, it will be necessary to make the resistance of the load equal to the plate resistance of the tube.

The manufacturer usually recommends a value for the load resistance, but it generally differs greatly from that of the plate resistance. This is not a contradiction to the considerations just presented. The value given by him is determined, not on the basis of maximum power for a given grid voltage, but on the basis of maximum ac power that the tube can furnish at all, regardless of what grid voltage may be required to do this. Furthermore, the question of distortion is also taken into account by him. These relations will be discussed in more detail in a later chapter.

In the two preceding paragraphs it was explained that in order to make a tube operate efficiently as an amplifier furnishing power to a load this load must have a definite resistance. It is often entirely impossible to construct the load with a resistance to match the particular tube that must be used. In Sec. 4–13 it was shown that, at least for the case of alternating current, it is possible to make a load with a given resistance appear as a different resistance. This is accomplished by means of a transformer. The same method is used in vacuum-tube circuits quite frequently. Thus, in ordinary radio receivers the final load to which the power must be delivered is the loud-speaker. It is impossible, or at least very inconvenient, to construct the coil in a loud-speaker with a resistance running into several thousand ohms, which is a usual value for the plate resistance of the tube used in this stage. The so-called "voice" coils of loud-speakers usually have a resistance from 8 to 10 ohms. This low resistance must be presented to the plate circuit of the vacuum tube as a resistance in the order of several thousand ohms. If the tube in question required, for instance, a load resistance of 2,000 ohms for its best performance and if the load had a resistance of 10 ohms, then it would be necessary to make the 10

ohms appear as 2,000. This is a ratio of 200:1, and the transformer necessary to accomplish this would have to have a turns ratio of $\sqrt{200}$, which is approximately 14. The actual and equivalent circuits are shown in Fig. 10-9.

Let us assume now that we were able to construct the actual load with a resistance of 2,000 ohms. It would seem that in such a case a transformer would be entirely superfluous since its ratio would have to be 1:1. But even in such a case it may be found desirable to use a transformer for the following reason. Suppose that the quiescent current of the tube at the operating point is 30 ma. This current will flow through the 2,000-ohm load resistance and will produce a voltage of 60 volts and a corresponding loss of wattage. Furthermore, the supply voltage must be increased by this amount over the value needed for the tube itself. Both these undesirable conditions can be avoided if we place a transformer with a turns ratio 1:1 between the load and the plate circuit of the tube. As far as the direct current is concerned, the resistance in series with the tube will be only the resistance of the primary winding, which in this case would probably be less than 100 ohms. But to an alternating voltage acting in the equivalent circuit, the transformer will appear like a resistance of 2,000 ohms. This example shows that whenever a tube is to be employed for the power amplification of alternating voltages it is desirable to use a transformer to couple the load to the tube.

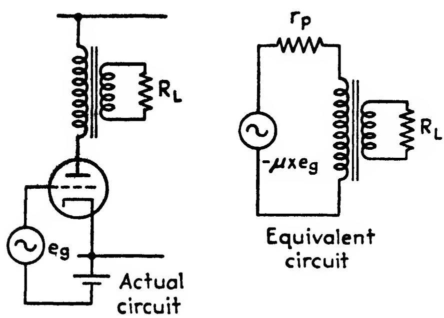

FIG. 10-9.—When the load is matched to the tube by means of a transformer, the equivalent circuit permits the calculation of the circuit performance.

10-11. The Tube as a Sensitive Relay.[1, 2]—In the preceding sections we have analyzed the performance of a tube when it was desired to reproduce an alternating voltage applied to the grid either in increased amount or at increased power across a load. In industrial applications it is quite often desired to use the tube as a sensitive relay. In order to appreciate fully what can be done with a tube in this field, it may be instructive to consider a typical example. Suppose that it is required to operate a relay from contacts which cannot carry any appreciable current and which may have at times a high contact resistance, say, around 100,000 ohms. Such a condition may be encountered if we want to operate a relay from contacts made by the pointer of an electric measuring instrument, for instance. Since the torque available in an indicating instrument is usually very low, it is evident that we cannot exert much contact pressure.

A relay such as is used in telephone circuits could be considered for this application. Telephone relays have been found very reliable in operation and may be obtained with various coil resistances, running from a few hundred to approximately 4,000 ohms. A relay with 4,000 ohms coil resistance will be found to require from 5 to 10 ma coil current for operation, depending on the number of contacts that must be actuated; the current must usually be reduced to 1 to 2 ma to make the relay drop out. The contacts of such relays usually handle 1 amp at 110 volts ac, which is more than sufficient to handle a fairly heavy contactor.

If the coil resistance is 4,000 ohms and the required operating current is given as 10 ma, it is evident that a direct voltage of 40 volts is required for the operation of the relay. But if the contacts that are to operate the relay, according to the statement made above, may have at times a contact resistance in the order of 100.000 ohms or so, it is quite obvious that they could not be used to operate the relay from a 40-volt source because the current might then be as low as 0.4 ma. On the other hand, increasing the operating voltage to a value sufficient to ensure operation, even under the conditions of highest contact resistance, would require a voltage of 1,000 volts; this, in turn, would cause a burnout of the relay coil if the operating contacts exhibited a low resistance at any particular operation and a burnout of the contacts when their resistance was high.

10–12. Use of the Load Line for Relay Problems.—In a vacuum tube, the plate current can be controlled by a change of grid voltage. As long as the grid is negative with respect to the cathode, we have seen that the voltage applied to the grid will not have to furnish any current. It is this property which makes the tube ideally suited for the solution of the above problem. We then proceed as follows. First, we select a tube capable of passing a plate current of 10 ma. Such a tube might be a 6J5 or a 6P5. The former is the more common one and is found on the preferred list. In order to make sure that no grid current will flow, we decide that the grid voltage should never be less negative than -2 volts. Inspection of the characteristic shows that in order to pass 10 ma with -2 volts on the grid, the tube must have a plate voltage of approximately 140 volts. If the relay coil has a resistance of 4,000 ohms, there will be a voltage of 40 volts needed to pass a current of 10 ma through it. The total voltage required for the operation of the tube and relay will therefore be $140 + 40 = 180$ volts, and the circuit will then look as shown in Fig. 10–10. Now we draw a load line (marked *a*) of 4,000 ohms into the tube characteristic, as shown in Fig. 10–11. This load

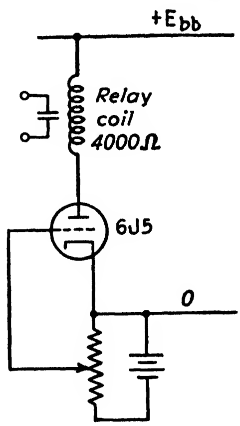

FIG. 10–10.—A dc relay is to be operated in the plate circuit of a 6J5.

line indicates that a grid voltage of −10 volts will bring the current down to approximately ⅛ ma and that with −11 volts on the grid the current will be reduced to zero. Therefore, any arrangement that permits us to change the grid voltage on this tube from −2 to −11 volts will result in the desired operation of the relay. Observe (1) that it now requires only a

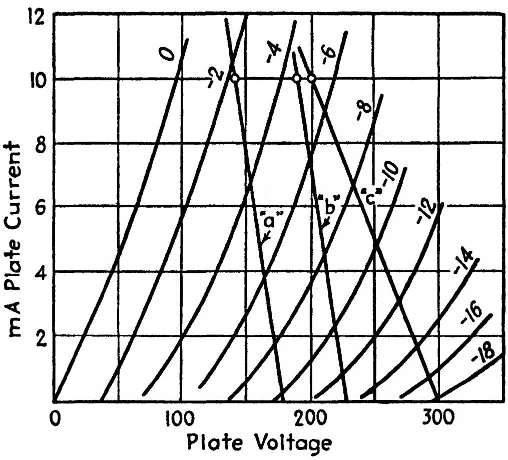

Fig. 10–11.—The plate characteristics and the load line corresponding to 4,000 ohms provide the answer to the problem shown in Fig. 10–10.

change of 9 volts to cause operation of the relay that would otherwise require a change from 0 to 40 volts; (2) that whatever source is furnishing this voltage to the grid will not be called upon to furnish any current. It can therefore be applied over a high resistance without fear of operational failure.

10–13. Range of Supply Voltage for Relay.—The direct supply voltage has to be 180 volts. This is a minimum value; it is entirely satisfactory to use a higher voltage if it should be easier to obtain. Thus, it may be possible to operate the tube from a 230-volt dc system already available. In this case we would draw a load line from the 230-volt point on the X axis, representing a 4,000-ohm resistance. This is also shown in Fig. 10–11, where the line representing this case is marked *b*. It is seen that in order to obtain a current change from 0 to 10 ma under this condition, the grid voltage will have to be changed from −14 to approximately −4.5. This is an entirely satisfactory method of operation as long as the maximum wattage dissipated in the tube does not exceed the value given by the manufacturer. The maximum permissible wattage consumed in the tube, also called the "plate dissipation," is given as 2.5 watts. This value is not exceeded in the second case, where a voltage of 190 volts exists across a tube with a current of 10 ma flowing, resulting in a plate dissipation of 1.9 watts. Suppose, however, that the available plate voltage is 300 volts or higher. Since the voltage across the relay with the rated current

is always 40 volts, there would then be left across the tube a voltage of 260 volts or higher, which is seen to result in a plate dissipation in excess of the permissible maximum. In such a case it may be found desirable to place in series with the relay an additional resistance of sufficient value to reduce the voltage across the tube at the rated current to the permissible value. Thus, we may place in series with the relay of 4,000 ohms a resistance of 6,000 ohms, making the total load in the plate circuit 10,000 ohms. With 10 ma flowing, the voltage across the total load will then be 100 volts; with a plate-supply voltage of 300 volts, there will then be left across the tube only 200 volts, bringing the plate dissipation down to 2 watts. This condition is indicated by the load line marked *c* in Fig. 10-11. It will be noted that under this condition the grid voltage will have to be changed from approximately -18 to approximately -5 volts in order to obtain operation. This is a total voltage swing of 13 volts, compared with the 9- or 10-volt swing of case *a* or *b*. In many cases this loss of sensitivity is of no consequence as far as the control voltage is concerned; in others it might be an undesirable feature.

10-14. Modes of Operation of the Grid Circuit.—To come back to the original condition indicated by the load line marked *a* in Fig. 10-11, the grid voltage had to be -2 volts to energize the relay and -11 volts to make it drop out. Figure 10-12*a* shows the fundamental circuit to ob-

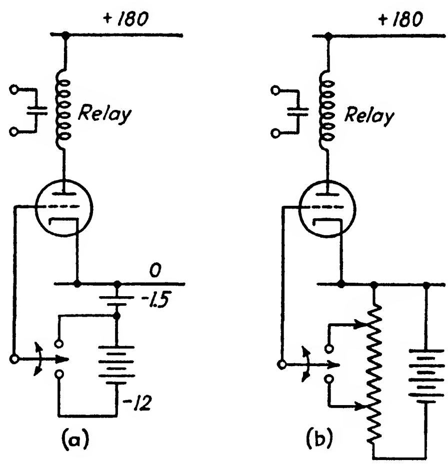

FIG. 10-12.—Changing the grid voltage between the two values found with the aid of Fig. 10-11 will energize and de-energize the relay

tain this result. A battery of 12 volts (11 volts would do, but with dry cells used for this purpose we cannot obtain exactly 11 volts) is connected with its positive terminal to the cathode of the tube, and the battery is tapped at 1.5 volts. A single-pole double-throw contacting mechanism is used to connect the grid to either the 1.5-volt tap or to the full 12 volts.

Figure 10–12*b* shows a connection where a source with any voltage higher than 12 volts can be used and where the exact desired operating voltages can be obtained by placing a voltage divider across the source. In both of these circuits it will be noted that the operating contacts will have to be double-throw, but the current that the contacts have to carry is only the grid current of the tube, which may be in the order of $\frac{1}{10}$ μa or less. The need of a double-contact mechanism to operate the relay in the plate circuit of the tube is a serious handicap. It would evidently be more desirable if operation could be obtained with a single contact only. If the contact mechanism in question is capable of carrying a few microamperes, then the desired result can be achieved by means of the two circuits shown in Figs. 10–13*a* and *b*. Suppose that in Fig. 10–13*a* the contacts *S* are open. The grid is then seen to be connected to the −11-volt

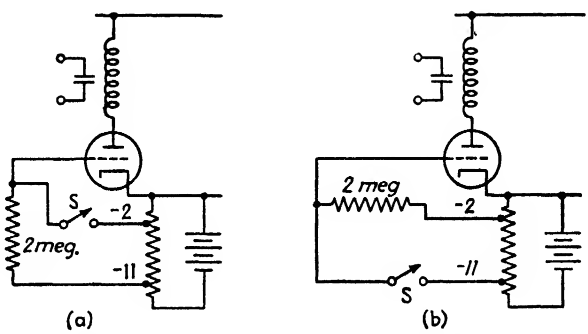

FIG. 10–13.—If the contact from which relay operation is desired can carry a few microamperes, the circuit shown in Fig. 10–12 can be replaced by the two modifications shown here.

potential through a resistance of 2 megohms. When contacts *S* close, however, the grid is connected to the −2-volt potential, and operation of the relay in the plate circuit will therefore take place. With contacts *S* closed, there will be a current flowing through the 2 megohms given by the voltage existing across this resistor, which is 9 volts in this instance, divided by 2 megohms. This is seen to be 4.5 μa. Let us assume now that the contacts *S* may have a contact resistance as high as 100,000 ohms. Under this condition there would then be a total resistance of 2.1 megohms connected across the 9 volts existing between the two operating taps, and the voltage across the contacts would be $9 \times 0.1/2.1$. This amounts to 0.475 volt, and the actual grid voltage with the contacts closed under this condition will not be −2 volts as required but approximately −2.5 volts, which would give a current less than required for the operation of the relay. In order to compensate for this possibility, it is therefore desirable to place the tap not at −2 but at least at −1.5, or preferably at −1, or even 0. It can be assumed that no harm will come to the relay if the operating current is a few milliamperes higher than 10. The arrangement

as just outlined will then ensure operation even under the adverse conditions of 100,000 or even 200,000 ohms contact resistance.

The arrangement shown in Fig. 10–13*a* would energize the relay in the plate circuit with the closure of the contacts *S*. If, for any reason, the opposite mode of operation fits better into the control scheme, the connection shown in Fig. 10–13*b* gives the desired result. In this case, with the contacts open, the grid is on the less negative tap and the relay, therefore, is energized. But with the contacts closed, the grid is seen to be connected to the more negative voltage tap, thus de-energizing the relay coil. Similar considerations, as made in the preceding case, indicate that the margin of safety of operation is increased by choosing the taps somewhat beyond the minimum values required for operation.

10–15. Common Supply for All Operating Voltages.—In the schemes so far discussed, the negative voltage required for the supply of the grid voltage was obtained from a battery. In commercial equipment this is not a desirable solution, since the deterioration of the battery obviously finally leads to failure to operate. There are various ways in which the required voltage for the grid circuit can be obtained. If the available system is a 230-volt dc system, for instance, it is possible to apply the scheme shown in Fig. 10–14. Here not only is the grid bias obtained directly from the line but the current for the heater is also obtained from the supply system. It has been shown that in the particular example the minimum plate-supply voltage had to be 180 volts. Since a total of 230 volts is available, the two resistances R_1 and R_2 in Fig. 10–14 can be made of such value that there are 180 volts across R_2 and the filament, and 50 volts across R_1. This is not meant to imply that a different split of the total voltage is not just as satisfactory. Thus, a division of the total 230 volts into 200 volts for the operation of the plate circuit and 30 volts to furnish the necessary grid voltage is just as satisfactory.

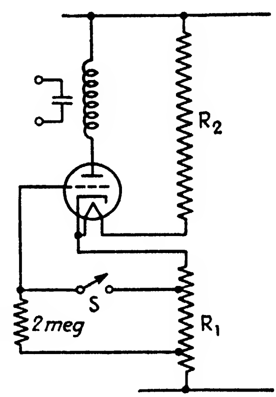

FIG. 10-14.—If a dc supply system is available, all operating voltages, including the filament supply, can be obtained from this system.

10–16. Use of a Voltage-doubler Circuit for Plate and Grid Voltages.—If a dc system is not available, the voltage-doubler circuit, discussed in Chap. VIII and shown in Fig. 8–12, offers a convenient solution to obtain the required operating voltages. In this particular example the voltage obtainable across one filter capacitor would be less than the required 180 volts, however, assuming that the doubler circuit is operated directly from the 110-volt line, as shown in Fig. 10–15*a*. A step-up transformer will therefore be required, as shown in Fig. 10–15*b*. Since with this circuit the voltages appearing across the two capacitors are equal, the volt-

age available for the operation of the grid circuit will, of course, be greatly in excess of what is needed and will have to be reduced to the desired operating voltages by means of a voltage divider, as shown in Figs. 10–15*a* and *b*.

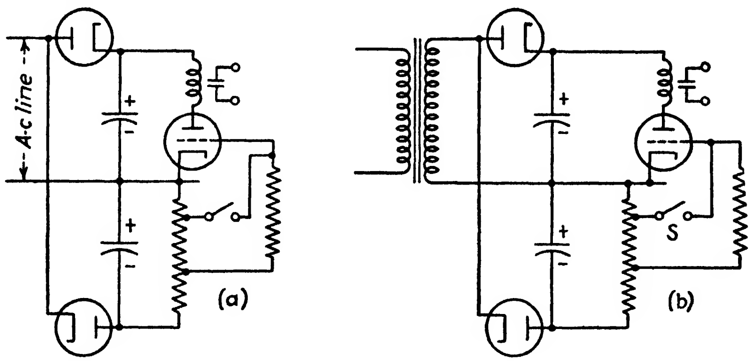

FIG. 10–15.—A voltage-doubler tube, with or without transformer, may be used to supply plate voltage and bias voltage for the vacuum-tube relay.

The use of a transformer is recommended even if the line voltage is sufficient to furnish the required operating voltage for the plate circuit because it is then possible to ground the cathode of the tube and thus have the contacts in the grid circuit also near ground potential, with a consequent reduction of the danger of "fireworks." If the line has one terminal grounded and it is desired to operate the circuit directly from the line without a transformer, then in the case of a permanent installation the desirable connection just described can, of course, be achieved by connecting the cathode of the tube to the grounded side of the line; but if the device is to be plugged in, such an operation obviously cannot be guaranteed.

10–17. Considerations Applying to Voltage Divider.—The connection shown in Fig. 10–14 suggests that the solution shown there would also be possible and even desirable if it is intended to use a simple rectifier circuit furnishing only a current in the order of milliamperes for the operation of the circuit. Thus, one might think that operation of the circuit shown in Fig. 10–16 would be identical with the operation of the circuit shown in Fig. 10–14; indeed this would be the case provided that the values of the resistors R_1 and R_2 were the same in both cases. However, since rectifier circuits used for the operation of tubes are not designed to furnish currents so high as would be required for the heating of the filament, the resistor values for the circuit shown in Fig. 10–16 have to be higher than those for Fig. 10–14. Now it is evident that the plate current of the tube

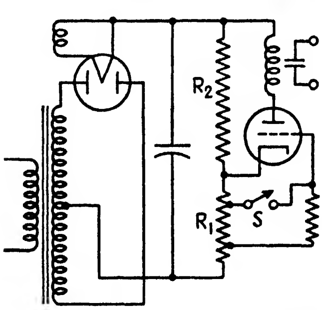

FIG. 10–16.—This circuit looks almost like the one shown in Fig. 10–14, but it is not so satisfactory.

will have to flow through resistor R_1 in order to return to the negative end of the supply system, whether this is a 230-volt dc system or a rectifier system. In the case of Fig. 10–14 the current through the resistors R_1 and R_2 must be equal to the value required for the operation of the filament or heater; for a 6J5 tube, such as was assumed in the examples discussed in the preceding sections, this is 0.3 amp or 300 ma. The additional 10 ma or so of plate current, when the relay is made to operate, does not change the voltage across the resistor R_1 appreciably. But if the current through the voltage divider—or "bleeder circuit," as it is sometimes called—is of the same order of magnitude as the operating current of the relay, such as 10 or 20 ma, then the voltage distribution is altered considerably by the change of current through this resistance. It will be seen later that the circuit shown in Fig. 10–16 is a special case of a resistor in the cathode circuit of a tube, and this case will be treated in great detail later.

10–18. Fundamental Timing Circuit.—In Secs. 2–14 to 2–17, the circuit performance of a series combination of capacitance and resistance was considered in detail. It was shown there that when such a combination is placed across a potential difference, the capacitor voltage will keep on changing until the current through the resistor is zero; it was also shown that in a time equal to one time constant—the latter given by the product CR—the voltage across the capacitor will change 63 per cent of the total voltage change necessary to reach the condition of zero current through the resistor. With the commercially obtainable capacitor values in the order of a few microfarads, it is evident that resistor values in the order of megohms are required in order to obtain time constants of a few seconds or more. In Fig. 10–17 a capacitor of 2 μf is shown connected over a resistance of 5 megohms to a source of 65 volts. The time constant of this combination is 10 sec. Consequently after this time the voltage across the capacitor will be 63 per cent of 65, or about 41 volts. Suppose now that we wish to observe the charging process and place a 100-volt meter across the capacitor. A good dc voltmeter of 100-volt range may have a resistance of approximately 100,000 ohms. It is obvious that, with such a resistance placed parallel to the capacitor, the current flowing through the 5 megohms, which according to the theory should all flow through the capacitor, will by-pass the latter and flow through the resistance of the meter. The voltage across the capacitor could therefore never rise beyond the value 65 × 100,000/5,100,000, or about 1¼ volts because this is the value of voltage that would exist across the 100,000 ohms of a voltage divider consisting of 5,000,000 ohms and 100,000 ohms placed across 65 volts. It is therefore clear that, if the voltage across the capacitor is to

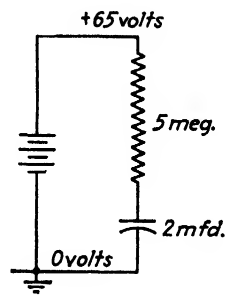

FIG. 10 17.—A series combination of resistance and capacity forms the heart of most electronic timing devices.

be used for the operation of a meter or relay, the device placed across the capacitor must draw a current that is small compared to the current used for charging the capacitor. Here again the vacuum tube is a device ideally suited to accomplish the desired purpose. Practically all electronic timing relays make use of capacitor-resistor combinations for the desired timing operation.

Let us analyze the simple circuit shown in Fig. 10–18. The operating voltage for the tube is obtained from a 230-volt dc system. Fifty volts are split off by means of the voltage divider R_1-R_2-R_3 to furnish grid bias. Let us assume again that the tube is a 6J5 operated under conditions outlined in Secs. 10–10 to 10–15. The relay will then operate at the instant when the grid of the tube becomes less than 2 volts negative. The resistor-capacitor combination R and C is placed across a total of 65 volts. (The significance of this will be discussed presently.) As long as switch S is closed, the capacitor voltage will be zero and the grid will be 50 volts negative with respect to the cathode. This will, of course, result in zero plate current. At the instant switch S is opened, the capacitor begins to charge. If for a moment we assume that the upper terminal of the capacitor is disconnected from the grid of the tube, then there can be no question that the capacitor voltage will rise and finally reach a voltage of 65 volts (which is the condition of zero current through the charging resistor). The conditions are exactly the same as for Fig. 10–17. With the grid of the tube connected to the capacitor, the behavior is the same as long as no grid current is flowing. Since operation of the relay takes place at the instant when the grid is 2 volts negative with respect to the cathode, it means that this instant will be reached when the capacitor voltage is 48 volts. This represents a fraction of $^{48}\!/_{65}$ of the total voltage, or approximately 74 per cent, and, according to the capacitor charge and discharge chart (Fig. 2–8), 74 per cent will be reached in 1.35 time constants. Thus, if the capacitor has a value of 2 μf and the charging resistor is 5 megohms, making the time constant equal to 10 sec, the operation of the relay in the plate circuit of the tube would take place 13.5 sec after the opening of the switch S.

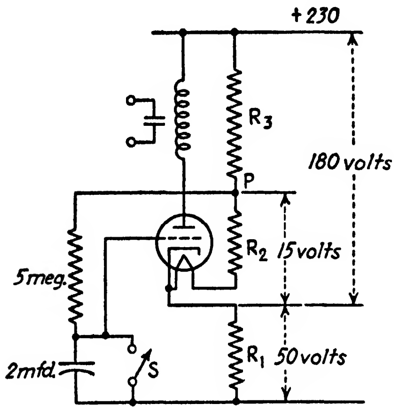

FIG. 10–18.—Combining the simple circuit shown in Fig. 10–17 with a vacuum-tube relay results in a timing relay.

10–19. Limiting Action of Grid Current.—We may ask whether current values may be reached in the plate circuit beyond the permissible limit when the grid finally becomes 15 volts positive with respect to the

cathode, as it seemingly does in this case. Fortunately, however, this condition will not be reached, owing to the fact that the grid begins to draw current when it becomes positive with respect to the cathode. As a matter of fact, owing to the initial velocity of the electrons emitted from the cathode, grid current begins to flow already when the grid is still slightly negative with respect to the cathode. With zero grid, this value may reach several hundred microamperes. Examination of the diagram shown in Fig. 10–18 shows that at the instant when the grid is -1 volt with respect to the cathode there will be a voltage of 16 volts across the 5-megohm resistors; the current through the resistor is then 3.2 μa. With zero volts on the grid, similar reasoning indicates a current of 3 μa. But we can be almost certain that the grid current of the tube will reach a value of between 3 and 4 μa somewhere between -1 volt and zero grid volts; at this point the current flowing through the 5 megohms will go on to the grid, and further charging of the capacitor will cease. Consequently, the plate current will not rise beyond a value given by approximately zero grid voltage. The principle just outlined is one of the utmost importance. Whenever there is a high resistance in series with a grid of a tube, the actual grid voltage cannot become positive to any marked degree no matter how high a positive voltage is applied on the other end of the resistor. It is the voltage caused by the grid current in this high resistance that prevents the grid from going positive. As we shall see later, the same principle is involved in the method of grid-leak detection.

10–20. Choice of Charging Voltage.—But why did we choose a voltage of 65 as the charging voltage of the capacitor? Could we not obtain the same time delay with a much smaller capacitor by arranging the circuit in such a way that operation would take place after a period of 4 or 5 time constants? Suppose that we were to change the circuit as shown in Fig. 10–19. The capacitor voltage would now tend toward a maximum value of 50 volts, and operation of the relay would take place when the fraction $^{48}/_{50}$, or 96 per cent, of the charging voltage was reached. Consulting the capacitor charge and discharge chart indicates that operation will now take place after 3.2 time constants. With the same capacitor and resistance values as before, a time delay of 32 sec could now be obtained or, if we wanted to have only $13\frac{1}{2}$ sec as before, the value of the capacitor could be reduced accordingly. Why is this solution not a more economical and preferable one? If tubes were always exactly identical, there would indeed be no reason for not using the latter solution, but characteristic values change as much as 10 per cent, and it will be found

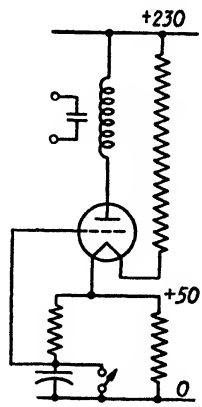

FIG. 10–19.—This circuit will give longer time delay than the circuit shown in Fig. 10–18, but it will not be so consistent.

that in order to obtain the same plate current in two tubes of the same type the grid voltages may differ as much as 1 volt. Suppose now that for any reason whatsoever the desired current of 10 ma required a grid voltage of -3 instead of -2 volts. Operation would then take place when the capacitor voltage has reached 47 instead of 48 volts. In the circuit shown in Fig. 10–18, the fraction will change from $^{48}\!/_{65}$ to $^{47}\!/_{65}$ = 72.3 per cent. Operation will now take place in 1.28 compared to 1.35 time constants. The change in actual time would consequently be 0.7 sec. In the case of Fig. 10–19, on the other hand, the ratio will change from $^{48}\!/_{50}$ to $^{47}\!/_{50}$ = 94 per cent. The capacitor charging chart indicates a drop to 2.8 time constants from the previous value of 3.2 time constants. In actual time this represents a drop of 4 sec, from 32 to 28 sec, which is not only a higher absolute value but also a higher percentage drop. From these considerations it becomes apparent that if fairly accurate repeat performance of a timing circuit is desired two conditions should be fulfilled in order that changes in tube characteristics may not cause undesirable variations. The first condition is that the total voltage swing on the capacitor should be made as large as conveniently obtainable, and the second is that the operation of the relay should take place while the capacitor voltage is still changing fairly rapidly. This latter condition is equivalent to saying that the circuit should be designed so that the point of operation will be somewhere around 1 to 2 time constants, with the upper value to be avoided if possible. These rules are flexible, of course, and the designer of timing circuits has to use his judgment. The all-important consideration is to design a circuit in such a way that a shift of the operating potential of a volt or two on the grid of the tube will not cause a change of the timing beyond the limits permitted by the particular problem.

Figure 10–18 showed the circuit as operating from a dc supply system. It is, of course, clear that the voltage-doubler system shown in Fig. 10–15 is just as applicable in this case as in the former. As a matter of fact, for the operation of a timing relay the high negative voltage available from the doubler circuit is a distinct advantage in the light of the remarks made in the preceding paragraph.

10–21. Adjustable Timing Range.—In the two circuits shown in Figs. 10–18 and 10–19, the grid started at a level 50 volts below the cathode and swung up toward an end level of 15 volts above the cathode (although the brakes were applied by the grid current before it could get that far) or to the cathode level itself, respectively. The tube does not know about this, however, and simply operates the relay when its grid is at a level 2 volts below the cathode. Therefore, any adjustments by means of which we can change the time that will elapse between the opening of switch S and the instant when the grid swings through the -2-volt level with respect to the cathode will also adjust the timing.

Evidently we have two possibilities of achieving this. Either we can change the time constant of the resistor-capacitor combination, or we can change the charging voltage. In Fig. 10–18 the charging voltage is obtained from the junction point P between resistors R_2 and R_3. If the two resistors are combined into one and converted into a potential divider, P can be moved up and down to any voltage level. Variation of timing is now obtained by changing the charging voltage. Reduction of the charging voltage should not be carried too far, however, because sliding P down in Fig. 10–18 changes the circuit toward the circuit shown in Fig. 10–19 with its objections. Sliding P up, on the other hand, makes the circuit operate in a smaller fraction of the time constant, which is entirely satisfactory.

The second method of obtaining adjustable timing consists of changing the time constant. This can be achieved by changing either the resistance or the capacitance values. No way has been found yet to produce variable capacitors in the range required for timing relays, and the only practical solution consists therefore of making the charging resistor variable. Owing to manufacturing difficulties in producing variable resistors in the megohm range, it is advisable to limit the working range to about 4:1; if a greater range is desired, a tap switch permitting the choice of various capacitors should be employed. The tap switch then gives a rough control in steps, while the variable resistor provides fine adjustment.

10–22. Effects of Leakage Resistances and Grid Current.—Capacitors are more expensive than resistors. If it is desired to obtain a combination with a time constant of 10 sec, will it then not be more economical to use a capacitor of ½ μf and a resistor of 20 megohms instead of a capacitor of 2 μf and a resistor of 5 megohms? If it were possible to insulate the tube circuits perfectly and if the grid current of the tube were zero, then indeed we could reason in this manner. There are, however, always leakage paths from one pin of the tube to the next, and these paths are generally in parallel to the charging resistor or to the capacitor. Their values usually change with humidity. If a given leakage path has a resistance of 200 megohms on a dry day and drops to 50 megohms on a humid day, the effect of this change will not be too serious if it is parallel to a resistor of 5 megohms since even at its lowest value it will not affect the discharging or charging current by more than 10 per cent. But if this same leakage path is placed parallel to a 20-megohm charging resistor, the operation will be influenced much more severely. In cases where very long timing periods have to be obtained, calling for high capacitor as well as high resistor values, special precautions must be taken. These will be discussed in a later chapter. The same reasoning applying to the leakage paths also applies to the grid current of the tubes. Different tubes of the same type may also show considerable differences in their grid currents, even with the grid at considerable negative voltage with respect to the cathode. Evi-

dently if the charging resistor of a timing combination is extremely high, which means that the charging current is extremely low, then any variations in the grid current will cause a considerable variation in the charging process. The manufacturer of tubes usually states a maximum value of grid resistance permissible in the tube circuit, usually not in excess of 1 megohm. The designer of timing circuits need not be afraid, however, to exceed this value considerably as long as the charging current is in the order of 1 to 2 μa. Nevertheless, very high values of resistance in the timing circuits should be avoided. As to the capacitors used in these circuits, it is quite evident that their leakage resistance also affects the performance of the circuit. If the leakage resistance were a constant value, the matter would not be too serious, but if for any reason humidity affects the latter, the performance of the timing circuit will not be satisfactory. The capacitors used should therefore be of the highest grade obtainable, and their resistance should be in the order of 100 megohms. The upper limit of these timing arrangements does not exceed about 40 sec if reasonable accuracy without special precautions is expected. If high accuracy is not required, on the other hand, or if it is possible to insulate the grid circuit of the tube specially, delays of several minutes can be obtained.

10–23. Two Special Timing Circuits.—Figures 10–18 and 10–19 show the simplest possible circuit to obtain a timing operation. Of the many modifications possible, however, the circuit shown in Fig. 10–20 may be of special interest. With switch S closed, the grid is at approximately zero potential with current flowing through resistor R_2 to the grid and through resistor R_1. Upon opening S, the current through this network will be maintained by the capacitor that begins to discharge. During this period the grid will remain essentially at zero potential. When the current flowing through resistor R_2 has decreased to such a value that with the tube removed from the circuit the grid terminal is negative; in other words, when point X on its own accord becomes negative with respect to the cathode, the current to the grid ceases under actual conditions, and the grid becomes progressively negative, finally resulting in zero plate current. The circuit is interesting from the point of view that the time constant governing the discharge is not the same for the whole timing process. During the first part, *i.e.*, when the grid is conducting, the effect is practically equivalent to connecting point X to the cathode, and the time constant is then given by the product of the capacitance and R_2. When the capacitor voltage has reached a value such that with the tube removed the grid would be just zero, from then on the discharge is governed along a curve with a time constant given by the product of C

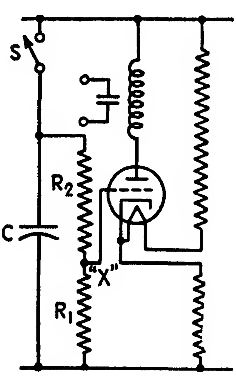

FIG. 10-20.—Another arrangement of an electronic timing circuit.

and the sum $R_1 + R_2$. The circuit has the advantage that the full plate voltage is being utilized for the charging of the capacitor.

We have seen that in these timing devices operation has been obtained when the capacitor has charged or discharged to a point where the tube conducts. An interesting application results when the relay in the plate circuit, upon closing, charges or discharges the capacitor to the original value. We have then a self-repeating timing relay. This circuit is also called an "interrupter." It can be used for operating signs or for testing apparatus. The simplest circuit incorporating this is shown in Fig. 10–21. One set of relay contacts is used to discharge the capacitor at the instant the tube operates. For the protection of the contacts, it is desirable to place in series with them a resistor of 10 to 50 ohms in order to limit the discharge current. This circuit would result in energization of the relay for only an instant. By having two relays in the plate circuit, one of them so adjusted that it operates considerably ahead of the time when the discharge relay operates, we can have control not only over the total cycle but also over the on-off periods within the cycle. Instead of two relays, it is possible to use a second tube and to arrange the circuit in such a way that operation of this tube will be obtained ahead of the tube that controls the discharge of the capacitor.

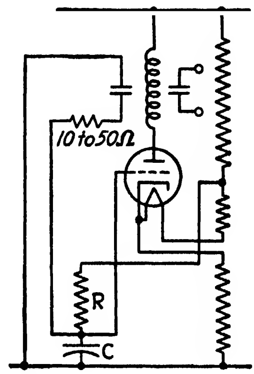

FIG. 10–21.—A self-interrupting timing relay.

PROBLEMS

10–1. Refer to the plate characteristics of a 6J5 tube shown in Fig. 14–15. By what voltage and resistance could the tube be replaced when operating at around 200 volts on the plate and −6 volts on the grid?

A resistor of 20,000 ohms is now placed in series with the tube, but the voltage across this series combination is to be raised to such a value that the actual voltage across the tube remains at 200 volts.

a. To what value must the supply voltage be raised?

b. In series with the −6 volts on the grid is placed an alternating voltage of 2 volts rms. What alternating voltage will appear across the resistor in series with the plate?

10–2. The resistor is now replaced by a choke coil of 5 henrys inductance and 1,000 ohms resistance, and the direct voltage applied to this series combination is readjusted so that the voltage across the tube is again 200 volts. An alternating voltage of 2 volts rms is applied to the grid, again in series with the −6 volt dc bias. The frequency of the alternating voltage assumes successively the following values: 20, 50, 100, 200, 500, and 1,000 cps.

a. What supply voltage is needed for the combination (*i.e.*, the direct voltage across tube and choke)?

b. What alternating voltage will appear across the choke for the various frequencies?

10–3. A type 6L6 tube in triode connection is operated at 250 volts plate voltage and −20 volts on the grid. Under this operating condition the plate current is 40 ma, the plate resistance 1,700 ohms, the amplification factor 8, and the transconductance 5,000 micromhos, as given by the manufacturer.

An alternating voltage of 5 volts rms is now applied to the grid in series with the −20 volts bias.

a. In order to obtain maximum ac power in the load, what resistance must the load have?

b. To what value must the voltage across the combination of tube and load be raised to ensure 250 volts across the tube itself?

c. What will be the ac wattage in the load?

d. Make the load resistor 500 ohms larger and 500 ohms smaller than the optimum value, and calculate the wattage in the load in each case.

10–4. A 6Q7 tube and a 100,000-ohm resistor are in series across a voltage such that the voltage across the tube itself is 200 volts with −2 volts on the grid. The plate characteristics of the tube are shown in Fig. 10–22.

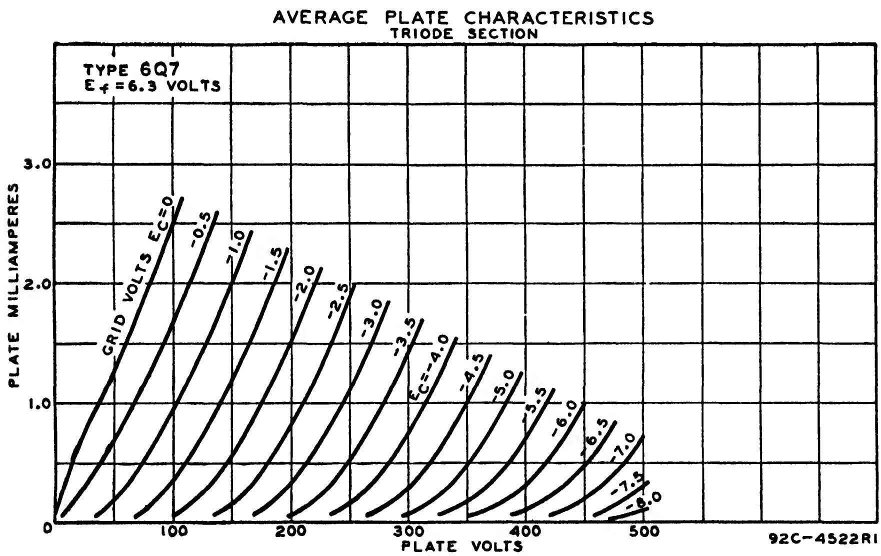

FIG. 10–22.—The plate characteristics of a 6Q7 tube. (*Courtesy Radio Corporation of America.*)

a. What total voltage is needed for the combination?

b. If the grid voltage is changed by 0.5 volt, how much will the voltage across the tube change, and how much will the voltage across the 100,000-ohm resistor change?

10–5. A 6J5 tube and a resistor of 100,000 ohms are placed in series across 400 volts. What voltage change will be produced across this resistor with a grid-voltage change from −2 to −4 volts? Compare the actual voltage gain, as obtained from the plate characteristics and the load line, with the value you would obtain by the use of Eq. (10–2). For plate characteristics see Fig. 14–15.

10-6. Given a 230-volt dc system and a relay with a coil of 6,000 ohms resistance. The relay requires 8 ma to close its contacts and 2 ma to drop out. It is desired to energize the relay when the pointer on an electric meter comes up against a fixed stop, but this contact has too high a resistance to energize the relay directly and can carry only a few microamperes. Design a tube circuit with all voltages obtained from the 230-volt source.

10-7. Two different types of tubes have the same mutual conductance of 1,000 micromhos, but one has a plate resistance of 10,000 ohms, the other a plate resistance of 50,000 ohms. Suppose that both tubes have a load of 40,000 ohms in series with the plate. What voltage gain will be obtained from the two combinations?

10-8. A type 6J5 tube, a relay, and a capacitor-resistor combination are connected in a circuit as shown in Fig. 10-23. The relay has a coil with a resistance of 4,000

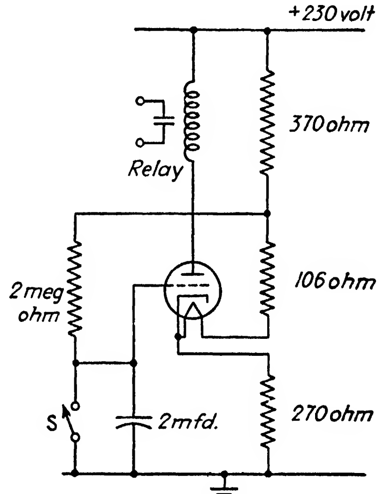

FIG. 10 23.—Circuit diagram for Prob. 10-8.

ohms, and it closes its contacts when the current in the coil reaches 8 ma. For plate characteristics see Fig. 14-15.

a. The manufacturer specifies 6.3 volts and 0.3 amp for the operation of the filament. Check whether the resistance values given in Fig. 10-23 will actually provide these values for the filament.

b. What time delay will there be between the opening of switch *S* and the operation of the relay?

c. If a time delay of 1.2 sec is desired between the opening of *S* and the operation of the relay, what resistance must be put in place of the 2-megohm resistor?

10-9. A type 6SF5 triode, the plate characteristics of which are shown in Fig. 10-24, is connected to a voltage divider, as shown in Fig. 10-25. If the voltage applied to the grid varies between -0.5 and -2.0 volts (which variation could be produced either by a change of the bias applied to the tube, or, for instance, by the placing of an alternating voltage with a peak of 0.75 volt in series with a fixed bias of -1.25 volts), how much will the potential of point *A* vary? If the grid-voltage variations between the above given limits were sinusoidal, what would be the rms value of the alternating voltage appearing between points *O* and *A* (or, for that matter, between points +420 and *A*)?

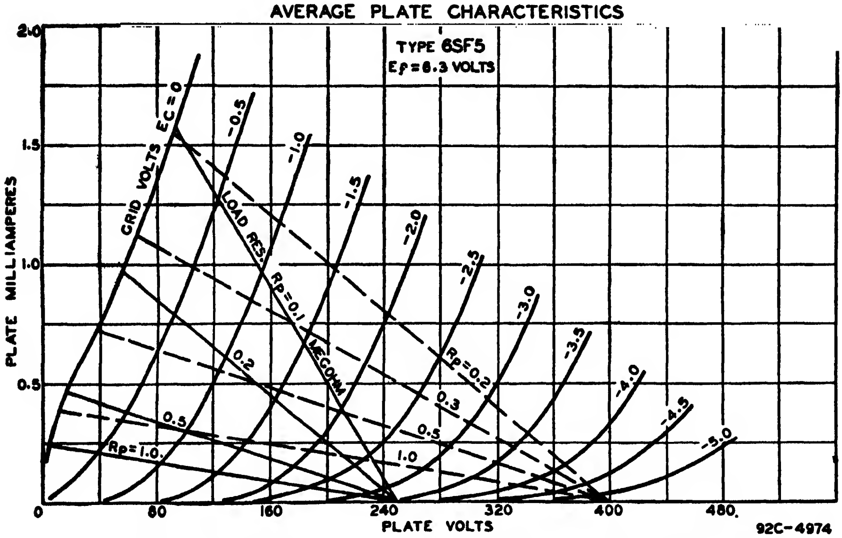

FIG. 10-24.—Plate characteristics of a type 6SF5 tube. (*Courtesy Radio Corporation of America.*)

10-10. Three volts rms are applied to the grid of a type 45 tube operating in a circuit, as shown in Fig. 10-26. At the quiescent operating point the tube is found to operate with a plate current of 31 ma, the amplification factor is 3.5, the plate resistance 1,650 ohms, and the transconductance 2,125 micromhos.

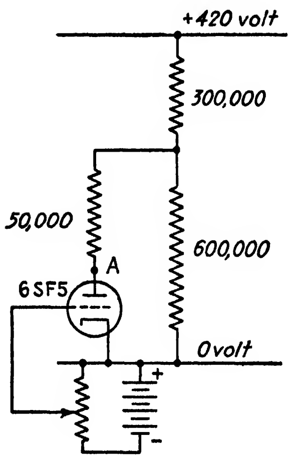

FIG. 10-25.—Circuit diagram for Prob. 10-9.

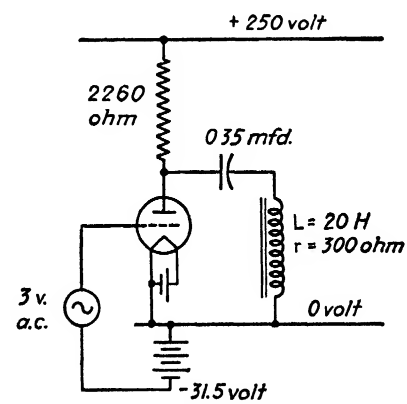

FIG. 10-26.—Circuit diagram for Prob. 10-10.

a. What are the alternating voltages appearing across *R*, *L*, and *C*, if the frequency of the alternating voltage applied to the grid is (1) 60 cps, (2) 120 cps?

b. If the voltage across the inductance is considered the output voltage, what is the actual voltage gain of the stage for the two frequencies?

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1. Boss, L. F.: Universal Electronic Relay, *Electronics*, May, 1942, p. 68.
2. Lawson, D. I.: An Analysis of a D.C. Galvanometer Amplifier, *Electronic Eng.* (London), August, 1944, p. 114.

CHAPTER XI

BUCKING OR COMPENSATING CIRCUITS; BRIDGE CIRCUITS; APPLICATION OF VACUUM TUBES FOR THE MEASUREMENT OF DIRECT VOLTAGE WITHOUT CURRENT DRAIN

11–1. Limitations of Vacuum-tube Voltmeters for Direct Voltages.—In Chap. X we saw the tube primarily in the role of a relay capable of operating from a voltage source not permitting any appreciable current drain. Suppose now that we wish to use it as a measuring instrument for voltages that permit no current drain. As long as we have to measure voltages in the neighborhood of a few volts with an accuracy of about ½ volt, or possibly a little less, there exists, obviously, no difficulty at all. All we have to do is to set the tube up with a meter in the plate circuit, provide a suitable bias of a size that just about reduces the plate current to zero, apply known voltages to the grid, and calibrate the plate-circuit meter in terms of grid voltages.[1, 3] Within limits we can predict the performance of such an arrangement with the aid of the characteristic curves. The most convenient curves for this purpose are, naturally, the transfer characteristics, since the indicating instrument in the plate circuit is a load with such a low resistance that for practical purposes we can consider the tube as operating with a fixed plate voltage. If the plate characteristics are used, then we have to draw a vertical line into this family at a voltage equal to the voltage of the battery used for the operation of the plate circuit. What we are then doing, after all, is nothing but obtaining a transfer characteristic for the plate voltage to be used.

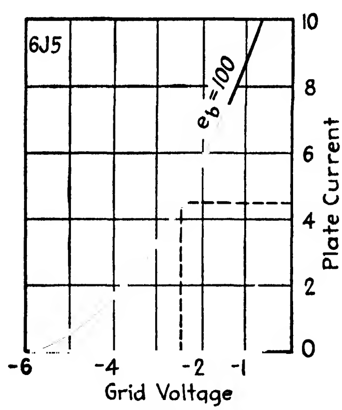

FIG. 11 1.—Transfer or mutual characteristic of a 6J5 for a plate voltage of 100 volts.

In Fig. 11–1 the transfer characteristic of a type 6J5 tube operating with 100 volts on the plate is shown. It will be noted that this characteristic, as it approaches the X axis, decreases its slope so that it runs into this axis almost tangentially. Suppose that we wish to use the tube as a device to measure a voltage in the order of 0.1 to 0.2 volt. Now, in the steeper part of the mutual characteristics, say, with -2.5 volts on the grid, the plate current changes at a rate of almost 3 ma per volt grid-

voltage change. This means that a change of 0.1 volt would produce a plate-current change of approximately 0.3 ma. But in order to make use of this, we shall have to bias the tube with 2.5 volts, which results in a current of 4.5 ma. We shall therefore have to use a meter of 5 ma, and the circuit takes the form shown in Fig. 11–2. An introduction of 0.1 volt at the terminals marked X will then change the plate current from 4.5 to about 4.8 ma. This, it must be admitted, is a very unsatisfactory meter for measuring voltages from 0 to 0.1 volt because we are using less than 10 per cent of the scale of the instrument. Now, if 0.1-volt grid-voltage change produces 0.3-ma plate-current change, it is obvious that the most desirable arrangement would be one where we could make use of a meter with a full-scale reading of about 0.5 ma. The first thought to accomplish this is to bias the tube so that the plate current will be zero, which, as the transfer characteristic indicates, takes about -6 volts. But the transfer characteristic also shows that such a solution would not be very satisfactory owing to the very gradual slope with which the characteristic runs into the X axis. In this region the plate current evidently does not change at the rate of 3 ma per volt of grid change, or 0.3 ma for 0.1-volt grid change. We find ourselves, therefore, faced with a dilemma. If we wish to use a low-scale meter, the tube will have to be biased to cutoff, in which case the transconductance, or the rate of change of plate current with a grid-voltage change, is very small, or, if we place the operating point in that portion of the characteristic where the transconductance is high, then we must use a meter that can carry the plate current at the operating point. Clearly neither one of these solutions is a very satisfactory one.

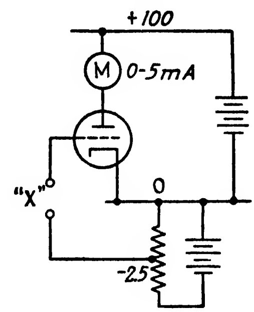

FIG. 11 2.—With zero volts applied at "X," the meter will read approximately 4.5 ma. The circuit is not suitable for the measurement of small input voltages.

11–2. Compensation of Steady Plate Current.—This is, however, one of the few situations where we can have our cake and eat it too. The solution consists in letting the tube operate on a point in the steep part of the characteristic. In the example above, for instance, we shall bias the tube to -2.5 volts, which causes a plate current of 4.5 ma. Instead of using a 5-ma instrument in the plate circuit, however, we now make use of an instrument with a much lower scale value, such as ½ ma, for instance; at the same time we arrange a battery and a resistor in connection with this meter in such a way as to produce a current through the instrument of exactly 4.5 ma in a direction opposite to the one in which the plate current of the tube is flowing through the meter. The fundamental circuit accomplishing this purpose is shown in Fig. 11–3. It is seen that the tube current passes through the meter from right to left while the bucking cur-

rent passes through it in the opposite direction. With the input terminals at point X short-circuited and with the bucking current adjusted by means of the variable resistor R to a value of 4.5 ma, the current flowing through the instrument is zero, and we can therefore use an instrument with a full-scale reading of ½ ma or even less if so desired. For all practical purposes, the bucking current is determined only by the value of the bucking battery and the adjustable resistor in series with this battery. If, therefore, the tube current changes now a small amount owing to the introduction

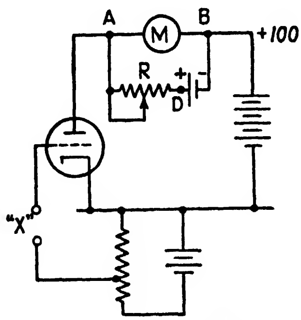

FIG. 11-3.—An auxiliary battery causes a current to flow through the meter in a direction opposite to that of the plate current. This permits the bucking out of the steady plate current.

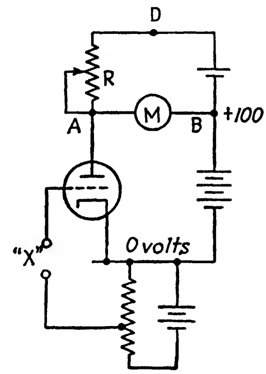

FIG. 11-4.—The circuit shown in Fig. 11-3 is drawn here in a manner making it easier to understand.

of a small voltage at the input terminals X, then the instrument will indicate only the *current change* that has taken place. This means that we have accomplished our purpose. The operating point of the tube is now on the steep part of the transfer characteristic; nevertheless we can make use of a low-reading instrument indicating only the *change* of plate current taking place because of the introduction of a small voltage on the grid of the tube.

11-3. Presentation of Compensating Arrangement in "Voltage-equal-height" Concept.—Figure 11-3 shows the fundamental principle of this "bucking" or compensating arrangement, but the method of presentation is not such as to promote a clear understanding of the circuit. In Fig. 11-4 the identical circuit shown in Fig. 11-3 is redrawn in our familiar way in which potentials are indicated as vertical distances. It will be noted that the bucking battery is simply in series with the main power supply for the operation of the tube, raising the potential of point D above the potential of point B by an amount equal to the voltage of the bucking battery. If the value of the variable resistor R is adjusted to such a value that the

voltage caused by the plate current of the tube flowing through this resistor is equal to the voltage of the bucking battery, then it is evident that the potential of A will be exactly the same as the potential of B. An instrument placed between A and B, no matter how sensitive, will therefore indicate zero current. Let us assume for a moment that the resistance of the indicating instrument is very low; under this condition the instrument can be considered as providing a short circuit between A and B. No current will flow through this short circuit, however, as long as the current flowing through the resistance R from D to A is equal to the current flowing through the tube itself. But since the short circuit provided by the instrument between A and B keeps these two points at exactly the same potential, the current that will flow through the resistance R will remain constant, regardless of what happens to the tube current. Therefore, if the tube current changes a small amount so that it is no longer equal to the current flowing through R, then this change in plate current will have to come through the instrument.

11–4. Additive and Subtractive Connection of Bucking Battery.—Before we attempt a mathematical analysis of this circuit, taking into account the internal resistance of the indicating instrument, another bucking arrangement, shown in Fig. 11–5, will be discussed. A comparison of the circuit arrangements shown in Figs. 11–4 and 11–5 reveals that in the former the bucking battery is connected additively to the main battery, while in the case of Fig. 11–5 its voltage is subtracted from the main battery. In Fig. 11–4 the tube operates with a plate voltage equal to the voltage of the main battery, while in Fig. 11–5 it operates with a voltage equal to the difference between the voltages of the main battery and the bucking battery. The advantage of the arrangement shown in Fig. 11–5 over that of Fig. 11–4 lies in the fact that the bucking battery will never have to furnish a current exceeding the small amount flowing through the meter itself. This is not the case with the arrangement shown in Fig. 11–4 where the bucking battery becomes a part, or an extension, of the main battery and will therefore have to carry not the small amount of current flowing through the meter, but the larger current flowing through the tube itself. This means that deterioration of the bucking battery will be faster with the arrangement shown in Fig. 11–4 than with the arrangement shown in Fig. 11–5. The fact that the tube is operating with a somewhat lower plate voltage in the arrangement shown in Fig. 11–5 may be considered a slight disadvantage. Since the bucking battery rarely exceeds a few volts, how-

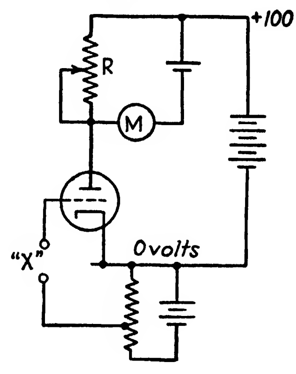

FIG. 11–5.—When the bucking battery is connected subtractively, only the small meter current will flow through it.

ever, this is not a serious drawback and the arrangement shown in Fig. 11–5 is therefore more desirable.

11–5. Wheatstone-bridge Circuit with Tube in One Arm.—In both the circuits shown in Figs. 11–4 and 11–5, the instrument is connected between two points that are at the same potential in the case of balance, *i.e.*, when the tube is operating with the chosen value of plate current. It will be remembered that a Wheatstone-bridge arrangement is also a circuit in which an instrument is connected across two points of the same potential in the case of balance and that an indication on the instrument results when this balance is disturbed. If we look at the circuit shown in Fig. 11–4 with this in mind, the similarity between it and a Wheatstone-bridge arrangement becomes immediately apparent. When we recognize the close similarity between these arrangements, the step from the circuit shown in Fig. 11–4 to the one shown in Fig. 11–6, which represents a Wheatstone-bridge arrangement with one arm replaced by a tube, is obvious. If, with the instrument removed, points A and B are at the same potential, which is the case if the voltage across the resistance R_1 caused by the current flowing through the tube is equal to the voltage across the resistance R_3, then the instrument, when connected between A and B, will register zero current. When a voltage is introduced at the input terminals X, the condition of balance is evidently disturbed, and a current will be indicated by the instrument.

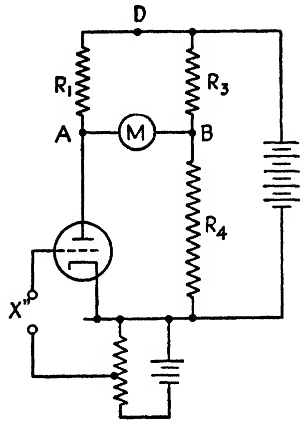

FIG. 11–6.—A Wheatstone bridge with one arm replaced by a tube. The meter reading will be zero in the case of balance.

11–6. The Equivalent-plate-circuit Theorem Applied to Compensating Arrangements.—The operation of the three circuits shown in Figs. 11–4 to 11–6, as well as the operation of many other compensating arrangements found in practice, can be analyzed very easily mathematically by making use of the equivalent-plate-circuit theorem. The very fact that a compensating arrangement is used for a particular problem indicates that it is desired to obtain operation of an indicating instrument from a very small grid-voltage change; under this condition it is quite obvious that the replacement of the tube by a battery and a resistance in series with this battery will be highly satisfactory for the analysis of the circuit. Let us first analyze the circuit shown in Fig. 11–4. Assume that we wish to set up the circuit shown in this figure with a type 1H4G triode. For the operation of the plate circuit we shall use a 90-volt battery; an indicating instrument with a full-scale range of 500 μa and an internal resistance of 400 ohms is available to us. As a bucking battery we intend to use a

single flashlight cell with a voltage of 1.5 volts. The manufacturer states that with a plate voltage of 90 volts and a grid voltage of -4.5 volts the plate current will be 2.5 ma, the plate resistance will be 11,000 ohms, and the amplification factor will have a value of 9.3. The circuit will then look as shown in Fig. 11–7*a*. The first thing we have to do is to calculate the value of the resistance R_1 in series with the tube. With the input terminals X short-circuited, the meter is supposed to read 0; this means that points A and B must then be at the same potential, which, in turn, makes the plate voltage of the tube exactly 90 volts. With a grid voltage of -4.5 volts, the tube takes a current of 2.5 ma, as stated by the manufacturer. Since no current is supposed to flow from B to A under this condition, the 2.5-ma plate current will have to be coming through the

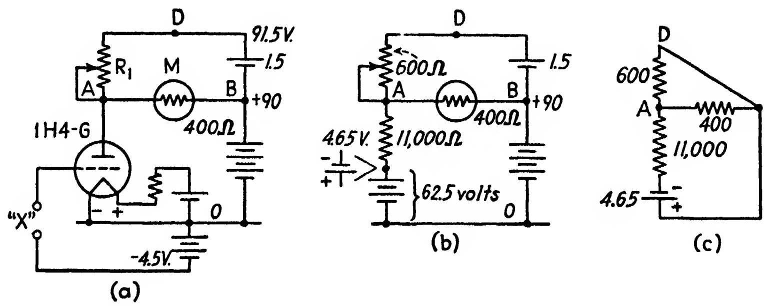

FIG. 11–7.—The analysis of a compensating arrangement can be carried out easily by replacing the tube with a resistance and a battery.

resistance R_1. The voltage across R_1 must therefore be 1.5 volts, the same as the voltage of the bucking battery. The value of R_1 is consequently $1.5/0.0025 = 600$ ohms. Since the characteristics of tubes vary within approximately 10 per cent, it is well to employ a variable resistance for R_1 with a value from 500 to about 750 ohms. The next step for the analysis of the performance is to determine the black-box substitution for the tube under the given operating condition. We know that the tube must be replaced by a resistance equal to the plate resistance at the operating point and a battery in series with the resistance. The plate resistance of the tube is given by the manufacturer under the conditions outlined above as 11,000 ohms; the plate current is 2.5 ma, and 2.5 ma flowing through a resistance of 11,000 ohms will produce a voltage of 27.5 volts. But since we have to apply 90 volts to the actual tube in order to obtain a current of 2.5 ma, the fictitious battery in series with the resistance must have a value of $90 - 27.5 = 62.5$ volts. The substitution for the actual circuit shown in Fig. 11–7*a* is the one shown in Fig. 11–7*b*. This substitution is valid only as long as the grid voltage on the tube is -4.5 volts. Now look at Fig. 11–7*b* carefully. There are three voltage sources acting in this circuit: the 90-volt battery used for the plate circuit, the 1.5-volt bucking

battery, and the fictitious battery of 62.5 volts. According to the principle of superposition, the actual current distribution in the circuit could be considered as the superposition of three current distributions found by short-circuiting successively all but one battery at a time. We are interested only in the current flowing in the branch AB. But we know that the circuit has been arranged in such a manner that the current in this branch will be 0, *i.e.*, that the three currents produced by the above-mentioned three batteries would cancel out in this branch. As a good exercise, the reader is invited to convince himself by calculation that such is indeed the case. If we short-circuit the 90- and the 62.5-volt battery, the current flowing in the 400-ohm resistance due to the 1.5-volt battery would obviously be in a direction from point A to B. The 62.5-volt battery with the other two batteries short-circuited would also cause a current flow in the direction from A to B, while the 90-volt battery with the others short-circuited would cause a current flow from B to A. If the three currents are calculated, it will be found that they add up to 0. Now, let us assume that we introduce a voltage of ½ volt at the input terminals X, changing the actual grid voltage on the tube in Fig. 11–7*a* to -4.0 volts. The equivalent-plate-circuit theorem tells us that the effect on the circuit in which the tube is operating is the same as if we changed the fictitious battery of 62.5 volts by an amount of $0.5 \times 9.3 = 4.65$ volts. The value of the fictitious battery would therefore change from 62.5 to $62.5 - 4.65$ volts, since the grid-voltage change is in such a direction as to increase the plate current of the actual tube circuit. But instead of changing the value of the fictitious battery from 62.5 volts to a lower value, we can obtain the same results by leaving it unchanged and placing in series with it a battery of 4.65 volts in such a direction as to oppose the polarity of the 62.5-volt battery. In Fig. 11–7*b* this battery is indicated as well as the place where it is to be inserted. Before the advent of this additional fourth battery in the circuit shown in Fig. 11–7*b*, the current through the 400-ohm branch was 0, as outlined above. Applying the principle of superposition to the newly arrived battery shows that the current flowing in the 400-ohm branch can be found by considering the new battery alone active in this circuit with all others short-circuited. Figure 11–7*b* therefore becomes now simply Fig. 11–7*c*. Owing to the short circuit of the bucking battery, the points D and B become one. This places the two resistances of 400 and 600 ohms in parallel. The parallel combination of 400 and 600 ohms is equal to $400 \times 600/(400 + 600) = 240$ ohms. The total resistance of the circuit is therefore 11,240 ohms and the current that flows in the circuit is $4.65/11{,}240 = 0.000414$ amp, or 0.414 ma. This current will divide itself in the two parallel resistances in a ratio inversely proportional to these two values. This means that 600 parts of it will flow through the 400-ohm resistance, and 400 parts will flow through the 600-ohm resistance. Through the instrument, which is the 400-ohm re-

sistance, there will then flow 60 per cent of the total current, or 0.248 ma, which is equal to 248 μa. For full-scale reading of our 500-μa instrument, the voltage that must be applied at the input terminals X of the circuit shown in Fig. 11–7*a* would therefore be slightly more than 1 volt.

11–7. Influence of Various Factors on the Sensitivity of a Bucking Arrangement.—The study of the equivalent circuit shown in Fig. 11–7*c* discloses the influence of the various factors. Out of a total plate-current change of approximately 400 μa, we obtained only approximately 60 per cent for the operation of the instrument. Evidently it must be our aim to obtain as much of the total plate-current change for the instrument as possible. From Fig. 11–7*c* it is clear that an increase of the 600-ohm branch will not affect the total current change appreciably. Even if we should increase the 600 ohms to the value of infinity, the total resistance in the circuit would change only from 11,240 to 11,400, which is less than 2 per cent change. But with the 600-ohm resistance made infinite, *all* the current would flow through the 400-ohm branch. An increase of the 600-ohm resistor is therefore obviously leading to an increased sensitivity of the whole circuit. How did we arrive at the value for this resistor? It will be remembered that the voltage across this resistor caused by the plate current had to equal the voltage of the bucking battery. The plate current of the tube at the chosen operating point is a fixed value and cannot be changed. Consequently, if we wish to increase the value of this resistance, the voltage of the bucking battery must also be increased. Increasing the voltage of the bucking battery from 1.5 to 7.5 volts, for instance, means that we shall have to increase the value of the resistance R_1 to 3,000 ohms. The reader may convince himself that under this condition the fraction of the total current passing through the instrument is given by the ratio $3{,}000/3{,}400 = 0.883$, which is considerably better than the value of 0.6 with the preceding arrangement. It also becomes clear that the resistance of the instrument itself is of great importance. The lower this value is, the larger will be the fraction of the total plate-current change that will pass through the instrument and thus be useful.

In the circuit treated in Fig. 11–7, the bucking battery was connected additively. If we connect it subtractively, as shown in Fig. 11–5, the analysis will proceed in exactly the same way, with one small difference. In Figs. 11–4 and 11–7 the tube operated at a plate voltage exactly equal to the voltage of the plate-supply battery, at least at the point of balance. If the tube used in Fig. 11–7*a* is connected in a circuit as shown in Fig. 11–5, however, the plate voltage at the point of balance will be not 90 volts, but $90 - 1.5 = 88.5$ volts. The plate current with -4.5 volts on the grid would therefore be, not exactly 2.5 ma as given by the manufacturer, but somewhat smaller. The amount by which it is smaller is easily calculated, however; since the plate resistance is given by the manufacturer as 11,000 ohms, a reduction of plate voltage from 90 to 88.5 volts

will decrease the plate current by an amount equal to 1.5/11,000 = 0.0001362 amp, which is equal to 0.136 ma. This is the same current change that we observe if a voltage of 90 volts across our black-box substitution of 11,000 ohms and 62.5 volts is reduced to 88.5 volts. Figure 11–8 shows the substitution valid for this case, and the reader may convince himself easily that the current in the 400-ohm branch will be zero with the resistor values as shown. It is also evident that this circuit performs for all practical purposes in exactly the same manner as the circuit shown in Fig. 11–7.

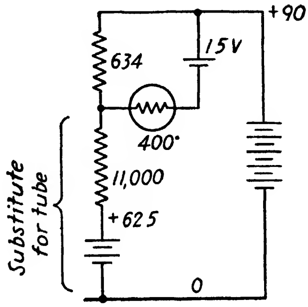

FIG. 11–8.—For subtractive connection of the bucking battery the circuit would appear as shown here.

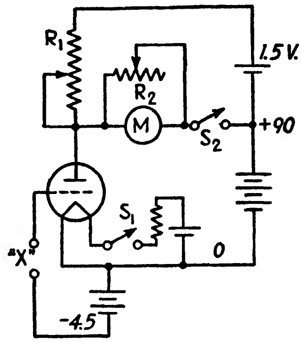

FIG. 11–9.—In practice certain precautions must be taken in the operation of circuits involving bucking batteries.

11–8. Precautions Required with Bucking Circuits.—A word of caution may be appropriate in connection with circuits containing bucking batteries. It was shown with the aid of Fig. 11–7 that zero current in the instrument was the result of three separate currents, produced by three separate batteries, canceling each other in the instrument. It is common practice to switch vacuum-tube circuits of this type on and off simply by opening the filament circuit, since with a cold filament the tube obviously represents an open circuit. Now this action obviously opens the circuit of the 90- and the 62.5-volt batteries shown in Fig. 11–7*b*, but it does not open the circuit of the 1.5-volt bucking battery. We shall consequently have a current flowing through the 400-ohm resistance considerably in excess of the full-scale value of the instrument. Proper precautions should be taken to prevent damage to the instrument. A switch should be provided in the branch *AB*, and a variable resistance should be provided either in series with the instrument or parallel to it and of such magnitude as to reduce the current through the instrument to a safe value, even under the most unfavorable conditions. A suitable arrangement is shown in Fig. 11–9. To put this circuit into operation we proceed as follows:

First, switch S_1 is closed. This heats the filament. After waiting a few minutes until the filament has reached a constant temperature, switch S_2 is closed. The arm of variable resistor R_2, which should at first be in

the extreme left-hand position, thus providing a short circuit for the instrument, should then be moved to the right. The instrument will probably show a small deflection since the balance point usually shifts from day to day. By adjusting the value of the resistor R_1, the reading of the instrument is now brought back to zero. R_2 can then be increased. Finally, this circuit may be opened by moving the arm beyond the right-hand end of resistor R_2. When the circuit is put out of operation, S_2 must be opened first so that no damage will come to the instrument as the tube current ceases owing to the opening of S_1.

11-9. Analysis of Wheatstone-bridge Circuit.—The analysis of the Wheatstone-bridge circuit shown in Fig. 11-6 proceeds along exactly similar lines. Suppose that we were to use the same tube as in the previous

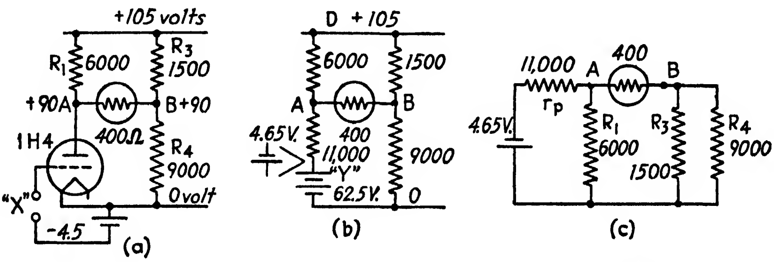

FIG. 11-10.—The analysis of a Wheatstone bridge with a tube serving as one arm. The substitute circuit shown in (b) indicates that the circuit will not remain in balance when the dc supply voltage changes.

example in such a circuit but that the voltage supply available for the operation of the circuit is 105 volts. (A regulated voltage of this value may be obtained from a voltage-regulator tube.) In order to be able to use the data given by the manufacturer, we decide to operate the tube with 90 volts on the plate. The plate current will then be again 2.5 ma and, since the voltage across the resistance R_1 will then be 15 volts, the value of R_1 will be $15/0.0025 = 6{,}000$ ohms. We now have to decide on the value of resistances R_3 and R_4. Their ratio is, of course, determined by the fact that point B must be at a potential of 90 volts. Since the current flowing through them seems to be a total loss, the first impulse is to make these two resistors of as high a value as possible. A little farther on the discussion will show that such a choice would seriously impair the sensitivity of the circuit and that, to the contrary, R_3 and R_4 should be made of as low a value as possible. The choice, therefore, will depend mainly on how much current the voltage supply for the whole circuit can furnish. Let us assume that we can bleed a current of 10 ma through R_3 and R_4. Since the voltage across R_3 must be 15 volts and across R_4 90 volts, the two values for R_3 and R_4 will be 1,500 and 9,000 ohms, respectively. The complete actual circuit now looks as shown in Fig. 11-10a. With the now familiar black-box substitution for the tube, the circuit assumes the form shown in

Fig. 11–10*b*. Convince yourself that with the 400-ohm branch open, points *A* and *B* both would be at a potential of 90 volts, which means that after closure of branch *AB* no current will flow in it. From reasoning similar to the discussion of the circuit in Figs. 11–7*b* and *c*, the current that flows in the 400-ohm resistor, when a voltage of ½ volt is introduced at the input terminals *X* of the circuit shown in Fig. 11–10*a*, can be found with the aid of the substitution shown in Fig. 11–10*c*. The solution is 295 μa for the meter current, but since only simple circuit theory is involved, the details of the solution will not be discussed further. The diagram shown in Fig. 11–10*c* indicates clearly why R_3 and R_4 should not have too high a value. A comparison of Figs. 11–10*c* and 11–7*c* shows clearly that the parallel combination of R_3 and R_4 is placed in series with the meter resistance. The higher these two resistances are, the lower will be the fraction of the total current that finds its way through the meter and these two parallel resistances.

11–10. Lower Limit of Voltages That Can Be Measured by Bucking Circuits.—In all the examples treated in the preceding paragraphs, the indicating instrument was assumed as a 0–500μa meter. Under this condition, the calculations show that an input voltage of approximately 1 volt at the terminals *X* would cause full-scale deflection of the instrument. But we originally asked whether we could have a circuit arrangement in which an application of $\frac{1}{10}$ volt to the input terminals would result in full-scale deflection. Since at balance the current in the branch *AB* of all these circuits is zero, there does not seem to be any reason why we should not be able to increase the sensitivity of the arrangement simply by placing a meter with a lower full-scale range at this spot. Thus, with a 50-μa meter in branch *AB*, we should be able to obtain full-scale deflection with only $\frac{1}{10}$ volt applied to the input terminals of the device, provided, of course, that the resistance of the instrument remains the same as the resistance of the instrument in the previous case. With a still more sensitive instrument, such as a galvanometer with a full-scale deflection of 5 μa, the sensitivity could be increased again tenfold and give us full-scale deflection with an input voltage of only 10 mv. If we try to carry this reasoning into practice, we shall very quickly find ourselves much bothered by the instability of the zero point of the galvanometer. We would find that the balance point must be readjusted almost continuously. There are many reasons why such a drift takes place. In the circuits shown in Figs. 11–4 and 11–5, it is not likely that the voltages of the plate-supply battery and the bucking battery will change relative to each other exactly in such a manner as to cause the current in branch *AB* to remain zero. Furthermore, the filament temperature of the tube will vary, owing partly to changes in the ambient temperature and partly to a change of voltage in the battery supplying the filament current. Changes in filament temperature will cause changes in the initial speed with which the electrons

are emitted. This will cause plate-current changes, even if the grid voltage as well as the plate voltage remains absolutely constant. Finally, of course, variation in the voltage of the grid-bias battery will cause plate-current changes. It is for these reasons that the range of such compensating arrangements cannot be extended further and further downward for the reading of smaller and smaller direct voltages.

The considerations just discussed apply to the schemes employing separate bucking batteries. It would seem that a Wheatstone-bridge arrangement should be considerably more free of some of these troubles. We know that the ordinary Wheatstone-bridge arrangement, with four resistors as the arms of the bridge, is insensitive to voltage variations of the supply; in other words, when the indicating instrument reads zero, changes in the supply voltage do not cause a deflection of the instrument indicating the balance of the bridge. Since the arrangement shown in Fig. 11–6 is a Wheatstone bridge with one arm replaced by the tube, it seems reasonable to assume that such a circuit will also be insensitive to changes in the supply voltage. When we look at the substitution for this circuit shown in Fig. 11–10*b*, it becomes apparent that the matter is not quite so simple. The application of Thévenin's theorem to the branch AB permits us to determine how much unbalanced current will flow in this branch if our supply voltage, in this case 105 volts, should change a certain amount, say, 1 volt. With branch AB open and with the supply voltage exactly at 105 volts, we know that the points A and B are at the same potential. When the supply voltage rises to a value of 106 volts, will A and B both rise in potential the same amount? The rise in potential of A, with a rise of 1 volt of the potential of D, is determined by the values of the voltage divider consisting of 6,000 and 11,000 ohms. The reader may convince himself easily that the potential of A will rise by $1 \times 11{,}000/17{,}000 = 0.646$ volt. In a similar way, the rise of potential of B, with a rise of the supply voltage from 105 to 106 volts, will be given by $1 \times 9{,}000/10{,}500 = 0.857$ volt. Point B rises, therefore, more in potential than A, and the voltage appearing across a break in the branch AB will be the difference between the two increases, *i.e.*, $0.857 - 0.646 = 0.211$ volt. The resistance that we shall measure across a break in the branch AB with all voltages short-circuited will be

$$R_t = 400 + \frac{6{,}000 \times 11{,}000}{6{,}000 + 11{,}000} + \frac{1{,}500 \times 9{,}000}{1{,}500 + 9{,}000} = 5{,}565 \text{ ohms}$$

According to Thévenin's theorem, the current that flows in this branch after the break has been closed is $0.211/5{,}565 = 0.0000379$ amp, which is equal to 37.9 μa. One-volt variation in the plate-supply voltage causes a drift of almost 40 μa on the instrument in branch AB. From this it becomes quite obvious why the sensitivity of such an arrangement cannot be increased indefinitely by using an increasingly sensitive indicating instru-

ment. This statement does not mean that we have reached the limit of usefulness of these circuits. In the literature will be found many circuit arrangements that minimize the influence of the variations found in the various voltages used for these circuits.

11-11. Wheatstone Bridge with Tubes in Two Arms.—Of the many circuits designed to minimize the effect of supply-voltage variations on the zero balance of compensating or bucking arrangements, one deserves particular attention because of its simplicity and practicability. Suppose that in Fig. 11–10*b* the 9,000-ohm resistor forming one arm of the bridge is replaced by a battery of 62.5 volts and a resistance of 2,750 ohms. Now the left-hand voltage divider, consisting of 6,000 and 11,000 ohms, and the right-hand divider, consisting of 1,500 and 2,750 ohms (Fig. 11–11), have arms of the same ratio; under this condition a voltage variation of the supply voltage will not cause an unbalance in the voltage existing between points *A* and *B*. Such an arrangement can be set up and will be found considerably more stable than the simple arrangement shown in Fig. 11–10*a*. The supplemental voltage of 62.5 volts required for this scheme must be obtained from a battery or from a voltage-regulated tap on a voltage divider across the total supply voltage, which represents an undesirable complication. Although this circuit is reasonably stabilized against variations in plate voltage, it still shows a drift if the filament current changes a small amount. Looking at Fig. 11–11 suggests, however, a quite obvious solution. Why not replace the 9,000-ohm resistor of Fig. 11–10*a* by another tube of identical type with the one that is to be used for measuring the desired voltage? The actual circuit is shown in Fig. 11–12*a*; the black-box substitution in Fig. 11–12*b*. The circuit was first proposed by P. I. Wold and was later improved by Wynn-Williams. This circuit, with proper precautions to keep the current in the filament as nearly constant as possible as well as with the necessary provisions to obtain accurate balance, can be made quite stable. It has been used in connection with the measurement of nerve potentials amounting to fractions of a millivolt. Figure 11–12*a* shows the tubes obtaining their grid bias from the same battery. With the input terminals *X* shorted, both tubes have the same grid voltage. The introduction of the unknown voltage at the terminals *X* changes the current through the left-hand tube without affecting the current in the right-hand tube appreciably. The indication of the meter can again be predicted by making use of the equivalent circuit shown in Fig. 11–12*b*. The introduction of a small voltage Δe

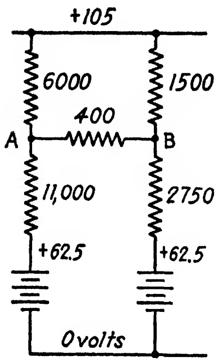

FIG. 11–11.—If the two resistors R_3 and R_4 of Fig. 11–10 are replaced by a combination of a battery and resistors as shown here, the circuit will remain in balance with changes of supply voltage.

at the input terminals X of the actual circuit shown in Fig. 11–12a will have the same effect as the introduction of μ times this voltage at point Y in Fig. 11–12b. Before the introduction of this fictitious voltage at point Y, the current in the meter branch was zero; therefore, any current in this branch will be due solely to the introduction of $\mu\ \Delta e$ at Y, and the amount of the current flowing through the meter branch can be calculated by making use of the circuit shown in Fig. 11–12c. In connection with Figs. 11–10a and c, it has been pointed out that for maximum sensitivity the two resistors R_3 and R_4 should be of as low a value as possible. A comparison of Figs. 11–10a and 11–12a makes it clear that with the latter

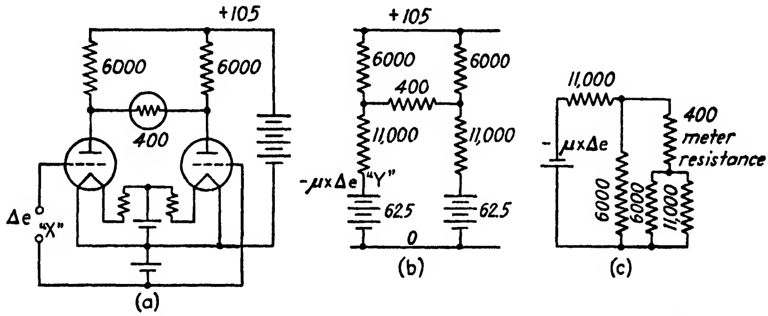

FIG. 11–12.—For best stability of the balance point, the Wheatstone bridge should have two tubes connected as shown here; the second tube acts as a dummy tube. The substitute circuits in (b) and (c) permit the calculation of circuit performance.

circuit we have no choice in this matter since the two arms are now identical. For a given input voltage the circuit shown in Fig. 11–12a will, consequently, not give so much current in the meter branch as the circuit shown in Fig. 11–10a. This small disadvantage, however, is more than compensated for by the much higher stability of the circuit shown in Fig. 11–12a.[2, 4–11]

11–12. Approximate Treatment of Bucking and Bridge Circuits.—The performance of these compensating circuits and bridge arrangements can be predicted with an accuracy of a few per cent without resorting to the accurate treatment given in the preceding paragraphs. Consider Figs. 11–7b, 11–10c, and 11–12c. Suppose that all resistors after the 11,000 ohms were short-circuited in these figures. (It will be remembered that the 11,000 ohms represent the plate resistance of the particular tube used in these examples.) An introduction of a voltage of 1 volt at the input terminals of all these circuits means that the battery shown in the equivalent circuits of Figs. 11–7c, 11–10c, and 11–12c will be μ volts. The current that then flows with all resistors, except the 11,000-ohm resistor short-circuited, is given by μ/r_p. It will be remembered that this is the transconductance of the tube. The current change produced by our fictitious battery, owing to the introduction of a 1-volt change on the grid of the

tube, can never exceed the value of the transconductance. For the type 1H4G tube, around which the circuits treated in Figs. 11–7 to 11–12 were designed, the manufacturer gives a transconductance of 850 micromhos for the values of plate voltage and grid voltage used as operating points of the tube. A 1-volt change on the grid of any of these circuits can therefore not produce a current change larger than 850 μa, or 0.85 ma. In the circuit shown in Fig. 11–7*c*, only 60 per cent of this maximum possible current change will flow through the meter branch of 400 ohms. This amounts to 510 μa. With a grid-voltage change of only ½ volt we would, consequently, arrive at a meter current of 255 μa by this method, compared with 248 μa obtained with the accurate method. If the bridge arrangement shown in Figs. 11–12*a* to *c* is analyzed in more detail, it is found that the current in the meter branch, *i.e.*, the 400-ohm resistor, with a 1-volt input voltage applied to the terminals X, can never exceed a value equal to one-half the transconductance of the tube being used in this circuit. When the meter resistance is zero, the current is found to be exactly one-half the transconductance. With the meter resistance not equal to zero, the meter current is the nearer to this maximum value of one-half the transconductance, the higher the value of the resistors chosen in series with the plate circuits of the tubes. With a problem of this kind, a few trial calculations with various values of resistors will quickly indicate when a point has been reached where further increase in their value will not materially improve the sensitivity of the arrangement.

11–13. Field of Applications for Dc Vacuum-tube Voltmeters.—The applications of the circuits discussed in the two preceding chapters are manifold. Wherever it is desired to measure a direct voltage without drawing any current during the process of measurement, or wherever it is desired to operate a relay from such a voltage, a tube circuit is obviously a convenient method of achieving this result. One of the better known applications is the determination of pH, or hydrogen-ion concentration of solutions, which is a measure of the intensity of acidity or alkalinity of a solution. From an electrical point of view, the problem is simply to measure a voltage ranging between 0.3 and 1.1 volt without drawing any current. The reason for this latter condition is that the voltage originates on two special electrodes connected to the solution. Any current drawn from this source during the process of measurement leads to polarization, which changes the actual voltage. Furthermore, the resistance of the solution may be so high that even a sensitive galvanometer will not give a satisfactory indication. As a matter of fact, some of these instruments make use of a glass electrode; in this case, the electrode consists of a thin-walled glass tube completely closed on one end, with the proper chemical solution on the inside. Evidently the current required by the measuring instrument has to flow through the wall of the tube and, although the latter is very thin and is usually of glass with a relatively high conductivity,

nevertheless, the resistance values of glass electrodes range from 5 to 75 megohms, and the current drawn by the measuring device will have to be extremely small. In such a case, the use of a vacuum tube is the only satisfactory method of obtaining a voltage measurement.

11–14. Increase of Precision by Slide-back Method.—In the circuits discussed in the preceding chapters, the indications of the meter in the plate circuit, either directly or in a bucking arrangement, were directly correlated to the input voltage. In other words, after such a device has been constructed, we can calibrate it by applying known direct voltages to the input terminals and noting the deflections produced on the indicating instrument. The use of such an instrument for precision measurements is open to very serious arguments, however. Any change in the transconductance of the tube incorporated in the instrument will necessarily lead to a variation in the calibration. If a tube has to be replaced, it is necessary to run another calibration curve. It is of importance, therefore, that the prospective user realizes that he pays for the advantage of having a direct-reading instrument drawing no current from the voltage to be measured by the disadvantage that the calibration of the instrument may change considerably with a change in various factors and that frequent recalibration is necessary if any degree of accuracy is to be maintained. If a high degree of precision is required, however, it is possible to make use of an arrangement known as the "slide-back" method. In this scheme the tube is not used to measure the unknown voltage directly but only to indicate when the unknown voltage is equal to a known voltage. It is evident that with such an arrangement the characteristic of the tube is of no importance; its function is then similar to the galvanometer in a Wheatstone-bridge arrangement. There, too, the actual current or voltage sensitivity of the galvanometer is of no importance (provided the sensitivity is high enough for the purpose on hand) since the instrument is used only to indicate that two points on the bridge are at the same potential. The circuit is shown in Fig. 11–13. A bucking arrangement is employed so that we may use a very sensitive meter M_2 in the plate circuit. By means of the double-throw switch S, the grid can be connected to either point F or point H. When connected to F, it is seen that the bias voltage E_{cc} only is acting on the grid of the tube. By adjusting the value of the

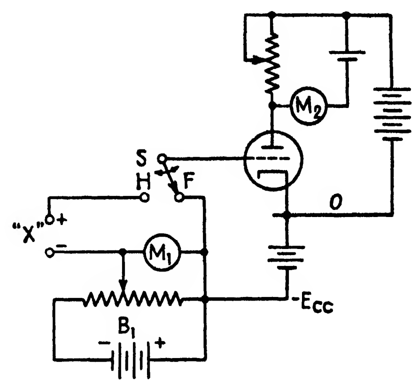

FIG. 11-13.—In the slide-back method the unknown voltage introduced at "X" is compared to an adjustable voltage indicated by the meter M_1; the tube itself does not do the measuring but indicates only when the two voltages just mentioned are equal in magnitude.

resistor in series with the plate of the tube, it is possible to bring the reading of meter M_2 to zero, or to any other value within the scale range of the instrument. Throwing the double-throw switch S to the left-hand position will connect the grid to H. If this operation does not change the reading of M_2, it is evident that H and F are at the same potential. The circuit diagram shows that the potential of H with respect to F is determined by the value of two voltages arranged in such a manner as to oppose each other. One of these voltages is obtained from a voltage divider placed across the auxiliary battery B_1. This voltage can be adjusted to any desired value by means of the sliding arm on the voltage divider, and it is measured by M_1, which may be an instrument of high precision. If this voltage were acting alone on the grid, it would obviously tend to make the grid of the tube negative. The unknown voltage is now placed in series with this adjustable voltage with a polarity such as to oppose the voltage existing across M_1. Switch S is thrown back and forth from right- to left-hand position, and the slider of the adjustable voltage divider is adjusted to a point when the operation of S will not cause any fluctuation of the reading in the plate-circuit meter M_2. When this condition has been reached, it is clear that the unknown voltage introduced at the terminals X must be equal to the voltage indicated by the meter M_1. The precision of this measurement will depend only on the accuracy of M_1 and the smallest grid-voltage change that will cause a noticeable deflection on M_2 in the plate circuit of the tube. Let us assume, for instance, that the tube and the compensating arrangement are sensitive enough to give a noticeable deflection on M_2 with a voltage of 5 mv applied to the grid of the tube. With such an arrangement, we could always be sure that the unknown voltage introduced at terminals X is within 5 mv of the voltage indicated by M_1. If we use the arrangement to measure voltages in the order of approximately 2 volts, then 5 mv represents ¼ per cent, which means that the accuracy of the indicating instrument M_1 is probably the limiting factor. If the unknown voltage is ½ volt, on the other hand, it cannot be determined with an accuracy higher than 1 per cent, regardless of the accuracy of the instrument M_1. It is evident that these circuits are not so convenient as direct-reading instruments, but they offer a highly precise way of measuring voltages not permitting any current drain. Another advantage lies in the fact that they permit the measurement of voltages much higher than could be applied directly to the grid of a tube. A voltage exceeding 50 volts or so cannot ordinarily be placed directly on the grid of a tube. If we place a voltage divider across it in order to reduce it to a suitable value, we destroy the outstanding property of this method: not to draw any current from the voltage to be measured. With the slide-back method, on the other hand, the grid of the tube is always operating at the same point since the action of the unknown voltage is canceled by the adjustable voltage. It is therefore

possible to measure voltages of any magnitude by this method, provided, of course, that an auxiliary source of direct voltage is available of at least as high a value as the voltage to be measured.

11–15. Measurement of Small Currents with the Vacuum-tube Voltmeter.—Vacuum-tube voltmeters, either of the direct-reading kind, or of the kind employing the slide-back method, offer also a convenient way of measuring small direct currents. It is interesting to note that the method is very closely related to the measurement of very large direct currents. The measurement of large direct currents is usually accomplished by letting the current flow through a resistance and by measuring the voltage produced by the current in this resistance, which is usually called a "shunt." If we wish to measure small currents, we let these currents flow through a resistance of high value; there will then appear across this resistance a voltage, the measurement of which cannot be accomplished with an ordinary voltmeter. The truth of this statement can best be shown by an example. Suppose that we wish to measure a current of 1 μa. If we let this current flow through a resistance of 1,000,000 ohms, or 1 megohm, a voltage of 1 volt will appear across the resistor. A good ordinary voltmeter with a range from 0 to 1 volt might have a resistance of approximately 1,000 ohms. If we try to use this instrument for the measurement of the 1 volt existing across the 1 megohm, the current of 1 μa flows no longer through a resistance of 1 megohm but through the parallel combination of 1 megohm and 1,000 ohms. Using the well-known formula pertaining to the parallel combination of two resistances, we find that the actual resistance is now only approximately 999 ohms. A current of 1 μa flowing through a resistance of 999 ohms will produce a voltage of 999 μv, or slightly less than 1 mv. We now have only 1 mv left in place of the expected 1 volt, and our 0–1 voltmeter will not give a satisfactory reading, of course. With a vacuum tube, however, either in direct connection or in the circuit using the slide-back scheme, it is possible to measure this voltage without upsetting its value by the presence of the measuring device. Such problems may arise, for instance, if it is desired to measure insulation resistances. Suppose that it is desired to measure the resistance of a sheet of fiber with a resistance of about 50,000 megohms. If we apply a voltage of 500 volts to two tin-foil electrodes glued to the two sides of this sheet, the current will be $\frac{1}{100}$ μa. If this current is made to flow through a resistor of 10 megohms, a voltage of 100 mv will result across the resistor. This is in the range where we could use a direct-reading bucking arrangement; if a higher accuracy is demanded, it would be advisable to use a slide-back method. Let us assume that we are going to use a tube in a way similar to the one shown in Fig. 11–13 and that the sensitivity of the meter M_2 is sufficient to give an observable deflection of the pointer when the grid voltage deviates 2 mv from the voltage at balance. The meter M_1 must be an instrument with a full-scale range of 100 to 200 mv.

The voltage appearing across the 10-megohm resistor in series with the fiber sheet is introduced at the terminals X of the circuit shown in Fig. 11–13. With a sensitivity of 2 mv at the tube, the accuracy of the measurement will be 2 per cent in case the voltage to be measured is approximately 100 mv. It is evident, of course, that the accuracy will depend also on the accuracy of the 10-megohm resistor. A single resistor with as high a value of resistance as this is usually not guaranteed to more than 5 per cent. But since it is, of course, entirely possible to build up this resistance from a number of resistors, each with a smaller ohmic value, the problem of obtaining a resistance of 10 megohms is not a serious one.

Exactly identical circuits are used in connection with photoelectric cells. As will be seen later, the current that flows through a photoelectric cell is rarely more than a few microamperes and quite often amounts to only a fraction of one. In order to measure such small currents or to obtain relay operation from them, it is only necessary to make them flow through a resistance of extremely high value and use the voltage appearing across this resistance to act on the grid of the tube. For the measurement of low light intensities, resistors as high as 500 to 1,000 megohms have been used in series with the phototube in order to develop sufficient voltage for the operation of the amplifier tube.

11–16. Lower Limit of Current That Can Be Measured by Vacuum-tube Voltmeter.—It was shown that small currents can be measured by making them flow through a high resistance and then determining the voltage produced across this resistance by means of a vacuum-tube setup. How small a current can we measure with such an arrangement? A 1-volt drop across our shunt can be had with either 1 μa flowing through 1 megohm, or $\frac{1}{100}$ μa flowing through 100 megohms, or by 1/1,000 μa flowing through 1,000 megohms. Can we expect equally satisfactory results from all three arrangements? So far, we have assumed that the grid of a tube will not draw any current from the source to which it is connected, provided that it is at a potential negative with respect to the cathode. In most practical cases, this assumption is entirely satisfactory and does not lead to any appreciable errors; but if our shunt assumes a value of 1,000 megohms, then we must make sure that any device that we use for the determination of the voltage across it will draw a current very much smaller than the one flowing through the shunt. In other words, if we wish to measure 1/1,000 μa by letting it flow through a shunt and observing the voltage across the latter, we must make sure that the instrument to be used for the determination of this voltage will draw a current very much smaller, say, one-twentieth or preferably less, than the current that we wish to measure. This means that we must demand that the tube will not draw a grid current in excess of a fraction of a millimicroampere. For the intelligent application of tube circuits to such problems, it is therefore necessary to have an idea of the amount of grid current that

any given tube will take and of the amount that will be tolerable in the particular problem.

11–17. Range of Grid Current and Methods to Minimize Same.—The grid current of vacuum tubes varies greatly, not only from type to type, but also from one tube of the same type to the next. Grid current is caused by a number of factors, all contributing to various degrees. Before even discussing what goes on inside the tube while it is operating, consider in Fig. 11–14 a tube with grid and plate voltages applied but with the cathode still cold. The three electrodes are brought out of the glass or metal envelope to three pins. It is obviously impossible to avoid all leakage from the plate to the grid pin as well as from the cathode to the grid pin because the grid pin is the most negative electrode of the tube structure. These two currents will combine and flow through the shunt resistor, as indicated in Fig. 11–14. The grid will, therefore, not be at the potential of the negative end of the grid-bias battery. Two ways are open to minimize this effect. One is to choose a tube where the grid wire is brought out on top of the tube, completely away from the other pins. Another method consists of opening the tube base (which is usually made of plastic) and bringing the grid wire out through the side of the base, filling the base after this operation with wax or a good insulating compound. This procedure is, of course, impossible with metal tubes; but, as will be seen later, metal tubes will hardly be used in cases where an extremely low grid current is of importance. It seems superfluous to mention that it is hardly consistent to go through a lot of trouble to insulate the grid terminals on the tube and then fail to apply the same precaution to the wire that connects this terminal with the shunt. In all these circuits, the grid wire must always be treated with special care, and a high value of insulation resistance is of paramount importance.

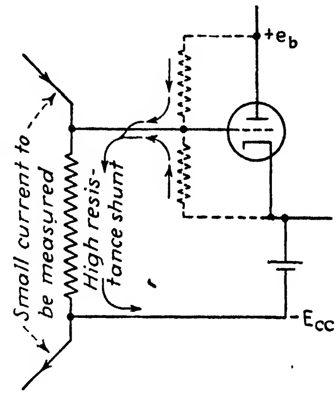

FIG. 11-14.—The leakage current flowing from anode to grid and from cathode to grid must be reduced to a minimum if a resistor of very high value is placed in the grid circuit for the purpose of measuring very small currents.

As soon as the cathode becomes heated and electrons are emitted from it, there will be an additional current to or from the grid. In the first place, a tube cannot be completely evacuated, and, therefore, there are gas molecules that may become ionized by the colliding electrons. A gas ion represents a positive charge and will, consequently, be accelerated toward the most negative region in the tube. Since the grid is the most negative point in the tube (remember our rubber-sheet model where the grid pins are sticking up highest), the positive ions will therefore flow to

the grid. To minimize this effect, it is, of course, the most desirable thing to have a tube evacuated to as high a degree as possible. However, there are always gases trapped in the metal parts of a tube. During the process of evacuation these occluded gases must be driven from the metal structure. This is usually accomplished by heating the metal parts within the tube to at least red heat. In glass tubes this can be very conveniently accomplished by placing a high-frequency coil around the tube that heats the internal parts by induction. In the case of metal tubes, it is the usual practice to play a gas flame on the shell itself. It is quite obvious that this method will not heat the internal structure to as high a temperature as the previously mentioned one. A better job can therefore be done in the evacuation of glass tubes than of metal tubes, although the fact that it *can* be done does not necessarily mean that it always *is* done. Another method to minimize the positive-ion flow to the grid is to operate the tube with so low a plate voltage that no ionization can take place. This means operation with approximately 12 volts on the plate or less.

Under certain conditions the grid may become an emitter itself and thus produce an electron stream. This phenomenon may take place when some of the oxide coating of the cathode, either during manufacturing or during operation, evaporates and becomes deposited on the grid. In modern tubes the grid is usually very close to the cathode and thus becomes heated by radiation. If there is any emitting material deposited on it, emission will take place, which represents a grid current. This effect can be minimized by operating the cathode of the particular tube at a temperature well below the rated value. Thus, it is not uncommon to have a filament rated at 6.3 volts operate with a voltage of 4.5 volts, or even less. It must be understood, of course, that under such a condition we shall not be able to have a plate current in the tube equal to the rated value but that we shall have to reduce the plate current to a fraction of a milliampere. This is not a serious handicap, however, since in circuits of this kind we either use a very sensitive meter in the plate circuit or let the plate current flow through a resistor of high value, applying the drop across it to another tube.

11–18. Summary of Methods to Reduce Grid Current.—To summarize our discussion, if we wish to use an ordinary receiving-type tube in a circuit where it is of paramount importance to have as low a grid current as possible, the following precautions should be taken:

1. Choose a glass tube. The chances of obtaining a well-evacuated tube are better with a glass tube.
2. Choose a tube with the grid terminal brought out on top of the tube; clean the tube with alcohol, heat it, and dip it in ceresin wax. If for any reason it is impossible to find a tube with the grid brought out on top, open the tube base, disconnecting it from its regular pin, and bring the grid terminal out separately.
3. Operate the tube with the plate voltage reduced to approximately 12 volts.

4. Reduce the filament voltage to approximately two-thirds its rated value. It may be mentioned here that tests conducted with a great number of tubes reveal that the type 38 pentode shows a smaller grid current than any other type of tube and will be found very satisfactory if used in circuits of the kind described in the preceding chapters.

When the precautions enumerated above are applied to a type 38 tube, the grid current is in the order of 10^{-10} amp, or $\frac{1}{10}$ millimicroampere. Grid resistors or shunts of as high a value as 500 to 1,000 megohms have been used in connection with this tube and have given entirely satisfactory operation.

11–19. Electrometer Tubes.—There are cases, however, where even a grid current of such a low value will interfere with the measurements to be taken. For such problems, there are available to the electronic engineer special tubes, called "electrometer" tubes, in the design and construction of which the manufacturer has taken all precautions to prevent grid current. The General Electric type FP-54 is an example of this kind of tube. The supporting structure inside the tube is made of quartz in order to obtain an extremely high insulation resistance between the electrodes; the operating voltage for the tube is only approximately 6 volts. The plate current is in the order of 50 μa. The tube does not give any voltage amplification, its amplification factor being unity. With this tube it has been possible to measure currents in the order of 10^{-15} amp. There will be found in the literature the description of many circuits employing either standard or special tubes for the measurement of very low currents.

11–20. Summary of Methods of Analysis.—The applications discussed in Chaps. IX and X have shown the use of a single tube for the determination of small or large direct voltages not permitting any current drain. It was shown further how to obtain relay operation from fairly large direct voltages not permitting any current drain. Two methods are available for the solution of the problems connected with these applications. If the grid-voltage variations are rather large, it is best to use the load line and the family of plate characteristics; if the variations of voltage applied to the grid are small so that the tube characteristics could be considered as straight lines, the best and easiest way is to consider the tube as a black box with a resistance and a battery hidden in it. Even in the case of larger variations, the latter method gives fairly accurate results.

If relay operation is desired from a voltage that is too small to produce sufficient current in the plate circuit of a tube, then it is necessary to resort to several tubes in cascade. Before dealing with the methods available to accomplish this purpose, we shall familiarize ourselves with other important members of the family of vacuum tubes.

PROBLEMS

11-1. For a type 1H4G the manufacturer gives the following data: plate voltage, 90 volts; plate resistance, 11,000 ohms; grid voltage, −4.5 volts; amplification factor, 9.3; plate current, 2.5 ma; transconductance, 850 micromhos.

The tube is to be used in a compensating circuit as shown in Fig. 11-15.

a. How must point X divide the 2,000 ohms of the potential divider so that, with points 1 and 2 short-circuited, the instrument in the plate circuit, a 200 microammeter, will read zero?

b. If we now leave the setting of the potential divider undisturbed, what will the instrument read when we introduce a voltage of 100 mv between points 1 and 2? The resistance of the meter has been determined as 600 ohms.

c. Assuming that the instrument is not a zero-center instrument, *i.e.*, cannot deflect to both sides, must the positive or the negative terminal of the instrument be connected to point X if the voltage introduced between points 1 and 2 makes point 2 negative with respect to point 1?

d. What drift of the zero point will result from a 1-volt deterioration of the 90-volt battery?

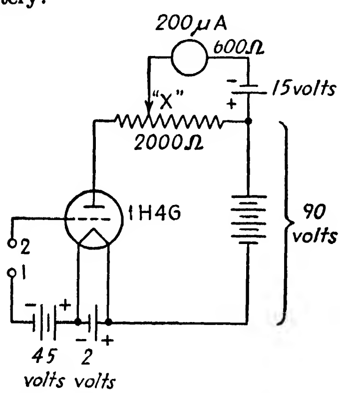

Fig. 11-15.—Circuit diagram for Prob. 11-1.

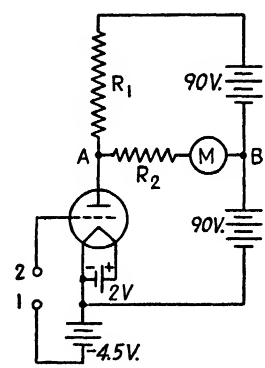

Fig. 11-16.—Circuit diagram for Prob. 11-2.

11-2. A type 1H4G tube (for characteristic values see Prob. 11-1) is connected in a circuit as shown in Fig. 11-16.

a. What value must R_1 have so that with the input terminals 1 and 2 shorted the meter will read exactly zero?

b. For ease of reading it is desired to have the instrument read 200 μa with 400 mv applied to the input terminals. The meter resistance is 600 ohms. What value must R_2 have? Hint: Apply Thévenin's theorem to branch AB.

c. Assume that, owing to age or current drain, each of the two 90-volt batteries drops to 85 volts. With R_1 and R_2 unchanged from the values found under *a* and *b* and with points 1 and 2 shorted, to what indication will the meter drift? Hint: Black box and Mr. Thévenin!

11-3. A voltage of 500 mv originating from a source not permitting any current drain is to be measured by means of a bucking arrangement with a milliampere meter. Available are a 6C5, a 6J5, and a 6C8G. The 6C8G is a twin-triode tube, for each section of which the manufacturer gives the following operating data: E_b = 250 volts, E_{cc} = −4.5 volts, I_b = 3.2 ma, r_p = 22,500 ohms, μ = 36, and g_m = 1,800 μmhos; the two

sections may be used in parallel. Which of the three tubes is the best choice for this particular case?

11-4. A 6SN7 tube consists of two separate triodes in one envelope; each section has characteristics identical with those of a 6J5 (see Fig. 10-11). Such a tube is connected

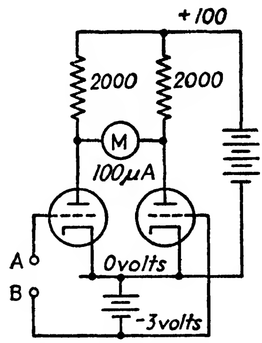

FIG. 11-17.—Circuit diagram for Prob. 11-4. The two separate tubes shown here are actually the two sections of a type 6SN7 tube.

in a circuit as shown in Fig. 11-17. The 100-μa meter has a resistance of 800 ohms. What voltage will have to be applied to the terminals A and B in order to obtain full-scale deflection of the meter?

SUGGESTED ADDITIONAL READING

Henney, K.: "Electron Tubes in Industry," Chap. 3, pp. 72-95, 104-107, McGraw-Hill Book Company, Inc., New York, 1937.

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CHAPTER XII

MULTIGRID TUBES

12-1. Purpose of Additional Grids.—In 1919, Schottky introduced another grid between the actual control grid and the plate. Consistent with the name "triode" for a tube with three elements, such tubes are called "tetrodes"; they are also known under the name of "screen-grid" tubes. Tetrodes did not come into general use until about 1927. The reason for introducing an additional grid between the anode and the control grid was the desire to reduce the capacity existing between these two elements. The capacity existing between grid and anode of an ordinary triode is not of much consequence when the tube is to be used in connection with the amplification of low-frequency voltages, but when it is desired to construct a multistage amplifier for radio frequencies, *i.e.*, frequencies ranging from several hundred thousand to several million cycles per second, the capacity existing between these two elements plays rather strange tricks. In the chapters dealing with oscillators, it will be shown that this capacity may cause a tube with a highly tuned circuit as a load to produce self-generated oscillations. In radio-frequency amplifiers using ordinary triodes, it was necessary to employ complicated "neutralizing" schemes, as they were called, to overcome the effect of this capacity. The introduction of a screen between the control grid and the anode of a tube and the connection of this screen to a point of fixed potential will obviously reduce the capacity existing between these two electrodes.

12-2. Rubber-sheet Model of the Tetrode.—In order to visualize what such an arrangement might do to the characteristic values of a tetrode, we shall return to the rubber-sheet analysis that served so well in the visualization of the operation of a triode. In Fig. 12-1 is shown the cross section through a rubber-sheet model representing a tetrode. An additional row of pins has been added between those representing the control grid and the anode. We have seen that the field near the cathode with the cathode cold, *i.e.*, not emitting any electrons, was the factor determining the amount of current that will flow in the tube after the cathode becomes emitting. It is true that this field is reduced to zero by the action of the space charge. Nevertheless, it evidently will take a larger space charge—which means a larger current—to reduce a stronger field near the cathode to zero. In the case of a triode, this field is produced by the combined effects of the grid and the anode voltage. As a matter of fact, since the grid is negative—which means "up" in the rubber-sheet model—any slope

away from the cathode can exist only if the anode is able to deform the rubber sheet between the grid pins in such a way as to form saddles resulting in a slope away from the cathode, at least at some points of the latter. When we look at Fig. 12–1, it becomes evident that the position of the anode will have little influence on the slope existing near the cathode; it would have to deform the rubber sheet between the two rows of grid pins. This will be the more difficult, the more closely the pins are spaced on the second row. The electric field existing near the cathode is obviously determined essentially only by the potential of the two grid

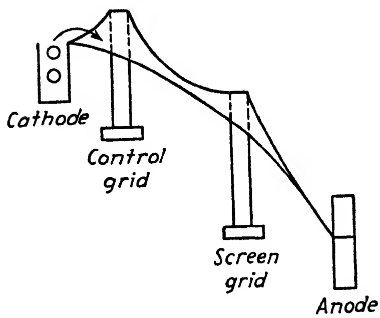

FIG. 12–1.—The cross section through a rubber-sheet model of a tetrode.

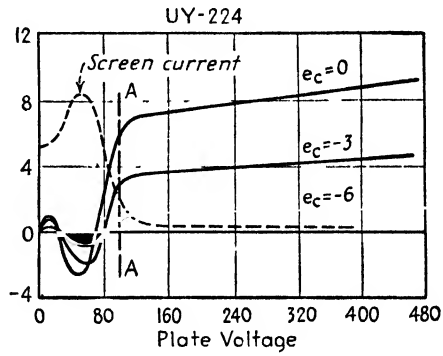

FIG. 12–2.—The plate characteristics of a tetrode show a peculiar behavior of the plate current for plate voltages less than the screen voltage. (*Courtesy Radio Corporation of America.*)

structures but will be influenced very little by the voltage applied to the anode. We can, therefore, expect that a tube of this type will have a high plate resistance as well as a high amplification factor. We can reason as follows. The plate resistance was defined as the ratio of a plate-voltage change to the plate-current change resulting from it. The considerations just made indicate that the plate voltage has very little influence on the plate current. A large plate-voltage change will, consequently, produce only a very small plate-current change, which means that the plate resistance of the tube is high. The correctness of this reasoning is proved by the fact that tubes of this type have plate resistances in the order of 1,000,000 ohms or more, compared to the values of 10,000 to 20,000 ohms found in triodes. If a certain tube has a plate resistance of 1,000,000 ohms, it means that a plate-voltage change of 250 volts, for instance, will produce a plate-current change of only ¼ ma. For the same reason, the amplification factor must be high. This value has been defined as the ratio of the effectiveness of a grid-voltage change to a plate-voltage change. Since a plate-voltage change has so little influence on the plate current, it takes a very large plate-voltage change to cancel the effect of a very small grid-voltage change. The correctness of this rea-

soning is also borne out by the facts. Tubes of this type are found to have amplification factors in the order of 1,000 and more. The characteristics of a typical tetrode are shown in Fig. 12–2. If we disregard at the moment the part of the curve lying to the left of the line AA, it will be noted that the characteristics are extremely flat. Thus, with -3 volts on the grid, the plate current will be 3.75 ma with 150 volts on the plate; raising this voltage from 150 to 350 volts, or a voltage change of 200 volts, causes the plate current to increase only by approximately ½ ma. The plate resistance of the tube in this region is, consequently, $200/0.0005 = 400{,}000$ ohms.

12–3. Plate Characteristics of Tetrodes.—The characteristics shown in Fig. 12–2 were taken with a voltage of 90 volts applied to the second grid, or screen grid. It will be noted that a peculiar behavior of the plate characteristics begins with a voltage near this value. To return to the rubber model of the tube, it seems that particular phenomena begin to take place as soon as the anode is at the same level as the second row of pins and that extremely strange things are happening when the voltage of this electrode becomes considerably less than the screen-grid voltage. Examination of the plate characteristics in the region between approximately 20 and 50 volts indicates that the current to the anode not only is decreasing while the voltage applied to it is increasing but that it actually reverses!

12–4. Secondary Emission.—The cause of this peculiar behavior is secondary emission. In Sec. 6–5, it was stated that there are several methods available to speed up the electrons within a metal to a point where they can break through the surface layers, *i.e.*, where emission takes place. Chapter VI dealt, however, only with thermionic emission, *i.e.*, emission produced by heating the metal. A second method of obtaining emission, as was stated there, is through bombardment of a metal with electrons. Emission resulting from this bombardment is called "secondary" emission. A rapidly moving electron on striking a solid substance may have sufficient energy to dislodge one or more electrons from the solid, and these dislodged electrons are known as "secondary" electrons. Much experimental work has been done in this field.

The essential facts concerning secondary emission, as summarized by Compton and Langmuir, are as follows:

1. The number of secondary electrons per primary bombarding electron depends greatly on the physical characteristics of the surface. Thoroughly degassed surfaces usually yield less than contaminated surfaces. The maximum number of secondary electrons per primary electron reaches a maximum of from 1 to 1.5 for degassed surfaces, 3 to 4 for untreated surfaces, and 8 to 10 for metals to which a special coating has been applied.

2. As the velocity of the primary electron is increased up to that corresponding to a few hundred volts, the number of secondary electrons per primary electron increases up to a maximum but then decreases again as the velocity of the primary electrons is further increased.

3. The velocity with which secondary electrons are emitted from the metal is slow and corresponds to only a few volts.

4. The secondary electrons leave the surface in all directions, the angular distribution curve depending somewhat upon the characteristics of the surface.

5. Secondary electrons may be derived from insulators as well as from conductors.

12-5. Explanation of Lower Part of Plate Characteristics with Rubber-sheet Model and Secondary Emission.—In an ordinary triode the conditions existing at the anode are most certainly conducive to secondary emission and, indeed, such emission does take place. We are never aware of this, however, because the secondary electrons find themselves in a field directed toward the anode. Looking at the rubber-sheet model of a triode, we see that any marbles knocked out from the receiving electrode would find themselves on an incline that would bring them right back to

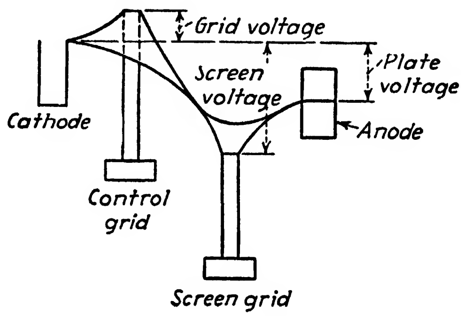

FIG. 12-3.—For a plate voltage less than the screen voltage, the rubber-sheet model takes the form shown here. If there are any secondary electrons knocked out of the anode, they will find themselves in a field directed toward the screen.

the anode. Now look at the condition existing near the anode of a tetrode, with the anode at a voltage less positive than the voltage applied to the screen grid. Under such a condition, a secondary electron emitted from the anode finds itself in a field directed toward the screen grid; or, in the analogy shown in Fig. 12-3, the rubber sheet has a slope directed away from the anode when the anode is "higher" up than the row of pins representing the screen grid. Under such a condition, the secondary electrons, instead of returning to the anode that emitted them, will go toward the screen grid. The correctness of this analysis can easily be ascertained by placing a meter in the lead going to the screen grid. It will be noted that the current to this electrode will increase materially as soon as the voltage on the anode becomes less than the voltage applied to the screen grid. In Fig. 12-2, the dotted curve shows the screen current. This then explains the rapid drop and even reversal of anode current when the voltage of the anode becomes less than the voltage of the screen grid. Why should the current increase again if we reduce the voltage on this electrode further? As just pointed out, the reduction in plate current is not due to the fact that the number of electrons arriving at the anode de-

creases when the anode voltage becomes less than the screen voltage, but it is due to the fact that many of the arriving electrons knock out secondary electrons, which then flow to the screen grid. The net current registered by a meter in the anode circuit will then, of course, be the difference between the current due to the electrons arriving at the anode and that due to those leaving as secondary electrons. As stated in Sec. 12–4 under (2), the ability of electrons to produce secondary electrons decreases when the speed with which they arrive decreases from that corresponding to a few hundred volts. Since the speed with which an electron arrives at any electrode depends only on the voltage existing between this electrode and the emitting surface (convince yourself by the rubber model that this statement is correct), the speed with which the electrons arrive at the anode decreases as the voltage applied to it is decreased. This means that secondary emission will become less and less, and, although it is true that the number of primary electrons also decreases with a decrease in anode voltage, the reduction of secondary emission far outweighs this effect, with the result that the current to the anode is actually increasing with a decrease in voltage applied to it.

12–6. Negative Resistance.—If we apply our definition of dynamic resistance to a point on this part of the characteristic, it is evident that the value turns out negative. A negative resistance is a rather interesting phenomenon. We shall see that it is possible to use it for canceling the effect of a positive resistance in oscillating circuits, so that these circuits will remain in oscillation of their own accord. Oscillators using a tetrode for the purpose of furnishing a negative resistance are called "dynatron" oscillators and formerly enjoyed great popularity. The method of putting the circuit to use will be discussed in the chapter dealing with oscillators.

Secondary emission has also been put to use recently in so-called photoelectric "multiplier" cells, a description of which will also be deferred until later.

12–7. Pentodes.—The fact that the region with plate voltages lower than the screen voltage is unstable, and therefore cannot be used for amplification purposes, cuts down the useful part of the characteristic to an undesirable extent. We have seen that this was due to the field existing between plate and screen that was of such a direction as to drive the secondary electrons toward the screen grid instead of back toward the anode. (The incline on the rubber-sheet model was away from the anode.) This undesirable feature can be overcome by the introduction of an additional grid structure between the screen grid and the anode. The tetrode thus becomes a pentode. This type of tube has replaced the tetrode in almost all important applications. The third grid is a rather coarse mesh and is held at the potential of the cathode; as a matter of fact, it is connected at the cathode inside of the tube in many cases. Figure 12–4 shows the family

of plate characteristics of a typical pentode. Examination of these curves shows that the dip in the plate current observable on the tetrode characteristics has now completely disappeared and that the flat parts of the

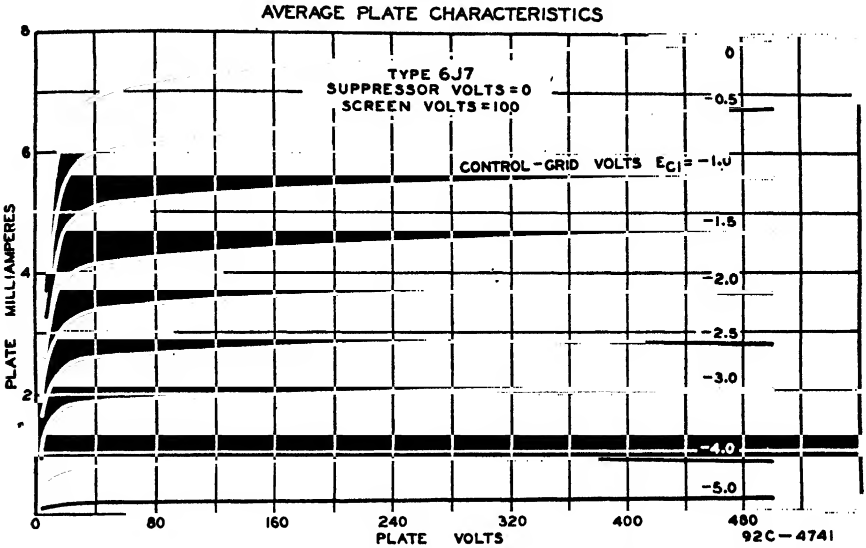

FIG. 12-4.—The plate characteristics of a standard pentode show that the introduction of a third grid has overcome the objectionable features of the tetrode. (*Courtesy Radio Corporation of America.*)

characteristics extend almost to the Y axis. A cross section through a rubber-sheet model representing this type of tube is shown in Fig. 12-5. The condition shown in this figure represents the case where the anode is less positive—that means not so far down in the model—than the screen

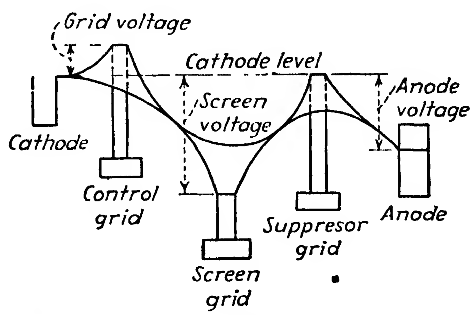

FIG. 12-5.—The rubber-sheet model of a pentode shows that, even with a plate voltage less than the screen voltage, any secondary electrons knocked out of the anode will find themselves in a field returning them to the anode.

grid. In the case of a tetrode, this condition would result in a field directed away from the anode, as shown in Fig. 12-3. But Fig. 12-5 shows that in the case of a pentode we now have a field near the anode in such a

direction as to make any secondary electrons return to the anode from which they were emitted. It is seen that the marbles rolling down on the rubber-sheet model enjoy a veritable coaster ride in their travel from the cathode to the anode. It may be argued that many of them must be falling in the funnel-shaped region formed by the screen grid. It must be remembered, however, that the speed of any marble is determined only by the vertical distance it has at any given instant from the level of the cathode. Consequently, if it misses the hole in the bottom of the funnel at all, it has sufficient momentum to climb the opposite side of the funnel and to find its way through the saddles formed between the pins of the third grid. This third grid in a pentode is called the "suppressor" grid. The reason for this name is obvious since it suppresses secondary emission. Strictly speaking, the statement is not correct, however. This grid *does not suppress secondary emission but the effects of secondary emission;* it simply creates a field near the anode in such a direction as to make the secondary electrons return to the anode from which they were emitted.

12–8. Characteristic Values of a Pentode.—In the case of a tetrode, we could predict, on the basis of the tube structure, that this kind of tube would have a high plate resistance as well as a high amplification factor. This prediction was based on the recognition that the plate voltage would have very little influence on the field established near the cathode, which, as we have seen, is the determining factor as far as the amount of plate current is concerned. It is quite obvious that with a pentode the plate voltage will have even less influence on the field established near the cathode because there are now three intervening grid structures. A glance at Fig. 12–5 makes it quite clear that a motion of the anode end of the rubber sheet does not change the incline of the rubber sheet near the cathode appreciably. These tubes therefore have plate-resistance values and amplification factors even in excess of those found in tetrodes.

12–9. Beam-power Tubes.—Within recent years, a new method has been discovered to suppress the action of secondary emission from the anode. It was found that the space charge in the region between the screen grid and the anode can be employed instead of a suppressor grid to eliminate the effects of secondary emission. So-called "beam-power" tubes make use of this principle. Although these tubes are really tetrodes, either the plate is far enough removed from the screen grid or the electrons are confined to a sufficiently intense beam over a shorter distance to enable the space charge in the region between the screen grid and the plate to modify the potential distribution between these two electrodes in such a way as to cause the field to be directed toward the anode. In other words, the effect of the concentrated space charge is the same as the effect of the suppressor grid; secondary electrons emitted from the anode find themselves in a field that returns them to the anode. Figure 12–6 shows a cutaway view of the internal structure of a beam-power tube; it shows

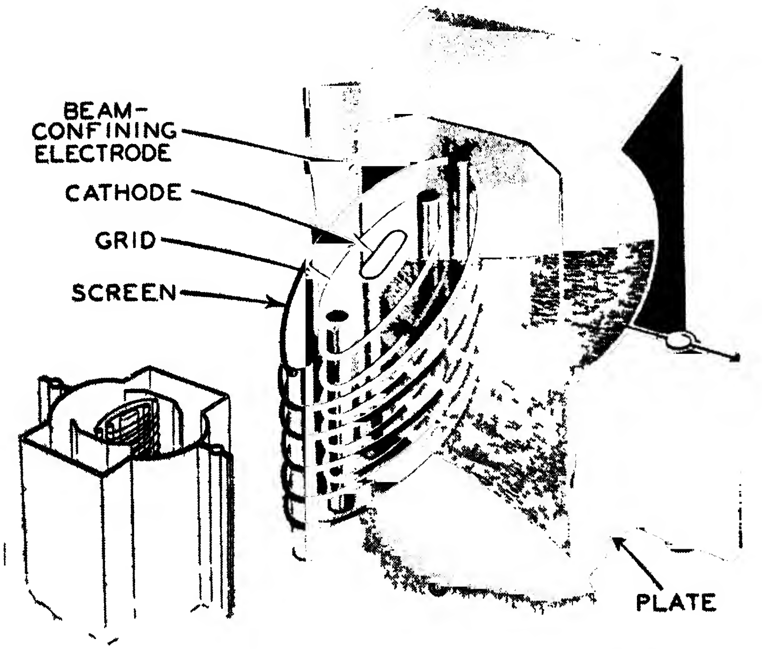

FIG 12-6 —In the beam-power tube an action similar to that produced by the suppressor grid in a pentode is obtained by the space-charge action of the electron stream between the screen grid and the anode. (*Courtesy Radio Corporation of America*)

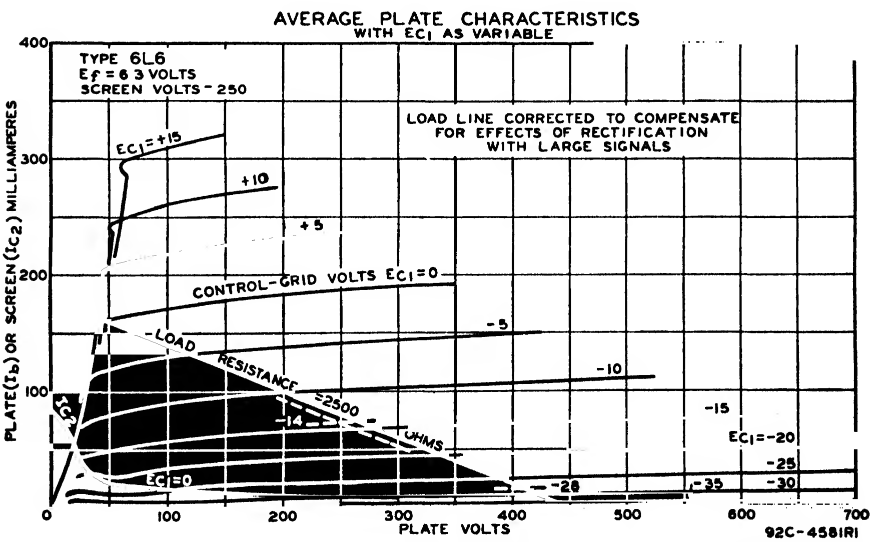

FIG 12 7 —The plate characteristics of a 6L6 beam-power tube (*Courtesy Radio Corporation of America*)

also how the beam of electrons is confined to a narrow space, which gives rise to the action just described. The plate characteristics of a typical beam-power tube are shown in Fig. 12–7. They are seen to be not quite so flat as those of a pentode (one would expect this since these tubes are really tetrodes), but the reasonably flat part of the curves extends almost as far as in the case of pentodes. A comparison of Figs. 12–4 and 12–7 furthermore discloses the fact that the plate currents obtainable with a beam-power tube are considerably larger than those of a tetrode or pentode. The latter fact makes these tubes particularly suitable when it is desired to obtain a fairly large amount of power. Pentodes are usually employed if the main purpose is to obtain a high voltage amplification from the tube.

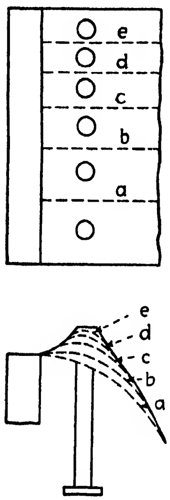

FIG. 12–8.—By varying the spacing of the grid pins, the action of a remote cutoff can be visualized.

12–10. Variable μ Tubes.—There are other modifications of the electrode structure in vacuum tubes by means of which the designer of tubes can produce certain desirable results. In the rubber-sheet model of the various types of tubes, it was assumed that the distances between the pins on any given row were constant. With such an arrangement, it is quite evident that the saddles formed in the rubber sheet by these pins are all of equal height. This means that, in raising the row of grid pins, a point will be reached where the current through all saddles will be interrupted at the same time. Now assume that we were to space the pins not with equal distances any more but with a pitch that increases. Figure 12–8 shows the top view and the cross section through the saddles formed on the rubber sheet between the various pins. It is clear that current will begin to flow at first between the saddles formed by the pins that are spaced farther apart. In most tubes of the receiving type the cathode consists of a cylinder covered with the oxide coating, and the grid is a wire spiral placed around the cathode. The electrical equivalent of Fig. 12–8*b* is obtained simply by winding this spiral not with a constant but with a varying pitch. Tubes having such a grid structure are called "variable μ" or, more recently, "remote cutoff" tubes. At some time, the advertising department of some tube manufacturer must also have been permitted to give this kind of tube a name and, giving their enthusiasm free reign—as advertising departments usually do—they named it "super control" tube. This entirely meaningless designation has fortunately been replaced by the really descriptive name "remote cutoff" tube. Any further improvement of this type will probably be called "super de luxe control" tube. The plate characteristics of a typical remote cutoff tube are shown in Fig. 12–9. It is of interest to compare the plate characteristics of an ordinary pentode,

as shown in Fig. 12-4, with those of the remote cutoff pentode, as shown in Fig. 12-9. At first glance the two figures seem to be nearly identical. An examination of the curves shows, however, that with the ordinary pentode the current is seen to decrease by almost equal amounts for every volt grid-voltage change, reaching almost complete cutoff at about -5 volts on the grid. The remote cutoff tube is still conducting with a grid voltage of -20 volts. A change of grid voltage from -10 to -15, for example, produces a smaller current change than a 1-volt change on the

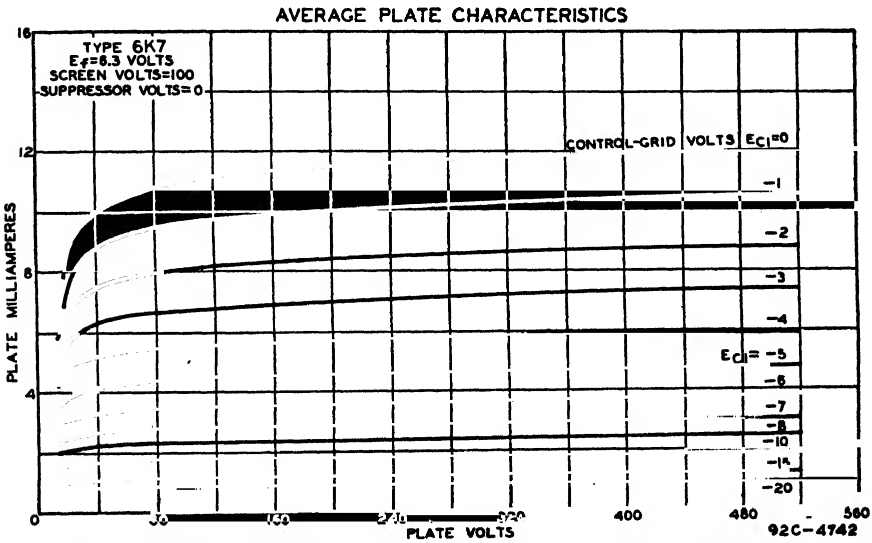

FIG. 12-9.—The plate characteristics of a 6K7 remote cutoff tube. (*Courtesy Radio Corporation of America.*)

regular pentode. The regular type of pentode is sometimes referred to as "sharp cutoff" tube, which emphasizes the difference in the behavior of the two types.

12-11. Calculation of Voltage Gain for Pentodes.—In Eq. (10-2), it was shown how to use the equivalent-plate-circuit theorem to calculate the performance of a single-stage tube circuit. To apply the equation it was necessary to know the amplification factor as well as the plate resistance of the tube at the chosen operating point. When we try to use this method in connection with pentodes, we find that the manufacturer has not committed himself too definitely on the values of amplification factor and plate resistance of these tubes. For instance, for a 6J7 tube operating with a plate voltage of 250 volts, the plate resistance is given as "greater than 1 megohm," but no figure is given for the amplification factor. The only value on which the manufacturer is evidently willing to commit himself is the transconductance, which for a plate voltage of 250 volts and a grid voltage of -3 volts is given as 1,225 micromhos. Is it possible to

predict the performance of a circuit containing a 6J7 tube with the aid of the transconductance only? It will be remembered that the transconductance tells us how much the plate current changes when the grid voltage changes by 1 volt. In the case of a triode this value is of use only when the tube has no load—or at least a load of very low resistance—in the plate circuit. As the characteristics of a triode show, the plate voltage has much to say about the current that will flow, and with a load in the plate circuit the plate voltage will, of course, change when the plate current changes. Let us assume a resistive load, for instance. Making the grid 1 volt more positive from a given bias voltage will try to increase the current; but this current flowing through the load will decrease the voltage on the plate so that the current will evidently not change so much as it would with the load short-circuited. But now compare the plate

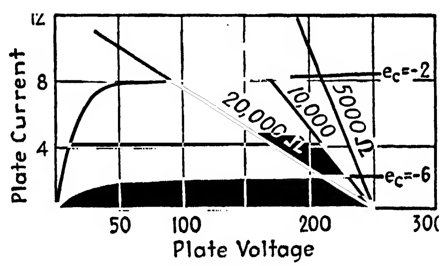

FIG. 12–10.—The plate-current changes taking place in a pentode are almost independent of the size of the load in the plate circuit, as shown here by the various load lines.

characteristics of a pentode with those of a triode. The extremely flat plate characteristics of a pentode indicate that the plate voltage will have very little influence on the amount of current that flows. In other words, the plate-current change that takes place with a given grid-voltage change is practically the same whether we have zero resistance in series with the pentode or a load resistance of appreciable value. This may be seen when we are drawing various load lines into the characteristics of a pentode. In Fig. 12–10 the plate characteristics of a particular pentode for grid voltages of -2 and -6 volts are shown. If the tube is operated with 250 volts on the plate and with zero resistance in series with it, a change of grid voltage from -2 to -6 causes a plate-current change from 8.3 to 2 ma; this is found by simply going vertically up from the 250-volt mark. Now, let us place in series with the tube a 5,000-ohm resistance. In Fig. 12–10 a load line is drawn for this resistance and marked 5,000; two additional lines representing 10,000 and 20,000 ohms load resistance are also shown. Examination of the intersection points of the three load lines with the two tube characteristics shows that the current changes occurring in the three cases are almost identical, owing to the extreme flatness of the plate characteristics. In other words, the current changes almost as much in the case of the 20,000-ohm load resistance in series with the tube

as it does with zero resistance in the tube circuit. This means that for the calculation of the voltage gain obtainable with pentodes it is only necessary to multiply the transconductance with the value of the load impedance or load resistance. Thus, if the transconductance of a particular tube is 1,000 micromhos, then with a load resistance of 100,000 ohms we shall obtain a voltage gain of 100. It must, of course, be realized that there is an upper limit to the value of the load impedance or load resistance when this method of calculating the voltage gain of a pentode gives increasingly erroneous results. After all, although the plate characteristics of a pentode are extremely flat, they are nevertheless not exactly parallel to the voltage axis, which means that the tube still has a definite value of plate resistance. For extremely high load resistances, such as 500,000 ohms to 1 megohm, the method will therefore not be suitable; in such cases the voltage gain is best determined by drawing a load line for these values of resistance into the plate characteristics.

Pentodes are usually employed when it is desired to obtain a voltage amplification as high as possible; beam-power tubes are used when relatively large amounts of power are to be obtained.

PROBLEMS

12-1. A type 6J7 tube (for plate characteristics, see Fig. 12-4) is operated with -2.5 volts grid bias, and a small alternating voltage (not exceeding ½ volt amplitude) is to be amplified. It is desired to obtain a voltage amplification of 200 from the tube. Assume that this amplification is to be obtained by the use of a plate resistor. What value must this resistor have and what plate supply voltage will be necessary? (In the interest of a common starting point, let us assume an actual plate voltage of 180 volts, with no alternating voltage on the grid.) What would happen if the grid-bias battery should deteriorate to -1 volt?

12-2. A type 6J7 tube is operated from a plate-supply voltage of 250 volts with a choke of 100 henrys in the plate circuit. It is biased to -4.5 volts. 0.2 volt rms sinusoidal alternating voltage is applied to the grid. What voltage will be across the choke for 100, 500, and 1,000 cps? (Neglect the ohmic resistance of the choke.)

What is the phase of this voltage compared to the phase of the voltage applied to the grid? Examination of the plate characteristics shows that they are reasonably flat down to a plate voltage of about 40 volts. With this value as a limit of plate-voltage swing, what would be the limit of frequency for the 0.2 volt rms applied to the grid before distortion of the alternating voltage appearing across the choke would set in?

12-3. A type 6K7 tube is connected across 500 volts dc with a resistor of 50,000 ohms in series with it. With various direct bias voltages, alternating voltages as given in the accompanying table are applied to the grid. Plate characteristics are given in Fig. 12-9.

Dc Grid Bias, Volts	Peak Value of Alternating Voltage Applied to Grid, Volts
-3.5	0.5
-4.5	0.5
-6.5	0.5
-9.0	1.0
-12.5	2.5
-17.5	2.5

Determine the resulting plate-voltage swing. If this swing is assumed to be sinusoidal around an average lying midway between the extremes of the plate-voltage swing, what would the actual voltage amplification be for the various dc grid-bias values?

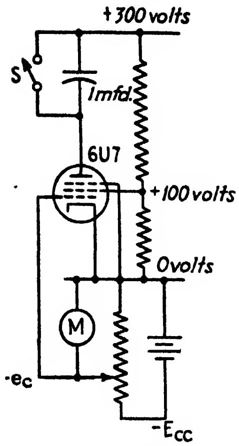

FIG. 12-11.

12-4. In certain timing devices it is desired to have a voltage change linearly with time (*i.e.*, proportionately with time). The so-called "sweep circuits" in cathode-ray oscilloscopes make use of such a voltage, for instance. It may be obtained by charging a capacitor with a constant current, and the extremely flat characteristics of pentodes have led to their use as constant-current devices. As shown by the plate characteristics of pentodes, the plate current for a given grid voltage is practically independent of the voltage applied to the plate of the tube.

The accompanying table gives the values of plate current for various grid voltages for a 6U7 tube; the plate currents given in the table were found to be practically constant for plate voltages between 50 and 300 volts.

The tube is connected in series with a 1-μf capacitor, as shown in Fig. 12-11. What time will it take to charge the capacitor to 250 volts after opening switch S, with the grid voltage adjusted to the various values given in the table?

e_c, volts	−3	−10	−20	−30	−40	−50
i_b, ma	8.2	2.2	0.9	0.4	0.15	0.03

SUGGESTED ADDITIONAL READING

Chaffee, E. L.: "Theory of Thermionic Vacuum Tubes," Chap. 27, pp. 588–621, McGraw-Hill Book Company, Inc., New York, 1933.

Dow, W. G.: "Fundamentals of Engineering Electronics," Chap. 6, pp. 126–140, John Wiley & Sons, Inc., New York, 1937.

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CHAPTER XIII

POWER RELATIONS IN ALTERNATING-CURRENT AMPLIFIERS; CLASSES OF AMPLIFIERS; PUSH-PULL OPERATION

13–1. Scope of Analysis of Ac Amplifiers.—In Chap. X the fundamental principles applying to single-stage amplifiers were discussed. Since pentodes and beam-power tubes have been added to our list, it becomes desirable to reexamine and expand our discussions on this subject. The field of ac amplifiers is so vast, however, that it will be entirely out of the question to cover more than a very small fraction of it here. Fortunately, the industrial engineer seldom is faced with the same problems that the communication engineer or the radio engineer has to work out. The internal capacitances of the tube elements are, for instance, of much importance if we wish to amplify voltages at radio frequencies; for the relatively low frequencies encountered in the usual industrial applications, these capacitances are of no consequence at all. In most of the industrial applications the distortion taking place in an amplifier is also of not too much consequence (except where the amplified output may be used for recording on an oscillograph). For the communication engineer this subject is of great importance. Our discussion of ac amplifiers will be confined to the very fundamentals, but the reader should have no difficulty in finding detailed information on a particular phase of this subject in the extensive literature.

13–2. Power Relations Derived from the Load Line.[2]—Figure 13–1 shows a tube with a load resistor R_L connected across a supply of direct voltage. The load line, drawn with a slope corresponding to the resistance value of R_L, intersects the characteristic of the tube with $-E_{cc}$ on the grid in the quiescent point Q. Connected in series with the bias voltage $-E_{cc}$ is a source of alternating voltage, the instantaneous value of which is designated as e_g. This source of alternating voltage therefore "bobs" the grid up and down from the level $-E_{cc}$ established by the bias battery. If the amplitude of the alternating voltage is E_{gm}, the grid will swing between the two levels $-E_{cc} + E_{gm}$ and $-E_{cc} - E_{gm}$. The plate characteristics for these two extreme grid-voltage values are shown in Fig. 13–1, and the intersection of the load line with these two characteristics gives us the points P_1 and P_2. The coordinates of the three points, Q, P_1, and P_2, give us all the information that we could possibly want about the performance of this circuit. The abscissas give us the voltages across the tube, and the ordinates give us the current values. The maximum value reached by the plate current is given by the ordinate of point P_1. The

minimum value is given by the ordinate point P_2. During that half cycle when the alternating input voltage swings the grid positive, the plate current increases, and, consequently, the voltage across the load resistor R_L also increases; this means that the potential of the plate swings negative from the value given by the quiescent point. The alternating component of the plate voltage may, therefore, be said to be 180 deg out of phase with the grid voltage. The plate current may evidently also be considered as being composed of a dc component given by the ordinate of the quiescent point Q and superimposed on it an alternating current with an amplitude

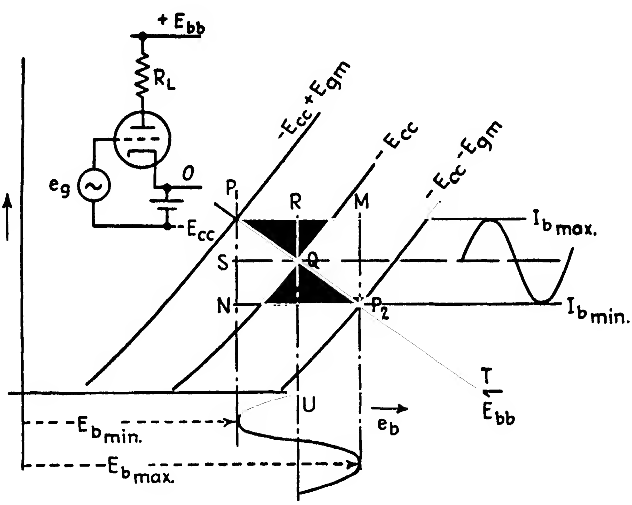

FIG. 13-1.—With the aid of the load line, the ac component of the voltage across the load and the ac component of the current flowing through it can be determined. The area of the rectangle P_1NP_2M is proportional to the ac power developed in the load.

given by the length QR in the diagram. Likewise, the voltage existing across the load resistance R_L can be considered as being composed of a dc component of the value given by the length TU and superimposed on it an alternating voltage with an amplitude given by the length QS in the diagram. The ac power appearing in a resistance is given by the product of the rms values of current and voltage. If these are of sinusoidal wave shape, the rms values are equal to the amplitude divided by $\sqrt{2}$, and the product of the two rms values is therefore equal to the product of the two amplitudes, divided by 2. In Fig. 13-1 this product will be represented by the area of the rectangle QRP_1S. The ac power is consequently equal to one-half the area of this rectangle. It is usually preferred to express the ac power in terms of the coordinates of the points P_1 and P_2 rather than to make use of the coordinates of the quiescent point also. Since the ordinates of points P_1 and P_2 represent the maximum and minimum values of current, respectively, it is evident from the diagram that the

amplitude of the alternating component of the plate current is one-half the difference between these two values, reasonably linear operation assumed. In a similar way the abscissas of the points P_1 and P_2 represent the minimum and the maximum voltage across the tube (or, if measured from point T, the maximum and the minimum voltage across the load), and the amplitude of the alternating voltage across the load is therefore one-half the difference between these two values. The ac power in the resistance is therefore given by

$$P = \frac{(E_{b\,\max} - E_{b\,\min})(I_{b\,\max} - I_{b\,\min})}{8} \tag{13-1}$$

An examination of Fig. 13-1 shows that the ac power is one-eighth the area of the rectangle P_1MP_2N. With a given alternating voltage applied

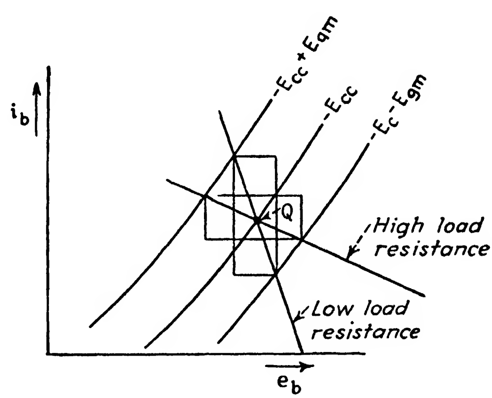

FIG. 13-2.—Different load resistors given different ac power.

to the grid, the maximum ac power will therefore be obtained with a load resistance R_L which will make this rectangle the largest. The meaning of this statement is made clear in Fig. 13-2. In this figure two load lines have been drawn through the quiescent point Q, the steeper one representing a small load resistance, the flatter one representing a higher load resistance. The two rectangles are shown determined by the intersection of the two load lines with the two plate characteristics corresponding to the extreme values of the grid swing as produced by the alternating voltage applied to the grid. With a low value of load resistance (the steeper line), we see that the variation in current is large, but the variation in voltage is small; with a high load resistance, the opposite is true. By intuition, we know that there must be a value of load resistance that will make this area a maximum. By the methods of differential calculus, it can be shown that this will be the case when the diagonal of this rectangle has the same slope as the plate characteristics which, in turn, means that

the load resistance must be equal to the plate resistance of the tube. This was arrived at previously by the use of the equivalent-plate-circuit theorem.

13-3. Load Resistance for Maximum Obtainable Power.[1]—In Sec. 13-2 reasonably linear operation was assumed. In other words, the alternating voltage applied to the grid was not so large as to move the operating point along the load line into a region of pronounced curvature of the characteristics. Under this condition the load resistance giving maximum power is equal to the plate resistance. As stated once before, the manufacturer usually specifies the resistance that the load should possess, and this value differs greatly from the value of the plate resistance. In order to clarify the reader's mind on this situation, the following two questions are presented. He should try to convince himself that they are entirely separate and distinct and that the answers to them will not necessarily be the same. First question: Given a small ac signal to be applied to the grid of a tube ("small" means much less than the total swing of the grid permitted on the particular tube), how large must the load resistance be in order to obtain maximum power? Second question: Suppose that we have available for the driving of the grid an alternating voltage as large as we may wish; how large do we have to make the load resistance in order to obtain the maximum power that the tube can furnish at all? It is evident that these are two entirely separate and distinct questions and that the derivations in Sec. 10-10 and the results arrived at in Sec. 13-2 gave an answer to the first question only. It was correctly given by our analyses: the load resistance must be equal to the plate resistance of the tube.

It also becomes clear that the second question is by far the more important. Tubes are low-power devices, and it is obviously our desire to obtain the maximum power that they are capable of furnishing. If the voltage available for the operation of the grid is not sufficient to make use of the full swing of the grid permissible on the particular tube, it is a relatively easy matter to employ an additional stage of amplification to step up this available voltage. The inquiry as to the size of the load resistance that will give the maximum obtainable power from the tube, regardless of the amount of voltage needed on the grid, is, therefore, the more important one. Unfortunately, it is not possible to answer this question in as straightforward a manner as the first question. There are several factors that cannot be taken account of in a mathematical manner. The allowable amount of distortion is, for instance, one factor entering into these considerations. Sometimes it is stated that a given tube can furnish so many watts of "undistorted power," and that in order to obtain this power the load resistance must have the recommended value. It should be realized that there is no such thing as undistorted amplification; no matter how small the input to the tube is made, the output will always show some degree of distortion. But in one problem a 10 per cent distortion may be perfectly allowable, while in another it must be kept below 1 per cent.

Load-resistance values satisfying the condition of maximum power in these two cases would differ materially. Other factors entering into these considerations are the choice of the operating value, *i.e.*, the quiescent point of the tube, the maximum voltages permissible on the plate of the tube and the desire to keep the grid always negative with respect to the cathode. Under these conditions, the only practical way of arriving at a solution is to draw into the family of plate characteristics a number of load lines and to determine the size of the rectangle as shown in Fig. 13–2. The rectangle with the largest area will represent the value of load resistance giving us maximum power. If distortion is of importance, it is necessary to check whether the distortion is within the limits dictated by the particular problem for which the tube is to be used. Several methods for the determination of the distortion are available, but the fundamental principle underlying all of them is simple. It is assumed that a sinusoidal voltage is applied to the grid, and from the load line corresponding values of the plate current are read. If there were no distortion at all, the plate current would also vary sinusoidally, and the deviations from this ideal condition therefore give a measure of the distortion. By choosing a few selected values of grid voltage and by manipulating the plate-current values corresponding to these selected grid voltages according to a set of given equations, the amount of the various upper harmonics introduced into the plate current by the distortion can be calculated. The reader interested in this subject will find an extensive literature covering it.

If an actual tube of the type under consideration is available, the most satisfactory method is, of course, to set the tube up with various values of load resistance and, by measurement, determine the value that will satisfy the condition of maximum power output with the distortion allowable in the specific application. In the case of triodes, it is found that the value of the load resistance satisfying all the above conditions and giving maximum power output is usually in the neighborhood of twice the plate resistance of the tube. A mathematical approach based on certain simplifying assumptions also leads to this result, but in view of the complexity of the problem, the presentation of the mathematical treatment is not considered worth while here.

13–4. Determination of Optimum Load Resistance for Pentodes.—Figure 13–3 shows the plate characteristics of a 6F6, a typical power pentode. Assume that we decide to operate the tube with a plate voltage of 250 volts and a grid bias of -15 volts. With this bias the alternating voltage applied to the grid should not have an amplitude in excess of 15 volts if we wish to prevent the grid from ever becoming positive. On the negative swing of the signal voltage, the grid will then become -30 volts. Four load lines are drawn through the operating point, the line A_1B_1 representing 1,000 ohms; A_2B_2, 2,000 ohms; A_3B_3, 4,000 ohms; and A_4B_4, 8,000 ohms. Which one of these load lines will give the maximum power when

an alternating voltage with 15 volts amplitude is applied to the grid? To answer this question, we have to complete the rectangles formed by the intersections of the various load lines with the characteristics for zero and -30 volts grid voltage. In order not to make Fig. 13–3 too confusing, this process is carried out only for the line A_3B_3. Mere inspection shows that the rectangle formed in this way is larger than the rectangle formed by using lines A_1B_1 or A_2B_2. Whether the rectangle formed by using the line A_4B_4 is larger than the one shown in the drawing would have to be checked more carefully; obviously this rectangle will be longer than the one formed by using A_3B_3, but its height will be less. In any case, the

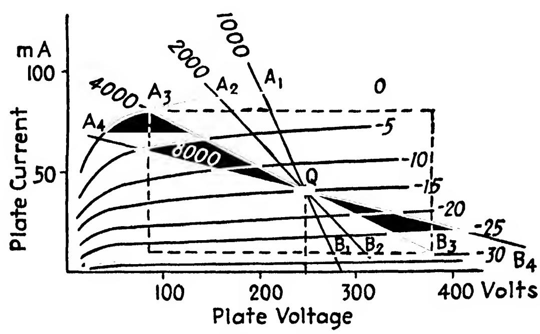

FIG. 13–3.—The load line giving the rectangle with the largest area (as shown for the line A_3B_3) represents the case of maximum ac power.

line A_4B_4 would be an undesirable one because (1) it hits the zero grid-volt characteristics below the knee and (2) point B_4 represents a plate voltage that is probably in excess of the permissible limit. From this we can conclude that the most desirable value for the load resistance will be somewhere between 4,000 and 8,000 ohms. It is of interest to note that the manufacturer gives a value of 7,000 ohms as the load resistance. However, since he chooses as operating point a value of 285 volts in place of 250 volts, which moves all the load lines somewhat to the right, the agreement between the value found in the above analysis and the value given by him can be considered as quite satisfactory.

Figure 13–3 discloses another fact of importance concerning pentodes. It will be noted that the quiescent point in this figure is not located at, or at least near to, the center of the rectangle, as it was in Figs. 13–1 and 13–2. Figure 13–3 clearly indicates that the plate current increases during the positive half swing of the alternating voltage applied to the grid a larger amount than it decreases during the negative half swing. This means that the positive half cycle of the alternating component of the plate current will be larger than the negative half cycle. A dc meter included in the plate circuit will, consequently, register an increase of current as soon as the alternating voltage is applied to the grid. Such a condition obvi-

ously indicates that a serious amount of distortion is taking place; indeed, the distortion is considerably more severe with pentodes than with triodes.

13-5. Path of Operating Point for a Reactive Load.—When the load in series with a tube consists of pure resistance, the voltage level of the plate obviously can never be higher than the voltage of the source supplying the combination of tube and load. This is due to the fact that the current through the tube is always in the same direction (since it is a rectifier), and the voltage across the resistor is therefore always of the same polarity, the negative end being the one next to the plate. The supply voltage E_{bb} was given by the intersection of the load line with the voltage axis; in other words, it had to be considerably higher than the voltage chosen for the quiescent point. Now consider the resistance replaced by inductance. According to Ohm's law for the inductance, the voltage across it depends, *not* on the *amount* of current, but only on the *rate at which it is changing*. In Fig. 13-4 the polarity of the voltage across the inductance depends, therefore, on whether the plate current is increasing or decreasing; during the time that it is increasing the voltage will be of such polarity as to oppose this increase, which the inductance tries to accomplish by reducing the plate voltage of the tube. A will therefore be negative with respect to B. When the current decreases, on the other hand, the inductance will raise the plate voltage; in other words, A will "bob" *above* the level of the supply voltage. The situation may also be looked at in a little different way. The plate current can be considered as consisting of a direct current with an alternating current superimposed on it. If the inductance has no resistance, the dc component cannot produce a voltage across it, and the voltage across it must be a pure alternating voltage. But if this be the case, A must be alternately positive and negative with respect to B.

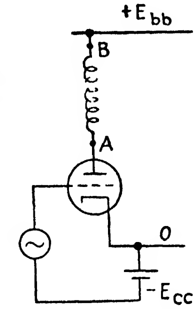

FIG. 13-4.—The polarity of the voltage existing across an inductance does not depend on the current but on the rate at which it changes. With alternating current flowing through the inductance, the potential of point A will therefore jump *above* the level of the supply voltage during the half cycle when the current is decreasing.

In the case of a resistive load the operating point slid along the load line as an alternating voltage was applied to the grid. In the case of an inductive load, the path of the operating point is no longer a straight line but becomes an ellipse, as shown in Fig. 13-5, which covers the general case that the load has resistance as well as inductance. The proof that the curve is an ellipse requires familiarity with analytical geometry and will not be attempted here. The reader should have no difficulty, however, in seeing that the operating path cannot possibly pass through the quiescent point any more, as in the case of a resistive load. At the instant when the alternating component of the plate current is zero, *i.e.*, when the plate current has the same value as given by the quiescent point, it will

be changing fastest, and there will consequently be a voltage across the inductance, with a polarity depending on whether the plate current is increasing or decreasing. These two instants are represented by points Q_1 and Q_2 in Fig. 13–5, which also indicates the direction in which the operating path is traversed during one cycle of the alternating voltage applied to the grid.

The shape of the path of the operating point in the case of an inductive load indicates that it is not possible to draw a "load line" for an inductance, as has already been pointed out. When we derived the method of the load line in connection with a Mazda bulb, it will be remembered that the procedure was based on the fact that the voltage across the nonlinear conductor plus the voltage across the resistor must always add up to the total voltage applied to the series combination. Regardless of the type of load, this statement will always be true for the *instantaneous* values of voltage appearing across the two circuit components; when the load is inductive or capacitive, in other words, when the voltage across it is not in phase with the alternating component of the current, we cannot draw a load line for effective values or for the reactance of the load at a given frequency. For the solution of problems involving reactive loads in the tube circuit, we are therefore forced to fall back on the use of the equivalent-plate-circuit theorem, which, of course, is applicable only if the tube remains operating in the linear part of the characteristics. When the grid voltage applied to the tube has an amplitude so large as to make the tube operate beyond the limits of linear operation, then we have no simple means at our disposal, such as the load line in the case of a resistive load. Figure 13–5 shows the path of operation for the case where the limits of linearity are not exceeded, *i.e.*, if the path of the operating point falls into a region of the plate characteristics where they can be considered as equidistant parallel lines. If the operating point travels beyond these limits, a graphical analysis becomes considerably more complicated, and a discussion of the methods to be applied then would be beyond the scope of this book. (An excellent treatment will be found in the book by A. Preisman; see list of Suggested Additional Reading at end of chapter.)

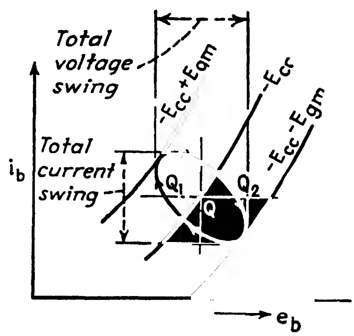

FIG. 13–5.—No load line can be drawn for an inductive load. The operating point describes an ellipse.

13–6. Different Load Lines for Dc and Ac Components.—In Sec. 10–10 it was shown that it is desirable to couple a load to the plate circuit of a tube by means of a transformer, when we are interested only in the ac output of the tube. If the load is purely resistive and the transformer used to couple it to the plate circuit can be considered as ideal, then to any al-

ternating voltage or current the load presented to the plate circuit will be purely resistive. To make this clear, consider a 110- to 220-volt transformer with negligible magnetizing current. If we connect a 220-ohm load to the 220-volt winding, a current of 1 amp is going to flow in the secondary. The current in the primary must, of course. be 2 amp. Since the voltages are in phase, owing to the fact that they are produced by the same changing flux, and since the secondary current is in phase with the secondary voltage, the primary current must also be in phase with the voltage; in other words, any voltage source feeding the primary winding is under the impression of feeding a resistor of 55 ohms (2-amp current at

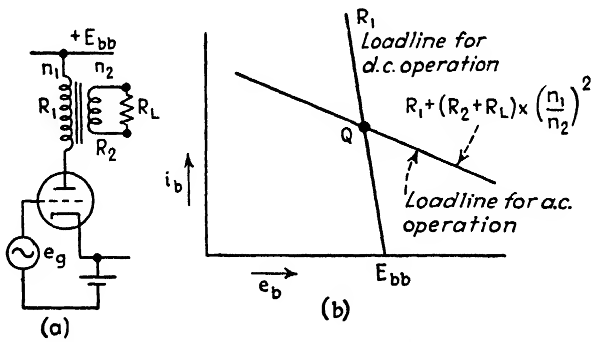

FIG. 13–6.—For direct current the load line is drawn with the resistance of the primary winding; for alternating current the load resistance connected to the secondary is reflected into the primary winding. The plate characteristics have been omitted in this figure.

110 volts applied to the winding). However, it is only to a source of alternating voltage with a frequency within the range where the transformer can be considered as ideal that it will appear like a resistance of 55 ohms. If we were to apply a direct voltage to the primary, the current, after an initial transient, would be determined only by the dc resistance of the primary winding, which will be only a fraction of 55 ohms. If we should include a transformer in the plate circuit of a vacuum tube, as shown in Fig. 13–6*a*, then the quiescent point (no ac signal applied to the grid) would be determined only by the dc resistance of the primary winding. It is with this value that a load line must be drawn if the supply voltage is given; it determines the supply voltage if the quiescent point is given. As soon as an alternating voltage is applied to the grid, the resistance of the secondary winding and the load resistance connected to this winding appear on the primary side, multiplied with the square of the turns ratio. Since voltage and current are in phase, however, it is permissible to represent this resistance by means of a new load line. The load line needed for the determination of the quiescent point is therefore drawn with the dc resistance of the primary winding, while the load line for the determination of the path of the operating point and the ac power output is drawn with

the above resistance plus the resistance reflected into the primary from the secondary. This situation is shown in Fig. 13–6*b*. The dc load line is seen to be much steeper than the one to be used for the analysis of the ac operation.

There are also circuits where the opposite condition is true. Figure 13–7*a* shows a circuit frequently encountered where the load line to be used for the purpose of determining the quiescent point is less steep than the one that must be used when an alternating signal voltage is applied to the grid. As far as dc operation is concerned, the capacitor C and the resistor R_2 may be considered as absent, and the load line must be drawn with the resistance of R_1. For an alternating current with a frequency

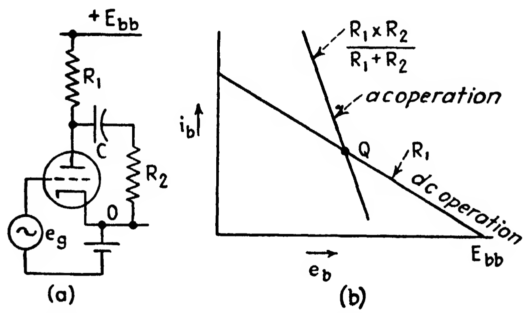

FIG. 13 7.—In the case shown here, the value of the load resistance is larger for dc operation than for ac operation. The plate characteristics have been omitted from this figure.

at which the reactance of the capacitor C is small compared to the resistance of R_2, the two resistors R_1 and R_2 are in parallel (replace the tube by a black box with $\mu \times e_g$ and r_p, short-circuit all other voltages, and you will see the truth of this statement), and the load line for ac operation must be drawn with a resistance value equal to the one of the parallel combination of the two resistors. This is shown in Fig. 13–7*b*.

13–7. Power Available in the Case of Dc Operation.—Equation (13–1) gave the ac power developed in a load resistance of given size. We have seen that the actual power was one-eighth the area of the rectangle determined by the intersection of the load line with the two plate characteristics corresponding to the two extreme grid-voltage values. In industrial applications the tube is quite often employed for the operation of a dc relay in series with the plate circuit. It should be pointed out emphatically that the power available for the operation of the coil of a relay placed in series with the tube has nothing to do with the value of ac power with which the preceding sections have dealt. In Fig. 13–8 a relay is shown in series with a tube. Let the load line FP represent the coil resistance of the relay; the length of the line OF represents the total direct voltage E_{bb} available for the operation of the combination of the two elements. By changing the grid voltage between appropriate values, it is possible to ob-

tain a current of the magnitude PC through the relay coil. The voltage across the coil under this condition will be given by the length CF. The wattage available for the coil is then given by the area of the complete rectangle $CPDF$, which represents therefore roughly eight times as much power as would be obtainable for ac operation. Here again the maximum obtainable power can be found by assuming load lines of different slopes in this figure and determining the size of the rectangle resulting from such a choice. After such a procedure, it is, of course, necessary to check the values of current and voltage against the maximum values permitted by

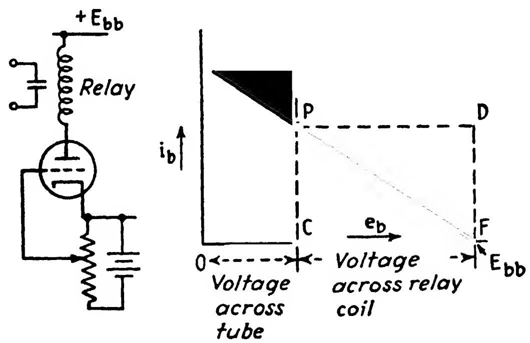

FIG. 13-8.—In the case of dc operation, the power available for the operation of the load is given by the full area of the rectangle $PCFD$. The plate characteristics have been omitted from this figure.

the manufacturer. Distortion, on the other hand, is of no importance in the case of relay operation.

13-8. Determination of Voltage Gain for a Remote Cutoff Tube.—In Sec. 12-10, a pentode with variable pitch of the grid structure was described. Such a construction results in variable amplification or remote cutoff. The degree of nonlinearity of such a tube is high, but if the amplitude of the input signal is kept low, the variable amplification feature may be put to good use. Figure 13-9*a* shows a circuit consisting of a pentode with a load resistor of 30,000 ohms placed in series with the tube. The tube is a type 6K7, and the combination is placed across a voltage of 320 volts. Suppose that the amplitude of the ac signal is ½ volt, which means that the total swing of the grid voltage is 1 volt; thus, if the bias is adjusted, for instance, to a value of $-2\frac{1}{2}$ volts, the extreme values of the grid swing will be -2 and -3 volts. The intersections of the load line with the characteristics for -2 and -3 volts, as shown in Fig. 13-9*b*, indicate that the total voltage swing on the plate will be from 80 to 120 volts, or 40 volts. Now let the bias be increased to -6.5 volts. The limits between which the grid voltage will now swing are -6 and -7 volts, and the intersections of the load line with the tube characteristics indicate a plate-voltage swing from 210 to 230 volts, or a variation of 20 volts. The

voltage gain of the stage is now only 20 instead of 40. Further increase of the bias will reduce the amplification still further. The intersections of the load line with the characteristics for −15 and −20 volts, for instance, indicate that in this region a 5-volt change on the grid produces a plate-voltage change from approximately 280 to 290 volts. In this region the voltage gain has therefore dropped to a value of 2.

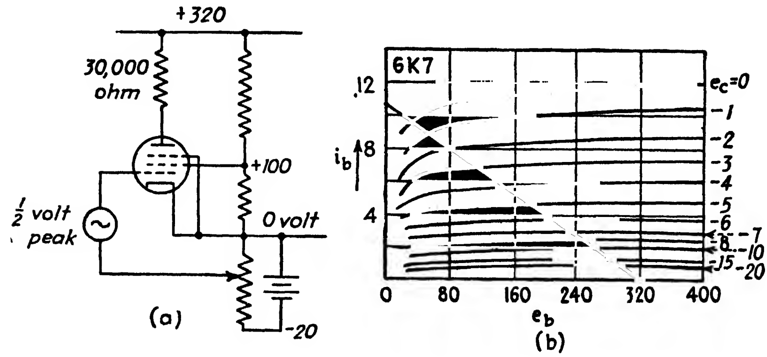

FIG. 13-9.—Determination of the voltage amplification obtainable with a remote cutoff tube.

13-9. Class A, B, and C Amplifiers.[4]—In our discussion of the tube as an amplifier so far, we have assumed—except where the tube is used in connection with a dc relay—that the grid will never be driven so far negative as to cut off the plate current completely. When it is desired to have the voltage variation appearing across the load resistor in series with the tube a reasonable reproduction of the voltage variation applied to the grid, then it is evidently necessary to let the tube operate in the linear part of the characteristics. This last statement is, of course, inaccurate since there is, strictly speaking, no part of the tube characteristic that is completely linear. Various problems solved by means of tube circuits will permit various degrees of distortion. When we wish to run a motor from a tube circuit, a distortion of 30 or 40 per cent may be perfectly tolerable; in the case of an amplifier used in connection with the analysis of wave shapes, a distortion of 3 per cent may not be tolerable. In the first case, we mean by "linear range" the region where the distortion does not exceed 30 per cent; in the second case, the linear region will be much smaller. In either case, however, we can hardly allow the plate-current flow to cease completely during any part of the cycle of grid-voltage variation. Amplifiers where this condition is fulfilled, *i.e.*, where plate current flows during a complete cycle of the input signal, are called "Class A" amplifiers, and the operation of the tube may be described as Class A operation.

When the bias of a tube is adjusted to such a value that the plate current is just zero and an alternating voltage is placed in series with this

bias, it is evident that plate current will flow only during the positive half cycle of the input voltage. An amplifier operating under such a condition is called a "Class B" amplifier. It would appear that the distortion produced by such an amplifier is entirely intolerable, but as will be shown shortly, it is possible to obtain satisfactory amplification from such an amplifier by the use of two tubes.

If the bias of a specific tube is adjusted to a value exceeding the cutoff point and an ac signal is placed in series with this bias, plate current will not flow during a complete positive half cycle of the signal voltage, as is the case with a Class B amplifier, but only during that part of the positive half cycle of the signal voltage when the total resulting grid voltage permits the flow of current. Amplifiers operating under such a condition are called "Class C" amplifiers. They are usually employed in connection with tuned circuits as load. These circuits will be described in detail in connection with oscillators. At this time a short reference to a mechanical analogy may be of help, however, to visualize the use of such amplifiers. The ringing of a heavy church bell is usually accomplished by pulling on a rope at the proper instant within the cycle of operation. The oscillation of the bell is therefore kept up by supplying energy in spurts instead of continuously. If a tuned circuit is made the load of an amplifier, it can be kept in oscillation by letting the amplifier furnish energy only during part of the cycle, and it is in such cases that Class C amplifiers are not only satisfactory but even desirable. The three classes of operation are pictured in Figs. 13–10*a*, *b*, and *c*. In all three figures the curve shown is the "dynamic" transfer characteristic, *i.e.*, the relation between plate current and grid voltage with the particular load that the tube is supposed to operate in the plate circuit (see Prob. 9–13). In Fig. 13–10*a* the bias is such and the signal voltage is of such magnitude that the plate current will flow during the complete cycle of input voltage.

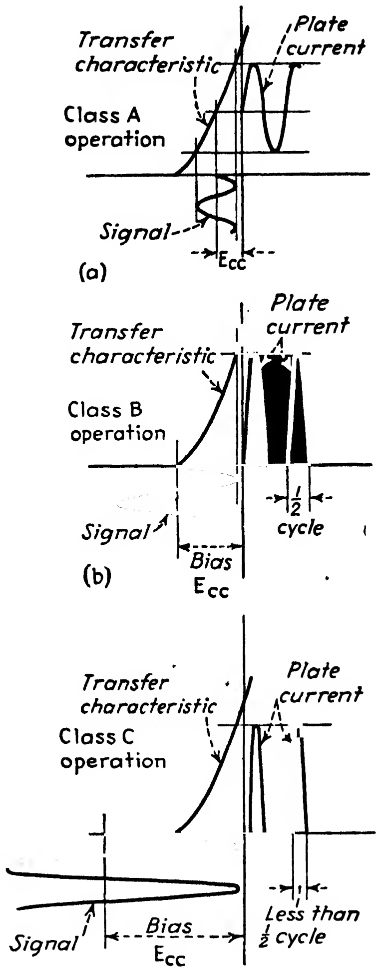

FIG. 13–10.—Grid bias, signal voltage, and plate current for Class A, B, and C operation of an amplifier tube.

It is left to the reader to analyze Figs. 13–10*b* and *c*, which picture Class B and Class C operation, respectively.

13–10. Operation with Positive Values of Grid Voltage.—When examining the plate characteristics for various tubes published by the manufacturer, we find that in most cases no curves are given for positive grid voltages. When operating with positive grid voltage, we obviously throw away the most outstanding characteristic of a vacuum tube: the ability to control the current flowing in the plate circuit of the tube without the expenditure of any energy at the grid of the tube. Consequently, there does

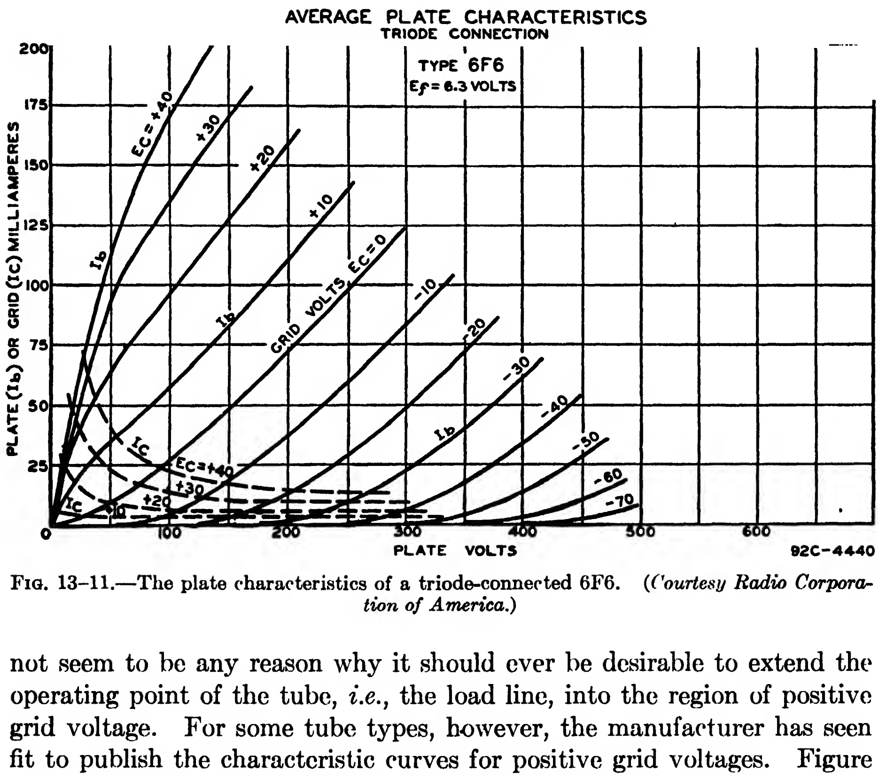

Fig. 13–11.—The plate characteristics of a triode-connected 6F6. (*Courtesy Radio Corporation of America.*)

not seem to be any reason why it should ever be desirable to extend the operating point of the tube, *i.e.*, the load line, into the region of positive grid voltage. For some tube types, however, the manufacturer has seen fit to publish the characteristic curves for positive grid voltages. Figure 13–11 shows the plate characteristics of a 6F6, for example, in triode connection, as published by the manufacturer. It will be noted that the curves for grid voltages of 0, +10, and +20 volts do not differ in character from those for negative grid voltages; they have about the same slope and are spaced approximately the same distance from each other as the characteristics for negative grid voltages. Often it is of more importance to obtain the maximum amount of power that a tube or a pair of tubes can furnish than to be able to exercise the control on the grid without having to furnish any current. In other words, if we have a source capable of furnishing whatever amount of current the grid is going to take

when it becomes positive with respect to the cathode, then this source may be used as a signal voltage, and the operating point of the tube and the load line can be extended into the positive region. It has become common practice to indicate whether a given amplifier tube is intended to operate with a positive grid voltage during part of the cycle of the signal voltage, or whether the grid is supposed to remain negative during the whole cycle of the signal voltage by adding the subscript 2 or 1, respectively, to the class of amplification. Thus, if it is stated that a given pair of tubes in push-pull arrangement operating in Class B_1 can furnish 5 watts, it means that the grids of the tubes remain negative at all times and that plate current flows in alternate tubes during only one half cycle of the signal voltage. If the tubes are said to operate in Class B_2, it still means that plate current flows only during one half cycle in each tube, but this time it is indicated that the grid will go positive at least during some part of the positive half cycle of the signal voltage.

Another mode of operation, which has found considerable favor, is one somewhat intermediate between Class A and Class B amplification; such operation is referred to as Class AB_1 or AB_2, depending on whether the grid becomes positive with respect to the cathode during any part of a cycle of the signal voltage. Since, in the case of Class A operation, the bias was placed at such a value that the signal voltage would never produce complete cutoff of the plate current, while in the case of Class B operation, the bias was of such a value as to permit plate-current flow only during the positive half wave of the signal voltage, then Class AB operation is a condition where the value for the bias is chosen in such a manner that the plate current of a tube becomes cut off during only part of the negative half cycle of a relatively large signal voltage. For small signal voltages the Class AB amplifier is, therefore, actually operating as a Class A amplifier, and only when the amplitude of the signal increases considerably will there be a cutoff of the plate current during part of the cycle.

If it is intended to obtain the maximum power from a tube by driving the grid positive during part of the cycle of the input voltage, careful attention must be paid to the source driving this tube. It must be able not only to furnish the average wattage required by the grid but also to deliver the relatively high peak of power required by the grid at the instant when it reaches its maximum positive voltage with respect to the cathode. It can be stated without too much fear of contradiction that if a large amount of power is required without appreciable distortion Class B_2 or AB_2 operation is to be avoided and that the problem will have a better solution by employing a higher power triode operating in Class A operation. Each problem, however, must be judged by its own requirement.

13–11. Push-pull Operation.[3]—The nonlinearity of the tube characteristics causes distortion even in the case shown in Fig. 13–10*a*, *i.e.*, Class A operation. There is, however, one particular arrangement by means of

which the effect of this distortion can be minimized to a considerable degree. The method is known as "push-pull" operation. The circuit diagram for this connection is shown in Fig. 13–12. Bias is supplied to the grid of both tubes by the battery E_{cc}. The signal is supplied to the grid by means of a center-tapped transformer, the center of which is connected to the negative end of the bias battery, and the two ends of the winding are connected to the grids. It is evident that with this connection, when the grid of tube I swings positive, for instance, the grid of tube II swings negative. This means that the current in tube I will be increasing and in tube II decreasing. Let us at first assume that the amplitude of the signal applied to either grid is so small that we can use the equivalent-plate-circuit theorem for the solution of the problem. Under this condition, each tube can be replaced by a black box, *i.e.*, by a resistance and a battery in series with it. The introduction of an alternating voltage in series with the grid bias in the actual circuit shown in Fig. 13–12 is equivalent to introducing a generator with a voltage μ times the alternating voltage applied to the grid in series with the fictitious battery. The equivalent of the circuit shown in Fig. 13–12 then becomes the circuit shown in Fig. 13–13. During the half cycle of the signal voltage, when the grids in the actual circuit shown in Fig. 13–12 have a polarity as shown there, the fictitious generators will have a polarity as shown in Fig. 13–13. It will be noted that the dc components of the two plate currents flow through the two sections of the transformer in the plate circuit in opposite directions. Their magnetic effect will therefore cancel, which is a decided advantage of the push-pull arrangement. As far as alternating current is concerned, the polarity or phase of the two fictitious generators is such as to make the two voltages add to each other; if we are interested only in the alternating current that will flow in the load R_L, the circuit shown in Fig. 13–13 becomes once more simplified to Fig. 13–14. In this circuit a total alternating voltage equal to μ times the total alternating voltage applied to the grids by the center-tapped transformer—sometimes called the "grid-to-grid" voltage—is seen to be acting over a total resistance equal to twice the plate resistance of each tube on a transformer, the secondary of which is connected to the load.

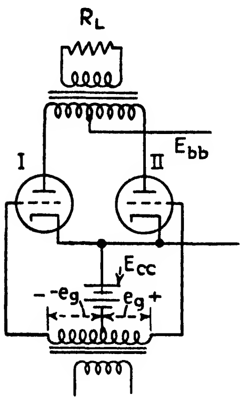

FIG. 13–12.—Push-pull operation of two tubes.

If the operation of the push-pull arrangement is limited to the reasonably linear part of the characteristics, as discussed in the preceding paragraph, the power obtainable will be exactly the same as if the two tubes were used in a parallel connection. The only advantage with which the push-pull arrangement can be credited in this case is the fact that with a

parallel arrangement the transformer core is subjected to a magnetization by the dc component of the plate currents, which, as we have seen, cancel

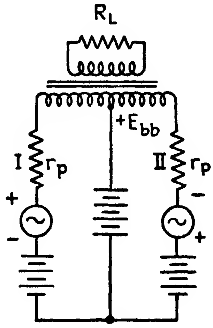

FIG. 13-13.—If the push-pull tubes shown in Fig. 13-12 operate in the linear part of their characteristics, their operation can be analyzed with the aid of the substitute circuit shown here.

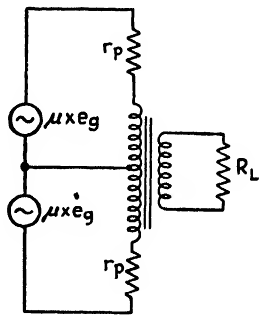

FIG. 13-14.—Short-circuiting all direct voltages in Fig. 13-13 results in the equivalent circuit for push-pull operation shown here.

out in the case of the push-pull arrangement. The real advantage of the push-pull arrangement lies, however, in the fact that the combination of the two tubes will minimize distortion so that, for a given amount of tolerable distortion, a larger grid signal can be applied to the grid of each tube than could be applied if only one tube or two tubes in parallel were used.

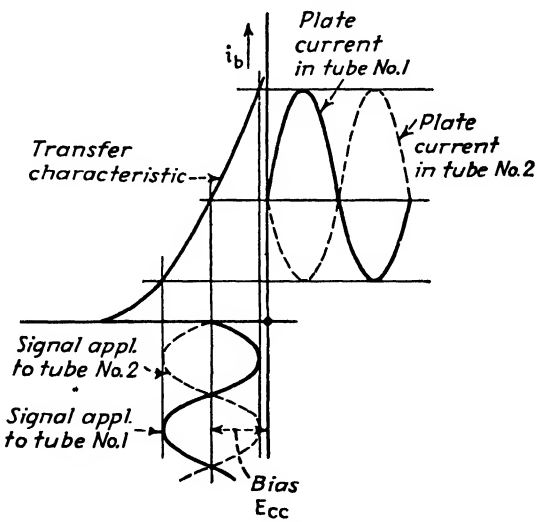

FIG. 13-15.—By combining the action of two tubes, the push-pull arrangement makes the positive and negative half waves of the output equal, thus eliminating the second harmonic.

To make this clear, consider Fig. 13-15. Let the curve shown in this figure be the transfer characteristic of each tube. If an alternating voltage

is applied in series with the dc bias, it is seen that the variations of the plate current from the quiescent value are much larger in the positive than in the negative direction. The output of a single tube or of two tubes in parallel therefore shows a considerable amount of distortion. In the push-pull arrangement, however, during the half cycle when the current is increasing in the one tube, it is decreasing in the other, and the total effect on the transformer is produced by the difference between the two. The conditions for the next half cycle are therefore exactly the same, except that the two tubes have reversed their roles. Successive half cycles of current in the load therefore look exactly alike. By the application of Fourier theory it can be seen that in such a case the alternating current wave will not contain even harmonics. It is therefore seen that in a push-pull arrangement, there will be no even harmonics in the output caused by the curvature of the individual tube characteristics, as is the case with a single tube. (Any even harmonics *in the signal applied to the grid* will, of course, appear in the output.) Since with a single tube the second harmonic created by distortion is by far the worst offender it is evident that the cancellation of the second harmonic by the push-pull arrangement permits us to apply much higher signal voltages to the individual tubes than we could in the case of a single tube. For a given distortion, the power output obtainable with the push-pull arrangement is usually in the order of three to four times the amount of power obtainable from a single tube.

13–12. Graphical Analysis of Push-pull Circuits.—The graphical analysis of the operation of two tubes connected in a push-pull circuit has received much attention and is a very interesting subject. By means of such an analysis, quite accurate prediction of the performance of such a circuit can be made. Suppose that in Fig. 13–12 the center-tapped transformer in the plate circuit, from now on referred to as the "output transformer," can be considered as ideal; with such an assumption the voltages appearing across each section of the center-tapped winding will always be of equal magnitude. Let us assume that the supply voltage for the two tubes is equal to 300 volts and that the resistance of the primary winding of the transformer is negligible; let the grid bias be -50 volts. With no signal applied to the grid, each tube will then carry a current given by the plate characteristic for -50 volts grid voltage and a plate voltage of 300 volts. These two currents flow from the center tap toward the two plates, and, as already explained, result in zero magnetization of the transformer core. Let us assume that this quiescent current in each tube is equal to 50 ma and that the number of turns to each side of the center tap of the output transformer is 1,000. Under this condition, each section produces 50 ampere-turns but of opposite effect. Now assume that the plate current of tube I increases to 65 ma and the plate current of tube II

decreases to 40 ma. The magnetic effect of winding I is therefore 65 ampere-turns, opposed by 40 ampere-turns in the other section. The net result is therefore a magnetic effect equal to the one that would be obtained if the primary winding consisted of 1,000 turns and the *difference* between the two plate currents were flowing through this winding. The secondary winding, to which the load R_L is connected, will therefore react in the same manner as if the primary winding consisted of a single winding with 1,000 turns and as if a current equal to the difference between the two plate currents were flowing through it.

Under these conditions, it is seen that the actual performance of the pair of tubes is the same as if we were to omit tube II and throw its effect into tube I by assuming that it will have a current equal to the difference between its actual plate current and the current of the omitted tube II. Tube I becomes a fictitious or "composite" tube, as it is called in the literature. At the quiescent operating point the two tubes in the actual circuit operate with the same plate voltage and grid voltage, and their plate currents will therefore be alike. The fictitious tube, which is supposed to have a current equal to the difference between the two plate currents, will therefore have zero quiescent current.

The characteristics of the fictitious tube I, or the composite tube, are constructed in the manner shown in Fig. 13–16. The characteristics of a single tube are traced on tracing paper and then rotated 180 deg; they are then lined up with the original characteristics in such a way that the supply voltages coincide, as shown in Fig. 13–16. As explained above, the assumption of an ideal transformer means that by whatever amount the instantaneous plate voltage of one tube is *less* than the supply voltage, the instantaneous plate voltage on the second tube will be *higher* than the supply voltage. The placing of the plate characteristics in the manner shown in Fig. 13–16 assures this condition automatically. If we draw, for instance, a vertical line at the 280-volt point of the original characteristic, this will be a line corresponding to the 320-volt point in the rotated characteristic. With the assumed bias voltage of -50 volts and a plate voltage OQ, *i.e.*, the plate-supply voltage acting on both tubes, the current in the two tubes will be alike and will be given by the lengths QR and QS. The difference between the two will be zero, and our fictitious or composite tube will therefore have zero current at these values.

Now let us assume that the plate voltage applied to tube I is increased from the value OQ to OQ_1. The plate current in this tube will therefore increase to a value given by the length of Q_1R_1. But since, owing to the presence of the ideal transformer in the actual circuit, the plate voltage on the second tube will be reduced by the same amount that it was increased on the first tube, the plate current through tube II will be reduced to a value Q_1S_1. The fictitious or composite tube will therefore now have a plate current equal to the difference of the two lengths Q_1R_1 and Q_1S_1.

By subtracting Q_1S_1 in Fig. 13–16 from Q_1R_1 (most conveniently carried out by means of a pair of dividers), we obtain point P_1. This will be a point on the plate characteristic of our fictitious tube for a grid voltage of -50. If the procedure just outlined is repeated for several points of the two plate characteristics, we obtain the plate characteristics for -50 volts grid voltage of the composite tube. This line is seen to be very nearly straight. Now let us construct the plate characteristics of the composite

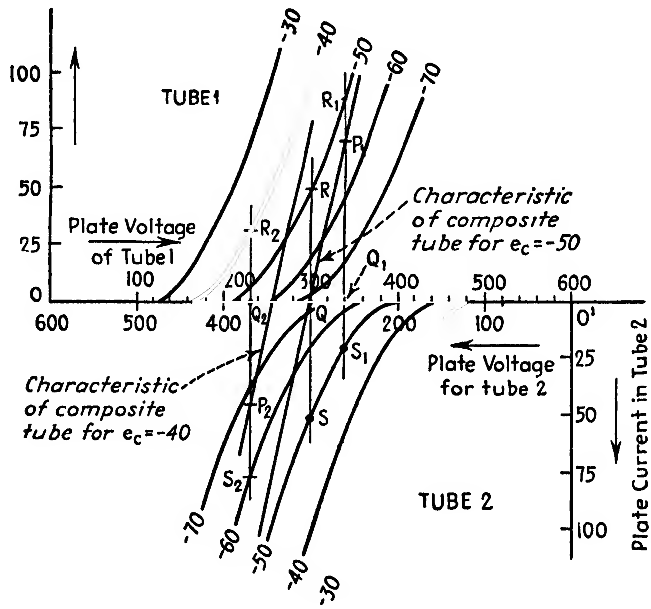

FIG. 13–16.—Method of constructing the plate characteristics of a "composite" tube, taking the place of the two push-pull tubes.

tube for -40 volts on the grid. In the actual circuit the presence of the center-tapped transformer in the grid circuit assures us that whenever the grid voltage on tube I is -40 volts, *i.e.*, 10 volts more positive than the grid-bias voltage, the grid voltage of tube II will be -60 volts, *i.e.*, 10 volts more negative than the grid-bias voltage. The plate characteristics of the composite tube for -40 volts on the grid will therefore be constructed with the aid of the -40-volt characteristic of tube I and the -60-volt characteristic of tube II in exactly the same manner as the -50-volt characteristic was constructed from the plate characteristics of the two tubes for a grid voltage of -50. The construction for one point P_2 of the plate characteristic of the composite tube for this grid voltage is shown in Fig. 13–16. For a plate voltage given by OQ_2 and a grid voltage of -40 on tube I, the plate current through this tube will be Q_2R_2; the plate voltage on tube II will be $O'Q_2$ and its grid voltage will be -60, as explained above. The plate current in this tube will therefore be given by the length

Q_2S_2. Subtracting Q_2S_2 from Q_2R_2 gives us point P_2, representing the current through the composite tube, which for these particular values of plate and grid voltage turns out negative because the current in tube II exceeds the current in tube I. Repeating this procedure for a number of grid voltages results in a number of characteristics that are considerably more linear than the individual tube characteristics.

From here on the manipulation of these plate characteristics of the composite tube is exactly the same as if we were using a single tube of any kind. We may draw a load line into this family of characteristics, using as the load resistance the value with which R_L would appear on the primary side of a transformer having a number of primary turns equal to the number of turns to each side of the center tap. The determination of the maximum possible power output and the value of the resistance that will provide it follows the principles outlined in connection with a single tube.

Although the discussion of this subject presented herewith is admittedly short and sketchy, it is hoped that it will enable the reader to get a better understanding of the more detailed literature covering this field.

13–13. Class B Push-pull Operation.—The push-pull arrangement can be carried to its ultimate and logical limit by making the two tubes involved in the circuit both operate under Class B conditions. This means that the grid bias is adjusted to such a value as to cause plate-current cutoff in both tubes. As long as the center-tapped transformer in the grid circuit of Fig. 13–12 does not furnish any signal, the plate current in both tubes will be zero, or at least nearly so; with the application of a signal, the two tubes will carry current alternately for one half cycle. Owing to the action of the output transformer, the current in the secondary winding and in the load will consist of both half cycles. If the characteristics of the tubes were linear clear down to the cutoff point, such an arrangement would make the wave shape of the output current an exact replica of the wave of the input voltage. It is clear that in such a case the amplitude of the signal applied to the grid can be equal to the cutoff voltage without ever causing the grid to become positive with respect to the cathode.

PROBLEMS

13–1. Figure 9–14 shows the plate characteristics of a type 2A3 tube. The tube is to operate with a grid bias of 30 volts, and the quiescent value of the plate voltage is to be 200 volts.

a. Determine the plate resistance of the tube at the quiescent point.
b. A sinusoidal alternating voltage with an amplitude of 20 volts is applied to the grid. What ac power will be obtained for load values of 2,000, 1,000, and 500 ohms?
c. If the grid is never to become positive and if the increase of current from the quiescent value is not to be more than 1.25 times the decrease from the quiescent current, what load resistance must be used, and what will the power output be?

13-2. A type 6F6 tube is operating in pentode connection with 260 volts on the plate and a grid bias of −15 volts. Under this condition the plate resistance is 80,000 ohms, and the amplification factor is 200. With an alternating voltage of 1 volt peak applied to the grid, find the value of load resistance that will give maximum ac power. HINT: Try 40,000, 60,000, 80,000, and 100,000 ohms.

If the peak of the alternating voltage applied to the grid increases to 15 volts but the actual plate voltage is never to exceed 400 volts, what must the load resistance be, and what will the approximate power output be? (For characteristics, see Fig. 13-3.)

Make the load resistance 2,000 ohms smaller than just found, and determine the output again.

13-3. A type 2A3 tube is to operate a dc relay in its plate circuit. The available supply voltage is 300 volts. Assuming the maximum permissible plate dissipation of the 2A3 as 9 watts, what resistance should the relay coil have in order to obtain the maximum wattage in it, and how much will the power be? Assume $e_c = 0$ when the relay operates.

13-4. In the push-pull circuit shown in Fig. 13-17, the two 2A3 tubes are operating from a dc supply of 250 volts and with a grid bias of −45 volts. If the resistance of

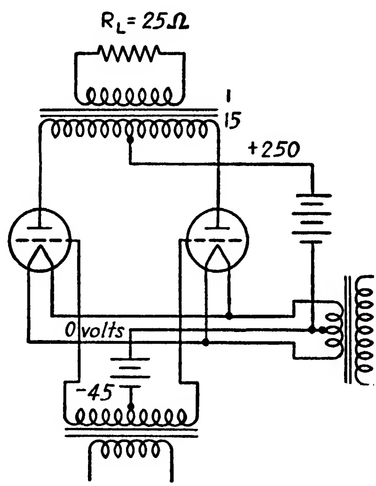

FIG. 13-17.

the primary winding of the output transformer is negligible, the quiescent plate voltage will also be 250 volts, and the current drawn by the tubes will be 60 ma (for each tube). The amplification factor of the tubes is 4.2, and the plate resistance is 800 ohms for these operating conditions. The output transformer has an over-all ratio of 15:1; the load has a resistance of 25 ohms.

Assuming linear operation, what power will appear in the load when a sinusoidal voltage with a peak of 35 volts is applied to each grid, the two voltages being 180 deg out of phase? (The peak voltage from grid to grid is then 70 volts.)

SUGGESTED ADDITIONAL READING

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2. Sarbacher, R. I.: Graphical Determination of Power Amplifier Performance, *Electronics*, December, 1942, p. 52.
3. Houghton, E. W.: Class A Push-pull Calculations, *Electronics*, June, 1937, p. 18.
4. Koehler, Glen: Class B and AB Audio Amplifiers, *Electronics*, February, 1936, p. 14.

CHAPTER XIV

MULTISTAGE AMPLIFIERS; COUPLING METHODS; SELF-BIAS

14-1. Need for Multistage Amplifiers.—In Chaps. X and XIII the performance of a single tube or of a push-pull stage was analyzed. Often the voltage available for control purposes is of such small value that the plate-current change resulting from the application of this voltage to the grid of a tube is too small to perform the intended task. When it was desired to obtain only an indication on a meter, we saw that by employing a bucking or compensating arrangement it was possible to use a very sensitive instrument, which would indicate even small voltage changes on the grid. If it is desired, however, to operate a relay in the plate circuit of a vacuum tube, which requires a definite current change between operating and drop-out value, then we must have sufficient voltage available for application to the grid to cause the desired current change. If the voltage available for control is not sufficient, then it is necessary to employ one or more additional stages of amplification before applying the voltage to the final tube, the plate circuit of which contains the load to be operated. The present chapter will deal with the various methods by means of which it is possible to couple the voltage output of one stage of amplification to the next.

14-2. Two Classes of Interstage Coupling.—The problem of interstage coupling divides itself into two parts: coupling methods that will transmit only the alternating component of a given signal voltage and those that are capable of transmitting the ac as well as the dc component of a given signal. From a practical point of view, it is much easier to design coupling methods that will transmit only ac components. Dc amplifiers have always been, and still are, considerably more difficult to construct and to operate. As far as the theory of dc coupling is concerned, however, it is actually considerably simpler than the theory of ac coupling. For this reason we shall consider first the various methods of obtaining dc coupling between amplifier stages.

14-3. Relay Operation Requiring More Than One Stage.—The various methods of dc interstage coupling can best be explained with the aid of an example. Assume that it is desired to operate a relay with a coil resistance of 4,000 ohms. It has been ascertained that it is necessary to change the current from 0 to 10 ma in the relay coil. Let us assume that a dc supply with a voltage of 180 volts is available for the operation of the intended circuit. Deciding that a type 6J5 tube is capable of furnishing

the desired amount of current, we draw a load line into the characteristics of this tube. The load line drawn for 4,000 ohms, as shown in Fig. 14–1, indicates that the grid voltage must be changed from −2 to −11 to obtain operation of the relay. Now let us assume that the source of control voltage furnishes only a variation of ½ volt for control purposes. This voltage may, for instance, be the voltage existing across a resistor in series with a phototube, and the light variation on the phototube may produce a current change causing only a voltage variation of ½ volt across this resistor, or the voltage may be the result of rectification of a small ac signal.

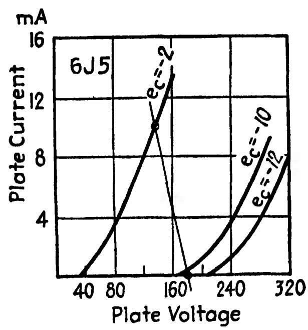


FIG. 14–1.—To make the 4,000-ohm relay operate in the plate circuit of a 6J5, a grid-voltage change from −2 to −11 is required.

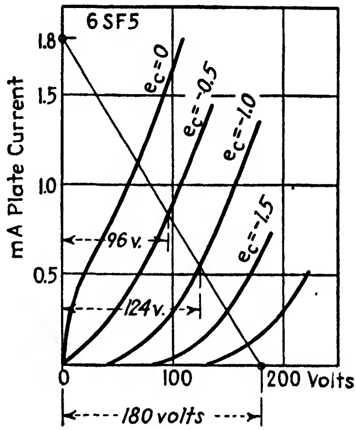


FIG. 14–2.—A 6SF5 tube would give a voltage gain of about 56 when operating with a load of 100,000 ohms in the plate circuit.

In any case, ½ volt is not sufficient to obtain the desired relay operation in the plate circuit of the 6J5 tube, and it will therefore be necessary to amplify the original voltage approximately twenty times. We have seen in the preceding chapters that the voltage gain of a stage can never be in excess of the value of the amplification factor of the tube; it is therefore necessary to look for a tube with an amplification factor in excess of 20 if we intend to obtain a voltage gain of 20. There are several tubes available with amplification factors in excess of 20, one of them being the 6SF5, which has an amplification factor of 100 and which is therefore more than sufficient to provide the required amplification. Suppose that we decide to place a load resistance of 100,000 ohms in series with this tube. Drawing a load line representing 100,000 ohms into the plate characteristics of this tube, as shown in Fig. 14–2, with a plate-supply voltage of 180 volts, will disclose that with a voltage of −0.5 volt on the grid the voltage across the tube will be 96 volts. If we decrease the grid voltage to −1.0 volt, the intersection of the load line with the characteristic for −1.0 volt indicates that the voltage across the tube will rise to approximately 124 volts.

A change of ½ volt on the grid of this tube therefore causes a plate-voltage change of 28 volts. The voltage gain of the tube in this region is therefore approximately 56. Incidentally, we can arrive at this value also by using the formula for calculating the voltage gain when the amplification factor and the plate resistance of a tube are given. For a 6SF5 operating with 100 volts plate voltage and -1 volt on the grid, the value of the amplification factor is 100, and the plate resistance is 85,000 ohms. With a load resistance of 100,000 ohms, the voltage gain of the stage will be given by

$$v_g = 100 \times \frac{100{,}000}{100{,}000 + 85{,}000} = 100 \times \frac{100}{185} = 54$$

This is certainly in close enough agreement with the value that we obtained by means of the load line. In order to make point A in Fig. 14–3 "swing" 9 volts, it is necessary to swing the grid of this tube by an amount equal to $9/56 = 0.16$ volts. A variation of grid voltage on the 6SF5 from a value of -0.5 to -0.66 volt therefore causes a variation of potential of A from 96 to 105 volts. In other words, when the grid of the 6SF5 "wiggles" or "bobs" between the limits 0.5 and 0.66 below the cathode potential, A wiggles or bobs between the limits 96 and 105 volts.

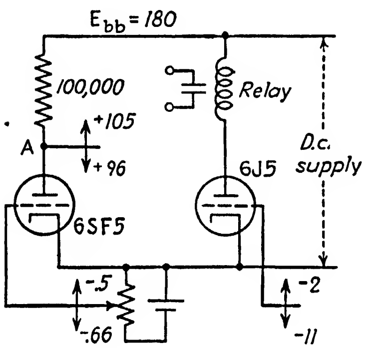


FIG. 14–3.—The potential variations of point A do not occur at a level needed for the operation of the grid of the 6J5.

14–4. Difference of Level of the Potential Variations.—The potential variation of point A is now seen to be 9 volts, which is exactly as much as we need for the operation of the 6J5. Our problem is not yet solved, however, because the variations of potential of A occur at a level differing from the level that is required for the operation of the 6J5. In other words, A bobs up and down from 96 to 105 volts above the ground floor—if we so call the level of the cathode. For the operation of the 6J5 its grid must bob around from -2 to -11 below the ground floor. It is therefore necessary to devise some means for transferring the voltage variation of A to the grid of the next tube. There are several methods available to achieve this result, none of them very satisfactory. The first method is shown in Fig. 14–4. Since voltages are equivalent to heights and since the problem is simply to transfer a voltage variation occurring at one level to a different one, which is a problem of height, it is only necessary to introduce between A and the grid of the following tube a battery with the proper voltage. When A is in its lowest position, the grid of the 6J5 must be at -11 volts. This means that the connecting battery must have a value of

96 + 11 = 107 volts. This is indicated in Fig. 14–4, and the reader may easily convince himself that a motion of A from a level of 96 to a level of 105 volts will cause a motion of the grid of the 6J5 from a level of -11 to -2 volts. The voltage variations of the various points in the circuit diagram are indicated in Fig. 14–4. A battery of 107 volts may be hard to obtain; a value of 112.5, on the other hand, could easily be obtained by the combination of two 45-volt and one 22.5-volt batteries. This means that A will have to swing between 101.5 and 110.5 volts. Evidently, this can easily be obtained by the adjustment of the bias of the 6SF5. The

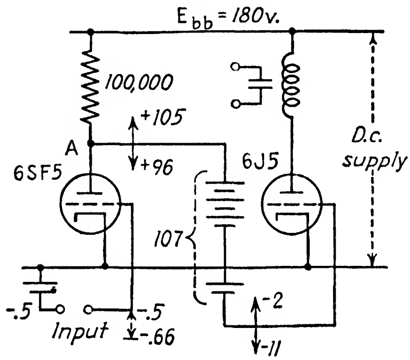


FIG. 14–4.—By connecting a coupling battery of 107 volts between point A and the grid of the second tube, the desired relation can be obtained.

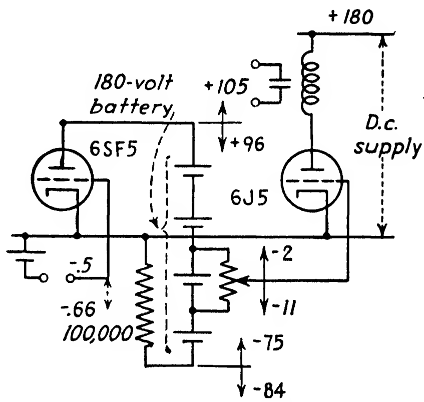


FIG. 14–5.—A second method of transferring the output of the first tube to the grid of the second tube utilizes a separate plate-supply source for the first tube.

voltage required for coupling A to the grid of the 6J5 cannot be obtained from the main source of voltage because the coupling voltage must be free to bob up and down with respect to the main source of voltage. On the other hand, this battery will not have to furnish any current since it is connected directly to the grid of the 6J5. Its deterioration will therefore be extremely slow; as a matter of fact, not any more rapid than the battery would deteriorate on the shelf. Nevertheless, the need for a battery in this circuit prevents it from being a practical solution to our problem, and the use of this circuit is therefore usually restricted to laboratory instruments.

14–5. Separate Dc Supply for Each Stage.—A second method of solving our problem is to give the first tube its own power supply and place the load resistance in the cathode circuit rather than in series with the plate. The connection diagram for this case is shown in Fig. 14–5. The connection of the 6SF5 to its load resistor looks rather confusing, but the reader may convince himself that the tube and its load resistor are placed across exactly the same voltage as they were in Figs. 14–3 and 14–4. With the same grid voltage applied to the tube as in Figs. 14–3 and 14–4, the volt-

age across the tube and the resistor will therefore also be the same as it was in the preceding circuit. Point A will swing in this figure between exactly the same values as it did in Fig. 14–4, and every point along the battery, including its lower end, will therefore exhibit the same amount of voltage swing as A. It is evident that the battery may now be tapped at a point that will swing between the values -2 and -11 and this tap may then be connected to the grid of the 6J5. At first glance, this circuit seems to be not so desirable as the one shown in Fig. 14–4 because the auxiliary battery must now furnish the plate current for the 6SF5. In order to show the similarity of operating conditions of Figs. 14–4 and 14–5, as far as the 6SF5 tube was concerned, it was assumed that the auxiliary battery in Fig. 14–5 had a value equal to the voltage of the main power supply—*i.e.*, 180 volts—in Fig. 14–4. This is by no means a necessity, however. The battery shown in Fig. 14–5 may be reduced to 90 volts, or even to 45 volts, and still be capable of furnishing the required voltage variation of 9 volts to the grid of the 6J5. The current drain on this battery will be less than ¼ ma, and it is evident that under such a condition quite a long life can be expected from it. Another advantage of the circuit shown in Fig. 14–5 over that shown in Fig. 14–4 will be found in the following considerations. In the circuit shown in Fig. 14–4, any variations of voltage in the main power supply—whether derived from batteries or from a rectifier system—will be transmitted partly through the coupling battery to the grid of the 6J5. (To recognize the truth of this statement, simply replace the tube by a black box.) In the circuit shown in Fig. 14–5, on the other hand, this undesirable feature is obviously avoided.

14–6. Coupling by Voltage Divider and High Negative Bias.—If there are more than two stages to be coupled, it is clear that the two methods of coupling shown in Figs. 14–4 and 14–5 will require a separate battery for every stage. A third method of coupling, shown in Fig. 14–6, is particularly adaptable when more than two stages are involved, although the diagram shows only the coupling between two stages. To use this method, it is necessary to have available an additional bias supply of a relatively large value, as shown in the circuit diagram. A voltage divider is placed between the anode of the first tube and the negative end of the bias supply. Any voltage variation of the anode will obviously be transmitted to the grid of the next tube; it is clear, however, that the voltage variation on the grid will not be so large as on the plate of the preceding tube. If, for instance, the values of the two resistors R_1 and R_2 in Fig. 14–6 are equal, say, 2 megohms each, then it is quite clear that the voltage variation on the grid of the second tube is only one-half the voltage variation on the plate of the first tube. In other words, if we wish to maintain the 9-volt swing between -11 and -2 volts of the previous example, the anode of the 6SF5 must now swing from 96 to 114. Since, with the grid of the 6J5 at -11 volts, the anode of the 6SF5 must be $+96$, the voltage across the

resistor R_1 must be 107 volts; with R_1 equal to R_2, the voltage across R_2 will also be 107 volts. The negative end of the bias supply must therefore be at a potential of $-11 - 107 = -118$ volts. The foregoing method of calculating the various values is approximate only since it does not take into account the effect that the connection of the voltage divider R_1R_2 will have on the anode potential of the 6SF5. By using our familiar substitution for the tube, however, the exact calculation of the performance of this circuit is not difficult. Although the circuit shown in Fig. 14–6 now requires a signal voltage twice as high as the preceding two circuits,

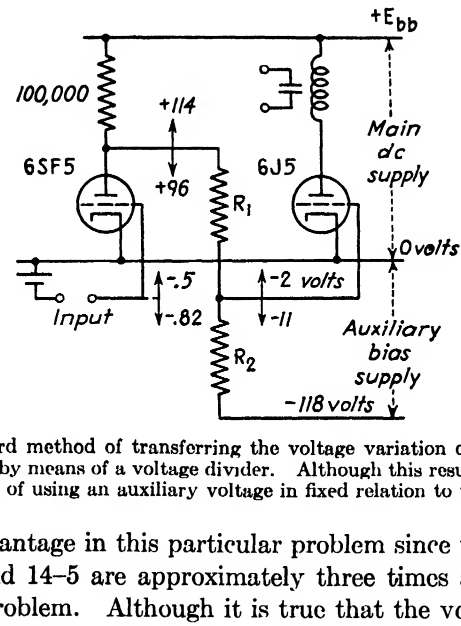


FIG. 14–6.—A third method of transferring the voltage variation of the first tube to the grid of the second is by means of a voltage divider. Although this results in a loss of voltage, it has the advantage of using an auxiliary voltage in fixed relation to the main supply.

this is no disadvantage in this particular problem since the circuits shown in Figs. 14–4 and 14–5 are approximately three times as sensitive as required for the problem. Although it is true that the voltage of the additional bias supply is actually larger than the voltage of the auxiliary batteries shown in the preceding two circuits, it has the great advantage that it has a common terminal with the main supply; thus, both voltages may be obtained from the same rectifier system. Furthermore, if additional stages are required, the same bias supply will, of course, be used for all stages.

14–7. Loftin-White System of Coupling.[5]—In the three methods discussed so far, the cathodes of all the tubes were held at the same potential. This is a necessity if the cathodes are of the filamentary type and it is desired to operate all the filaments from the same source. If the tubes are of the indirectly heated type, however, this condition will not necessarily have to be fulfilled. In the fourth and last method, instead of attempting to bring the variations of anode potential down to the appropriate level for the next stage, the grid of the second tube is simply connected to the anode of the first tube, but the cathode of the second tube is raised to a

level such that the grid will be negative with respect to it. This system of direct coupling is known as the "Loftin-White" system. Figure 14–7*a* shows a diagram of it. If the tubes used are the same as in the preceding examples and if the operating voltages of them are to remain the same, then the potential distribution will be as shown in Fig. 14–7*a*. The cathode of the second tube must be placed at a potential of +107 volts with respect to the cathode of the first tube so that the variation of anode voltage from 96 to 105 results in an actual grid-voltage change from −11 to −2 on the second tube. In order to have 180 volts available for the operation of the second tube and its associated load, the total supply must then

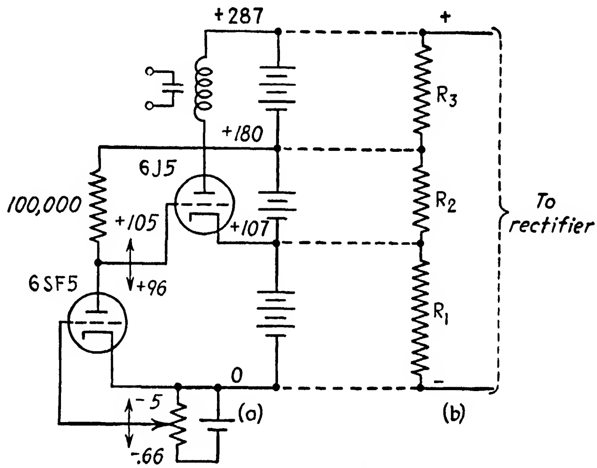


FIG. 14–7.—In the Loftin-White system of coupling, the plate of the first tube is directly coupled to the grid of the second tube, but the cathodes of the two tubes are displaced with respect to each other.

have a voltage of 287 volts. Batteries are shown in Fig. 14–7*a* as the source of the required direct voltages. If the direct voltage is supplied by a rectifier, it is quite natural to think of obtaining the required taps by means of a voltage divider, as shown in Fig. 14–7*b*. This will not be a very satisfactory solution, however, except if the current bled through the voltage divider is large compared to the operating current of the tubes, especially the second one. To see the truth of this statement, let us assume that the batteries of Fig. 14–7*a* were eliminated and the connections to the voltage divider shown in Fig. 14–7*b* had been made as indicated by the dotted lines between the two parts of the figure. The plate current of the second tube will now have to flow through the resistance R_1 of the voltage divider in order to reach the negative end of the rectifier system. The voltage across this resistance will change with a change in load current, and the cathode potential of the second tube will therefore not remain at the fixed value that formed the basis of our analysis. The reader may convince himself easily that the variation of potential of the cathode is in

such a direction as to oppose the action of a grid-voltage change. Thus, if the grid of the second tube, through the action of the first tube, is swinging up, the current through the second tube will increase. As stated above, this current increase will have to flow through the resistance R_1 of the voltage divider, and the voltage across this resistance will consequently increase. This means that the cathode of the second tube will also swing up, so that the actual grid-voltage change on this tube will be less than it would be if the cathode had remained at a fixed potential.

The original Loftin-White amplifier, also called "direct-coupled" amplifier, was designed for the purpose of amplifying alternating voltages only. It used a tapped voltage divider as a source of the required voltages, but in order to avoid the loss of amplification resulting from the conditions outlined in the preceding paragraph, large capacitors were placed across the various sections of the voltage divider. These capacitors, acting like storage batteries, keep the circuit performing as if it is operating from batteries, provided, of course, that the capacities are large enough to maintain their voltage during one cycle of the lowest frequency for which the amplifier is intended. For pure dc operation on the other hand, the conventional Loftin-White amplifier is not satisfactory.

14–8. Factors Limiting the Number of Stages.[1–4, 6, 9]—From the foregoing discussions, the reader may have gained the impression that with a sufficient number of stages operation of a relay could be obtained from as small a voltage as desired. When discussing bucking or compensating arrangements, we saw that the limit of sensitivity of such an arrangement is given by the unavoidable drift of the zero point on the meter, owing to many factors. It will be appreciated that the same factors that set a limit to the sensitivity of those arrangements also set a limit for the amount of amplification obtainable by cascading several stages. The slightest drift of the operating point in the first stage will naturally be amplified by all the following stages. It is for this reason that dc amplifiers are usually laboratory instruments rather than practical devices for the shop. Even in the laboratory they usually turn out to be a headache. The problem of a satisfactory dc amplifier is by no means solved, and it is for this reason that those familiar with the difficulties eagerly page through all the magazines in the hope of seeing a satisfactory solution published. One practical solution recently incorporated in a very successful industrial instrument consists of converting the small voltage at first into an alternating voltage, which, as will be seen presently, can be amplified much more conveniently.

14–9. Amplification of Alternating Voltages.[7]—The four circuits discussed so far will amplify direct voltages; to be more precise, if the grid voltage applied to the first tube changes a certain amount and remains at the new value, the plate current of the final tube will change and remain at the changed value also. Sometimes a dc amplifier is described as an

ac amplifier with a frequency response going down to zero. It is, of course, clear that the four circuits shown in Figs. 14–4 to 14–7 will also be capable of amplifying a fast-varying voltage, in other words, an alternating voltage. Thus, if in any of the four circuits the grid of the first tube is made to swing rapidly between the limits given in these figures, then an alternating current will flow in the plate circuit of the second tube. The rapid variation of the grid voltage on the first tube could be considered as the result of applying a steady bias with a value halfway between the two extremes of the swing, and an alternating voltage in series with this bias with an amplitude of half the total swing.

14–10. Coupling Methods for Alternating Voltages Only.—If we are interested only in the amplification of the alternating component of the grid voltage, then there are available to us additional methods of coupling the output of the first stage to the grid of the second. These coupling methods employ electrostatic or electromagnetic means to achieve the transfer of the ac component. If it is desired to transfer the variation of a direct voltage from one point in a circuit to another, a conducting connection between the points of the network is evidently necessary. This is accomplished in one way or another by the circuits shown in Figs. 14–4 to 14–7. When it is desired, however, to transfer an alternating voltage from one point of a network to another, the two parts of the network do not have to be connected conductively to each other. The reader knows that alternating current will flow through a capacitor that is an insulator as far as steady voltages are concerned. Furthermore, we know that application of voltage to the primary of a transformer will cause the appearance of a voltage across the secondary; here, too, there is no conducting path between the two windings. These two methods illustrate the use of an electrostatic or an electromagnetic field for coupling purposes.

14–11. Analysis of Capacitive Coupling on the Basis of Transient Response.—A good grasp of how capacitive coupling operates can be obtained by using the circuit shown in Fig. 14–4 as a steppingstone. In this circuit the variations of the potential of the anode were transferred to the grid of the next tube by means of a coupling battery. If the grid of the second tube takes no current at all and if all the insulation resistances involved were infinite, then this battery evidently would not have to furnish any current whatsoever. Under this condition, we could replace it by a capacitor, which we would then charge once and for all to a voltage of 107 volts before we wished to take the circuit into operation. The capacitor must, of course, be perfect, too, so that it will retain the charge we have placed on it. Such an arrangement would obviously perform in exactly the same manner as the circuit shown in Fig. 14–4, *i.e.*, direct voltage variations on the anode of the first tube will be transferred to the second tube. But owing to the unavoidable leakage and grid current, however small these two may be, the scheme is, of course, not practicable.

The fundamental circuit employing capacitive coupling as used in practice is shown in Fig. 14–8. Suppose that, owing to the application of a signal voltage to the grid of the first tube, its anode, *i.e.*, point A, swings between the values of 96 and 105 volts. With no signal applied to the grid, the anode will then be at 100.5 volts, and the application of the ac signal to the grid simply superimposes an alternating voltage with an amplitude of 4.5 volts on this steady value of 100.5 volts. Assume that the load line was drawn for the second stage and that it was found that for the particular operating condition a grid bias of -8 volts is desirable; the grid of the second tube is now connected over a resistance R_g of high value to a suitable source furnishing the value of grid bias just decided on. A capacitor is placed from the anode of the first tube to the grid of the second tube. With no signal applied to the first tube, a steady state will soon be reached; this state is characterized by the fact that no current now flows through the resistance R_g because as long as current would flow through this resistor and the capacitor the voltage across the capacitor would still change according to the fundamental law for a capacitance. When the steady state has been reached, the voltage across the capacitor will therefore be equal to $100.5 + 8 = 108.5$ volts. The charged capacitor now evidently takes the place of the coupling battery shown in the circuit of Fig. 14–4. Now let the potential of point A be suddenly changed from 100.5 to 105 volts, because of a sudden change of grid voltage on this tube. Since the capacitor is in series with a high resistance, it cannot change its voltage suddenly because a large current is necessary to do so and the high resistance in series prevents this. The grid of the second tube will, therefore, also be suddenly "yanked" upward by 4.5 volts. The similarity of action taking place in this circuit and in the one shown in Fig. 14–4 is self-evident. But, although in Fig. 14–4 the grid remains at this new value as long as the anode of the preceding tube remains at the new value, this will not be the case in Fig. 14–8. With point A at 105 volts and the capacitor charged to 108.5 volts, there will be a voltage of 4.5 volts across the resistance R_g. A current will flow through the series combination of C and R_g. If we wait long enough, the capacitor obviously will charge up to a new voltage of 113 volts, and the grid of the second tube will be back at its -8 volt level. How long this will take will depend entirely on time constant CR_g. If the potential "wiggles" of A are of a freq

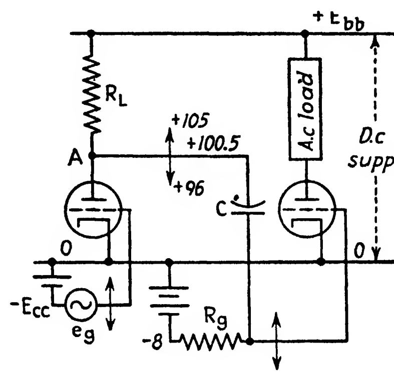


FIG. 14–8.—With the capacitor considered as a storage battery, the capacitive coupling shown here is essentially identical with the dc coupling shown in Fig. 14–4.

such that one cycle is small compared to the time constant CR_g, then the capacitor will not have time to change its charge appreciably between cycles. This means that the voltage variations of A will be transferred more or less completely to the grid of the next tube.

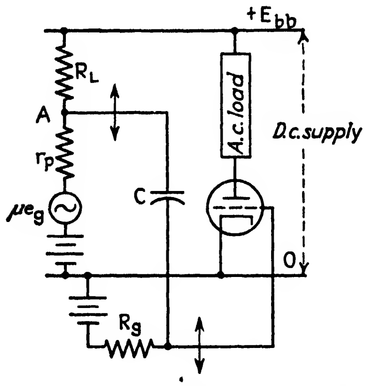


FIG. 14-9.—A battery and a resistance substituted for the tube in Fig. 14-8 give the circuit shown here.

14-12. Analysis of Capacitive Coupling on the Basis of Ac Theory.—It is felt that the description of capacitive coupling as given in Sec. 14-11 gives a clear picture of the fundamental processes involved and will be of great help in visualizing what happens if transient voltages are applied to the grid of the first tube. This method of presentation is not the common one, however, and it does not lend itself too easily to a mathematical treatment of the performance of such a circuit if the signal voltage applied to the grid of the first tube is not a transient but an alternating voltage of sinusoidal wave shape, or a mixture of such voltages. In that case, the analysis of the performance of the circuit can be made on the basis of ordinary ac theory. The use of the equivalent-plate-circuit theorem will be of great help, as long as the tube is operating in the linear part of the characteristics. Replacing the first tube by a battery and a resistance equal to the plate resistance of the tube in series with it converts the circuit shown in Fig. 14-8 to the one in Fig. 14-9. The signal applied to the grid of the second tube—apart from the fixed grid bias, of course—is evidently the voltage across R_g caused by whatever current may be flowing through this resistance. According to the principle of superposition, this current is the result of all voltages acting together in the circuit. Owing to the presence of the capacitor C in series with R_g, no direct current can flow steadily through R_g after the initial charging process of the capacitor has been completed. Consequently, as far as the steady-state condition is concerned, we can disregard all sources of dc potentials and consider them short-circuited. This refers to the main direct supply voltage, the fictitious battery in the tube substitution, and the battery furnishing the bias. With these three voltages short-circuited, the circuit shown in Fig. 14-9 assumes the shape shown in Fig. 14-10. The calculation of the alternating voltage appearing across R_g is now a simple problem of ac circuit theory and will not

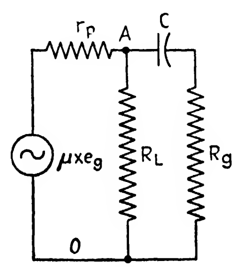


FIG. 14-10.—As far as the alternating component of the current is concerned, Fig. 14-9 simplifies further to the one shown here.

be discussed any further. A few important considerations in connection with Figs. 14–8 to 14–10 will be pointed out, however.

14–13. Choice of Values for Coupling Capacitor and Grid Resistor.—With the branch consisting of the capacitance C and R_g absent, the voltage gain of the stage, *i.e.*, the voltage variation of point A with a given signal applied to the grid, can be calculated by means of Eq. (10–2), as was shown in Chap. X. The branch consisting of C and R_g, according to Fig. 14–10, is in parallel with the load resistance R_L and will therefore reduce the voltage gain obtainable from this stage. In order to minimize this effect, it is desirable to make R_g as large as possible. But R_g is in series with the grid as shown in Fig. 14–8. Owing to the possibility of grid current, the manufacturer usually gives a limiting value for this resistor, varying from a value of 1 to 2 megohms to as low a value as 50,000 ohms for certain types of tubes. After having settled on a value for R_g, we must determine the size of the capacitor. It will naturally be our desire to make the alternating voltage across the capacitor as low as possible so that the alternating voltage appearing across R_g will be as nearly as possible equal to the voltage appearing across R_L. In order to achieve this result, the reactance of the capacitance C at the lowest frequency that it is desired to amplify should be small compared to the value of the resistance R_g. Since the alternating voltages appearing across C and R_g are 90 deg out of phase, adding up to the voltage across R_L, a ratio of x_C to R_g of 1:10, for instance, will make the voltage appearing across R_g still 99½ per cent of the voltage across R_L. Let it be required, for instance, that the amplifier will amplify a voltage of a frequency of 20 cycles without appreciable loss and that R_g has been made 500,000 ohms. The value of the capacitor C must then be such that the reactance at 20 cycles is around 50,000 ohms. Solving the equation $x_C = 1/2\pi fC$ for C, with the values $x_C = 50{,}000$ and $f = 20$, results in a value of $C = 0.16\ \mu f$. Since, according to the foregoing discussion, the value of the reactance of the capacitance C at the frequency that the amplifier is supposed to amplify is held low compared to the value of the resistance R_g, it is evident that for approximate calculations of the voltage gain the capacitor C can be considered as short-circuited. This simply places R_g parallel to R_L and makes the calculation of the voltage gain extremely easy. If a load line is being used, the steady operating point is found by using a load line representing R_L, while for the ac component the load will consist of the parallel combination of R_L and R_g. The reader is referred to Sec. 13–6 and Fig. 13–7*a*, where this case was discussed in detail.

14–14. Effect of Leakage of Coupling Capacitor on Bias.—As has been stated before, the upper limit for the value of R_g is given by the manufacturer's recommendation, and if it is required to design an amplifier for very low frequencies, it will be necessary to use a large value for the capacitor C. Theoretically, there are no objections to this, but all capacitors

exhibit, unfortunately, some leakage. Suppose that in Fig. 14–9 the value of R_g was made 2 megohms and that the value of the coupling capacitor C was chosen as 2 μf in order to give low enough frequency response. A 2-μf paper capacitor may exhibit a leakage resistance anywhere from 20 to 500 megohms, depending on the quality. Assume that the particular capacitor in question had a resistance of 100 megohms. Therefore, there now exists a path over which direct current will flow from point A to the lower end of the source of bias voltage. The 100 megohms of the capacitor and the 2 megohms of the grid resistor can be considered as a voltage divider placed across the two points just mentioned. Since the direct voltage existing across these is 108.5 volts, there will be a direct voltage of a little over 2 volts across R_g. The actual operating bias of the second tube is therefore not -8 volts, as intended, but only about -6 volts. This shift in operating bias is certainly undesirable, even if in a particular case it may not seriously interfere with the operation of a circuit. But there are tubes, especially high-μ tubes, that require operating bias values of approximately 2 volts; a shift of grid bias of even 1 volt would seriously interfere with the intended operation of the circuit. If it were possible to depend on the value of the leakage resistance of the capacitor, we could take it into account when designing the circuit. This value changes, however, with humidity and temperature to such an extent that the method just suggested does not represent a practical solution. O. H. Schmitt [10] has suggested the use of two capacitors for the purpose of coupling, connected as shown in Fig. 14–11. If the reactance of the two capacitors in series is low compared to the value of the resistors at the lowest frequency that it is desired to amplify, the circuit will perform as far as alternating voltages are concerned exactly in the same way as it would with a single capacitor with a value equal to that of the series combination of the two capacitors; as far as a direct voltage is concerned, it will be reduced to a fraction of what it would be with only one capacitor and resistor. In the above example we saw that a direct voltage of 2 volts remained, for instance. If we add another capacitor of 2 μf and a resistor of 2 megohms, these two volts will be reduced once more in the ratio of approximately 50:1, with the result that the shift in bias will be negligible. Of course, the effective coupling capacitance is now 1 μf, instead of 2, and it may be necessary to increase the capacitance values to twice their original value.

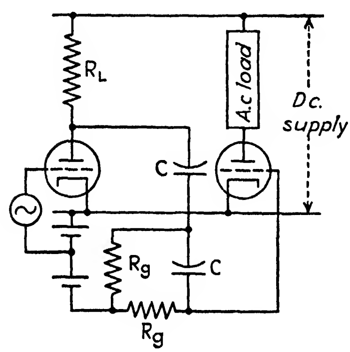
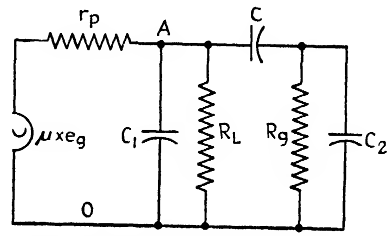


FIG. 14–11.—O. H. Schmitt's proposal to minimize the leakage of the coupling capacitors.

14–15. High-frequency Response of Capacitance Coupling.—In Sec. 14–14 the considerations applying to the low-frequency response of capacitive coupling were discussed. Industrial electronic engineers as a rule do not have to deal with higher frequencies, which are the domain of the radio engineer. The high-frequency response of an amplifier is, however, often of importance, even if the frequencies to be amplified seem to be only of low value. Thus, the amplification of a square wave, even if of low frequency, requires an amplifier with a frequency response at least twenty or thirty times as high as the fundamental frequency of the square wave. Square waves are quite often needed for the production of peaked waves, which are needed for the control of gaseous tubes. Generally speaking, whenever it is desired to amplify a wave with a steep front, it is necessary to have an amplifier with a good high-frequency response. For this reason, the upper frequency limit of any given arrangement is of interest to the industrial electronic engineer. From Fig. 14–10 it would seem that, for any frequency for which the reactance of the capacitor is negligibly small compared to the value of R_g, the voltage appearing across this resistor is independent of the frequency. This would indeed be the case if the circuit shown in Fig. 14–10 actually represented all the factors involved in Fig. 14–8. As already stated in preceding chapters, for high frequencies the capacitances of the tube elements with respect to each other begin to have an influence on the performance of the circuit. The circuit shown in Fig. 14–10 is therefore not the complete equivalent of the one shown in Fig. 14–8. Parallel to R_g must be placed a capacitance equal to that existing between the grid and the cathode of the second tube shown in Fig. 14–8, and from points A to O a capacitance equal to that existing between the anode and the other electrodes of the tube would have to be indicated. For high frequencies the equivalent of the first tube and its coupling circuit will be given by the circuit shown in Fig. 14–12. As a matter of additional complication, the grid capacitance of the second tube is not only that between the grid and the other electrodes of this tube when cold, but there appears an additional capacitance due to the interaction of the anode and the grid while the tube is operating, a phenomenon known as the "Miller effect." The design of amplifiers with a flat response over a wide band of frequencies has received much attention, and the reader interested in this subject is referred to books on radio



C_1 Anode capacity of first tube
C_2 Grid capacity of second tube

FIG. 14–12.—For operation at very high frequencies, the equivalent circuit shown in Fig. 14–10 does not represent the true state of affairs. The tube capacitances must be added as shown here.

communication and television, especially the latter, where amplifiers of these characteristics are absolutely essential.

14-16. Transformer Coupling.[8]—The second method of obtaining a coupling for the alternating component only of the plate current of a tube makes use of a transformer. In Sec. 10-8 the case of an inductive plate load was analyzed. Equation (10-4) permitted calculation of the voltage appearing across the inductance in the plate circuit when an alternating voltage was applied to the grid. The voltage across an inductance is due to the changing magnetic field produced by the changing current flowing through the winding. If we place an additional winding on the core, the inductance becomes a transformer, and the voltage appearing across this additional winding is given by the turns ratio of the two windings. Figure 14-13 shows the principle of transformer coupling. The voltage appearing across the primary of the transformer is given by Eq. (10-4), and the voltage across the secondary, which is seen to be applied to the grid of the next tube, is determined by the turns ratio of the transformer. It would seem that transformer coupling offers a convenient way of obtaining an over-all voltage gain larger than the amplification factor of the tube since, in addition to the voltage gain of the tube itself, the voltage may be stepped up by the transformer. In the early days of radio when tubes were the expensive part of the equipment, coupling transformers with step-up ratios as high as 10:1 were employed. They were anything but satisfactory, however, owing to the following considerations. As shown by Eqs. (10-4) and (10-5), the primary inductance of the transformer must be high if it is desired to obtain a reasonably good low-frequency response. This means a large number of turns on the primary winding, and if it is desired to obtain a step-up toward the secondary, the secondary winding must have an even larger number of turns. Quite apart from the mechanical difficulties of producing windings with many thousands of turns of extremely fine wire, capacitance effects become pronounced at even moderate frequencies. Such transformers have, therefore, an unsatisfactory high-frequency response; at present the ratio 3:1 is considered as a maximum consistent with good design. Even with a conservative ratio such as this, however, the upper frequency limit of transformer coupling does not anywhere near approach the values obtainable with resistance-capacitance coupling. In addition, transformers are considerably more bulky and expensive than the components required for a resistance-capacitance coupling network.

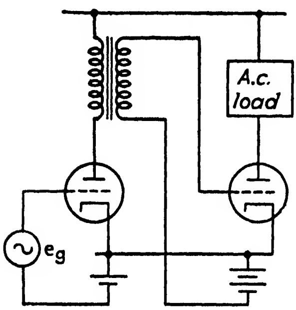


FIG. 14-13.—The principle of transformer coupling.

14–17. Advantages of Transformer Coupling.—A few features on the credit side of transformer coupling must be mentioned, however. With capacitive coupling, the load in the plate circuit of the first tube is a resistance. Since the dc component of the plate current produces a voltage across this resistor, it is necessary to have a supply voltage in excess of the operating voltage of the tube. This is not the case with a transformer, in which the direct voltage produced by the resistance of the primary winding is usually entirely negligible. This advantage, however, is usually not of very much importance, except in the case of battery-operated receivers, in which it is desired to keep all operating and supply voltages as low as possible. A second advantage is found in the fact that with transformer coupling it is possible to reverse the phase of the voltage applied to the grid of the second tube. In Fig. 14–13 the reversal of either the primary or the secondary leads produces a 180-deg phase reversal on the grid of the second tube. With capacitive coupling, each stage produces a 180-deg phase reversal in itself, and there is nothing we can do about it. Three or more stages of capacitive coupling often lead to instability of the amplifier; with transformer coupling, such a condition can easily be remedied by reversing the phase of one of the stages.

One application where the transformer has no equal in convenience is the operation of a push-pull stage. As was shown in Fig. 13–12 such a stage requires the application of two signals 180 deg out of phase with each other to the two grids of the push-pull stage. A center-tapped winding on a transformer is evidently the most convenient method of obtaining such a signal. In Fig. 13–12 the primary winding of the transformer is not shown connected to any source. This winding can be in the plate circuit of a tube, similar to Fig. 14–13, thus giving us the coupling between a single tube and a push-pull stage. A transformer is also practically a necessity when coupling to a Class B_2 or Class AB_2 stage. Whenever the grid is driven positive during part of the cycle, it takes an appreciable amount of current, and in such cases capacitive coupling leads to some detection that seriously interferes with the intended operation of the stage.

The superiority of capacitive coupling over transformer coupling, in respect to frequency response, size, weight, and cost of equipment involved, has led to many attempts to make it suitable for the operation of a push-pull stage. These attempts have led to several satisfactory solutions of capacitive coupling for push-pull stages, but their discussion will be deferred until the principles underlying them have been presented.

14–18. Production of Bias Voltage by Cathode Resistor.—In all the tube circuit diagrams shown up to now, the bias voltage necessary for the operation of the tube was derived from a separate battery. In the early days of radio when only tubes with directly heated filaments were available, this

was indeed the only way of obtaining the required bias, and the batteries furnishing this voltage were called "C" batteries. With the advent of the indirectly heated cathode, however, the cathodes in an amplifier did not all have to operate at the same potential. Another way of obtaining bias presented itself, shown in Fig. 14–14. A resistor R_C is included in the cathode circuit; the plate current of the tube is flowing through this resistor and produces a voltage making the cathode positive with respect to the negative end of the power supply. With the grid connected as shown, it is seen that it will be negative with respect to the cathode. The cathode resistor must be shunted by a capacitor. The reactance of this capacitor at the lowest frequency that the stage is expected to amplify must be low compared to the value of the cathode resistor. If this is not done, a severe reduction of amplification of this stage will result, as will be shown in the next chapter.

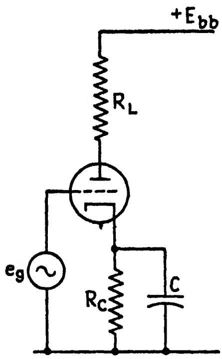


FIG. 14 14.—Method of obtaining self-bias of a tube.

14–19. Determination of Value for Bias Resistor.—In the case of self-bias, as the circuit shown in Fig. 14–14 is also called, the total supply voltage available must also furnish the bias voltage. It is therefore clear that the voltage remaining for the tube and its load will be diminished by the amount of the bias voltage, compared to the condition where the latter is obtained from a separate bias battery. The determination of the value of the required bias resistor is a simple procedure, which may best be shown by an example. Let us assume that the available supply voltage in Fig. 14–14 is 300 volts, that the load resistance is 20,000 ohms, and that a quiescent current of 5 ma is desired. Let the tube be a 6J5, the characteristics of which are shown in Fig. 14–15. The voltage across the load resistance with the required current is 100 volts, which leaves 200 volts for the plate voltage and the voltage across the bias resistor R_C. The problem resolves itself, therefore, into finding a plate voltage and a grid voltage (the latter considered as positive) adding up to 200 volts and giving a current of 5 ma. To obtain 5 ma with -4 volts on the grid, for instance, requires a plate voltage of about 135 volts, as disclosed by the -4-volt characteristic in Fig. 14–15; but 135 plus 4 does *not* add up to 200 volts, as required. To obtain 5 ma with -6 and -8 volts on the grid requires a plate voltage of 178 and 218 volts, respectively. The sum of the two corresponding voltages will therefore be $178 + 6 = 184$ and $218 + 8 = 226$ volts, respectively. This shows that 6 volts across the bias resistor will not be enough and that 8 volts will be too much. Assume linear characteristics over the small range of voltages in which we are interested. The correct value can now be found easily by interpola-

tion from the above found quantities in the following manner. The difference between 184 and 226 is 42 volts; 184 volts corresponds to -6 volts and 226 to -8 volts bias. We wish to obtain 200 volts, which is 16 volts more than 184. We must therefore go $^{16}\!/_{42}$ of 2 volts, or about 0.76 volt from -6 toward -8. This gives us a value of -6.76 volts for the required grid bias. In order to obtain this voltage with a current of 5 ma flowing through the cathode resistor, the resistance must be $6.76/0.005 = 1{,}350$ ohms. If this stage must amplify an alternating voltage of a frequency of 60 cps without too much loss, the reactance $1/2\pi fC$ of the bypass capacitor should be small compared to 1,350 ohms. At 60 cps a

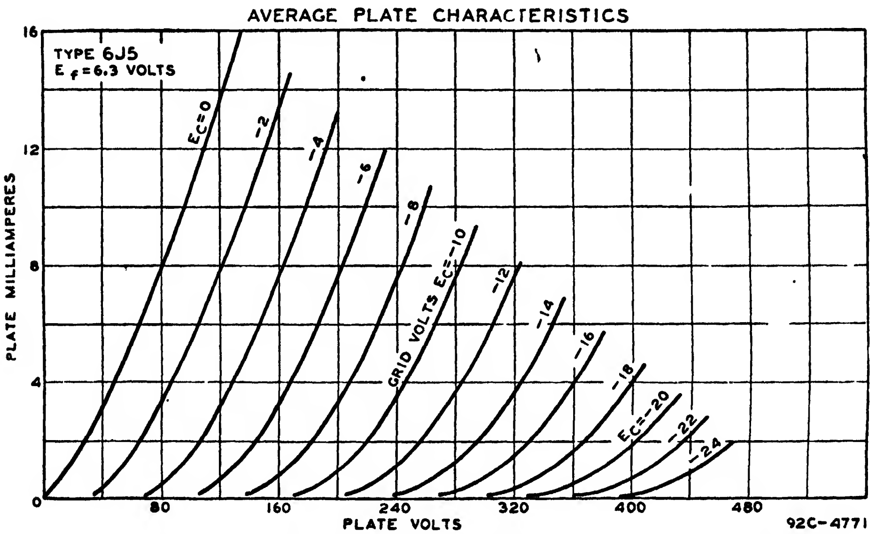


FIG. 14–15.—Plate characteristics of a 6J5. (*Courtesy Radio Corporation of America.*)

1-μf capacitor has a reactance of about 2,650 ohms (a figure to remember), and a 10-μf capacitor has 265 ohms. This is about one-fifth of the value of the cathode resistor and will give quite satisfactory results. However, since low-voltage electrolytic capacitors with standard values of 25 and 50 μf are commercially available, a 25-μf unit would obviously be the logical choice.

PROBLEMS

14–1. What are the two values of voltage required between points 1 and 2 of the circuit shown in Fig. 14–16 to energize and de-energize the relay in the plate circuit of the 6F6? The relay requires 25 ma to close its contacts safely, and the current should be reduced to zero to ensure opening. The plate characteristics of the 6J7 are shown in Fig. 12–4. The 6F6 is operated in triode connection, *i.e.*, with the screen grid tied to the plate. The plate characteristics for this type of operation are shown in Fig. 14–17.

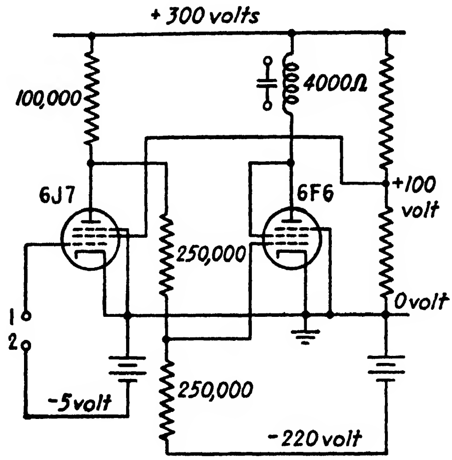


FIG. 14-16.—Circuit diagram for Prob. 14-1.

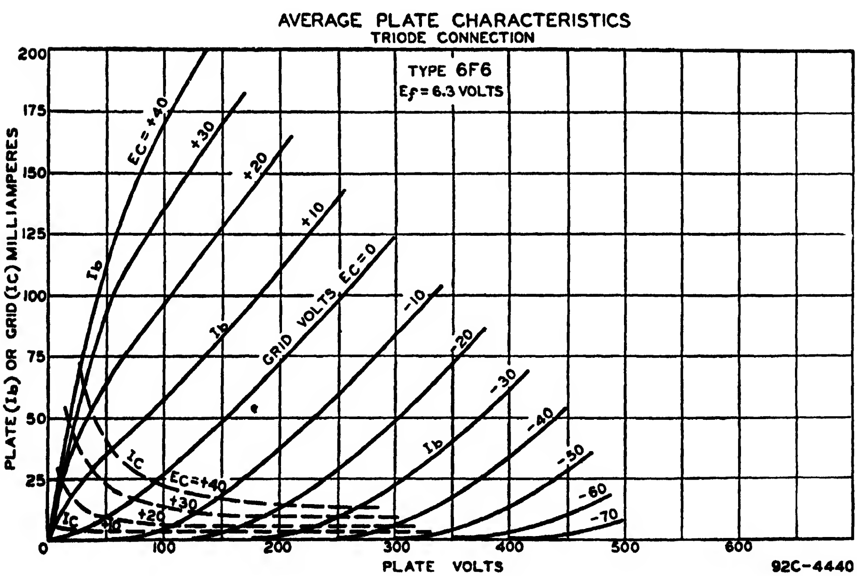


FIG. 14-17.—Plate characteristics of a triode-connected 6F6. (*Courtesy Radio Corporation of America.*)

14–2. A relay with 2,500 ohms coil resistance requiring from 0 to 50 ma current change for operation is connected in the plate circuit of a type 2A3 tube across 250 volts plate-supply voltage. It is desired to obtain operation of the relay when the voltage across a certain dc device (a resistor in series with a photocell, for example) changes from 16 to 17 volts, the relay to be energized when this voltage is 17 volts and to be de-energized when this voltage is 16 volts. (Note this requirement carefully.)

The control-voltage change must evidently be amplified before being applied to the 2A3. Give the values of the grid bias of the first tube, value of the coupling battery between first and second tube (using the coupling method described in Sec. 14–4), if

a. One type 6C5 is to be used ahead of the 2A3.
b. One type 6SF5 is to be used ahead of the 2A3.

Use the same plate-supply voltage for both tubes. Plate characteristics of the three tubes are shown in Figs. 9–5, 9–14, and 10–24.

14–3. According to the manufacturer's recommendations, the resistance in the grid circuit of a 2A3 under certain operating conditions should not exceed 50,000 ohms. If a 6J5 is to be coupled to a 2A3 in a circuit as shown in Fig. 14–18, how large does the

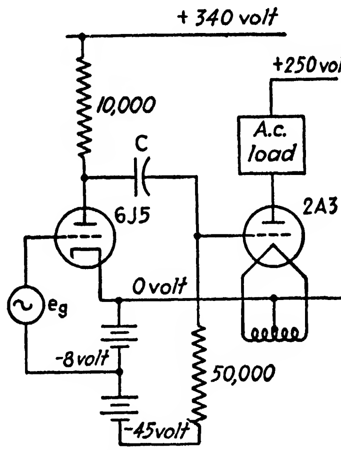


FIG. 14–18.—Circuit diagram for Prob. 14–3.

coupling capacitor have to be made so that the amplification at 50 cps is not less than 80 per cent of that at 400 cps? What voltage gain will be obtained from the grid of the 6J5 to the grid of the 2A3 at these two frequencies? (Under the operating conditions shown in the diagram, the plate resistance is 7,700 ohms, the amplification factor 20 for the 6J5.)

14–4. A one-stage amplifier is to be built that will amplify voltages of as low a frequency as 6 cps. The amplification for this low frequency is to be not less than 60 per cent of the amplification that a 60-cps voltage will receive. (The exact amount of amplification is not essential, as long as the voltage gain for a voltage of 6 cps is not less than 60 per cent of the voltage gain for a voltage of 60 cps.) Available is a transformer with a primary inductance of 20 henrys and a 1:1 ratio, the primary winding of which can carry 60 ma, and a transformer with a primary inductance of 70 henrys but able to carry only 1 ma.

Under consideration are the four circuits shown in Fig. 14–19. Investigate which of the four schemes, if any, will satisfy the specifications.

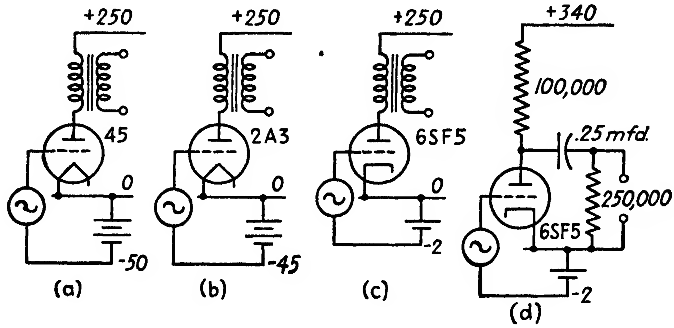


FIG. 14–19.—The four circuits to be investigated in Prob. 14–4.

14–5. Figure 14–20 shows a direct-coupled or Loftin-White amplifier, consisting of two 6J5 tubes. With the input terminals shorted, both tubes are to operate with a plate voltage of 90 volts and a grid voltage of -2 volts. Under this condition the plate resistance r_p is 9,500 ohms, the amplification factor is 20 (values obtainable from the characteristics shown in Fig. 14–15), and the plate current is 4 ma. The load in the

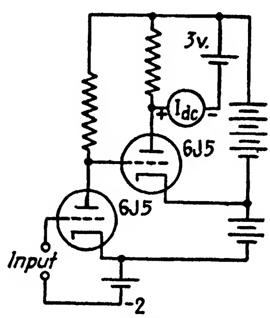


FIG. 14–20.—Circuit diagram for Prob. 14–5.

plate circuit of the second tube is a 1-ma meter, with a resistance of 200 ohms, which is to read zero with zero input voltage. A bucking battery is used to compensate for the zero input-plate current. Determine

a. The total supply voltage needed.
b. The voltage of the tap to which the cathode of the second tube must be connected.
c. The plate resistor in the plate circuit of the first tube.
d. The plate resistor of the second tube for a bucking battery of 3 volts.
e. The input voltage (amount and polarity) to make the 1-ma meter read full scale. (Note the polarity with which this meter is connected.)

14–6. Self-bias is to be provided for a type 2A3 tube (Fig. 9–15), operating with a load having a dc resistance of 200 ohms from a total supply voltage of 300 volts. The current through tube and load is to be 50 ma. Determine the value of the bias resistor and its wattage rating. Give a suitable value for a by-pass capacitor if amplification is to be maintained reasonably well down to 500 cps.

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CHAPTER XV

CATHODE FOLLOWERS; PHASE-INVERSION CIRCUITS; INVERSE-FEEDBACK AMPLIFIERS

15–1. Effect of a Load in the Cathode Lead of a Tube.—The production of a bias voltage by placing a resistor in the cathode lead is only a special case of the general problem of placing a load in series with the cathode of a tube. This arrangement has recently become very popular, and we shall analyze circuits of this kind in more detail. Consider the circuit shown in Fig. 15–1. With the points B and D short-circuited, the grid is seen to be at a positive potential with respect to ground. This does not mean that its potential with respect to the cathode, *i.e.*, the grid voltage, is positive. The current will adjust itself in this circuit to such a value that the voltage occurring across resistor R_c places the cathode at a potential even more positive with respect to ground than the grid. This is equivalent to establishing a negative grid voltage, as indicated in Fig. 15–1 by the levels at which the various electrodes are drawn. In the light of our previous concepts, this arrangement is so unusual that it may be desirable to show such an operation with an example. Suppose that the tube involved in Fig. 15–1 is a 6C5, that R and R_c are 20,000 and 10,000 ohms, respectively, and that a current of 3 ma through the combination is desired. Let the total voltage available for the operation of the circuit be 250 volts. With 3 ma flowing, the voltages across R and R_c will be 60 and 30 volts, respectively, leaving for the plate voltage of the tube a value of 160 volts. Reference to the plate characteristics of a type 6C5 (see Fig. 9–5) tube indicates that in order to obtain a current of 3 ma with a plate voltage of 160 volts a grid voltage of -6 volts will be required. Since the voltage across the cathode resistor produced by the specified current places the cathode at a level of $+30$ volts, it will be necessary to place the grid at a level of $+24$ volts in order to have an actual grid voltage of -6 volts. (Remember that the grid voltage of a tube is the voltage existing between *grid and cathode*, not between the grid and the hot-water faucet!) The bias voltage E_{cc} must therefore be 24 volts.

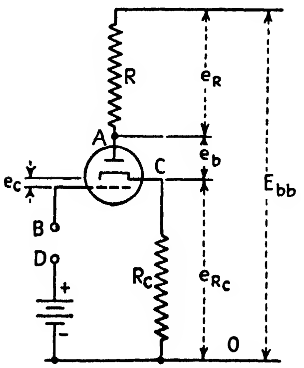

FIG. 15–1.—A resistance placed in the cathode lead of a tube modifies its performance considerably.

If we now introduce a signal voltage between points B and D in Fig. 15–1, what voltage variations will take place at A and C? Assume that we are making point B positive with respect to D by an amount Δe_s. The grid of the tube has now become more positive with respect to ground by this amount. In all previous arrangements this was also the change of potential between grid and cathode. In our present circuit, however, any plate-current change, flowing as it does through R_c, will also change the potential of the cathode with respect to ground. At the same time, an increase of plate current will, of course, also mean a decrease of plate voltage since the voltage across R and R_c will increase with an increase in plate current. Whatever changes of plate current, actual grid voltage, and actual plate voltage will take place, however, must satisfy the fundamental Eq. (9–8), developed in Sec. 9–19. This relation read

$$\Delta i_b = \frac{\mu\,\Delta e_c + \Delta e_b}{r_p} \tag{9–8}$$

In order to make this equation yield an answer to our problem, we shall have to express the three values appearing in it in terms of signal voltage change. When the potential of point B increases by Δe_s, a plate-current change Δi_b will take place. Reference to Fig. 15–1 shows that this increase in plate current will cause a decrease (or negative increase) in plate voltage given by

$$\Delta e_b = -\Delta i_b(R + R_c) \tag{15–1}$$

The actual grid-voltage change that we must introduce in Eq. (9–8) is evidently not Δe_s but this latter value minus the amount that the cathode moved in the *upward* direction. In other words,

$$\Delta e_c = \Delta e_s - \Delta i_b R_c \tag{15–2}$$

Substituting the values Δe_b and Δe_c, as given by Eqs. (15–1) and (15–2), into Eq. (9–8) results in

$$\Delta i_b = \frac{\mu(\Delta e_s - \Delta i_b R_c) - \Delta i_b(R + R_c)}{r_p} \tag{15–3}$$

In this equation Δi_b, the unknown value, appears on both sides. Solving for Δi_b results in

$$\Delta i_b = \frac{\mu\,\Delta e_s}{r_p + R + R_c(\mu + 1)} \tag{15–4}$$

Equation (15–4) was derived under the assumption of resistive loads in the plate as well as in the cathode lead. Equations (15–1) and (15–2) would, however, be just as valid if impedances instead of resistances were used. Equation (15–1) is therefore true also in the case where R and R_c are replaced by impedances. Equation (15–4) states a very interesting

fact. The inclusion of an impedance in the cathode lead of a tube has the same effect on the plate-current change taking place with a given signal-voltage change as if an impedance $(\mu + 1)$ times as large had been included in the plate circuit of the tube. As in so many other cases, it is not very hard to arrive at the same result by plain reasoning, after seeing it presented in mathematical form. A glance at Fig. 15–1 shows that R_c is, of course, a component of the plate circuit just as much as R, but it is also included in the grid circuit. Since the grid circuit is μ times as effective as the plate circuit, its inclusion there makes itself felt as if it had been added an additional μ times in the plate circuit.

This makes it also clear why we must have a capacitor by-passing the cathode resistor, when it is desired to obtain a dc bias, as discussed in Secs. 14–18 and 14–19. In the example analyzed there we found that the bias resistor had to have a value of 1,350 ohms. Since the amplification factor of a 6J5 is 20, this resistor will have the same effect as if $21 \times 1{,}350 = 28{,}350$ ohms was included in the plate circuit in addition to the assumed load resistance of 20,000 ohms. This fictitious resistance can be considered as an increase of plate resistance and will therefore reduce the voltage gain obtainable from the tube. The alternating current that will flow with an application of a signal voltage to the grid will be less than half of what it would be with fixed bias. The shunting of the resistor with a large capacitor reduces the impedance, as far as the ac component is concerned, so that even after multiplication with $(\mu + 1)$ it will still be small compared with the useful load in the plate circuit. Another way of looking at the capacitor is to consider it again as a storage battery. The reduction of amplification, or degeneration, as it is sometimes called, is due to the fact that the cathode does not stay put at a fixed level but bobs up and down with the grid. Placing a battery across the bias resistor holds it at a fixed level so that signal-voltage changes will at the same time be grid-voltage changes.

15–2. Bias Resistor for a Push-pull Stage.—In Secs. 14–18 and 14–19 self-bias of a single tube was analyzed; in Sec. 15–1 the reason for having a large capacitor shunting the bias resistor became apparent. If a push-pull stage is to be biased in this manner, the conditions are different. The signals applied to the grids of such a stage are 180 deg out of phase, and if operation of the tube is confined to the reasonably linear parts of the characteristics, the current in one tube will decrease when the current in the other is increasing. This means that the sum of the two tube currents is essentially constant, and with a constant current flowing through the cathode resistor, the voltage across it will remain constant. The two cathodes will therefore not bob up and down, as the cathode of the single tube in Fig. 15–1 will do with the application of a signal to the grid. The capacitor may therefore be omitted in the case of self-bias of a push-pull stage, if the operation of the stage is reasonably linear; for nonlinear oper-

ation, however, the analysis given above no longer holds true, and a by-pass capacitor will be required.

15-3. Phase Inversion by Means of Cathode Resistor.[4, 5]—Equation (15-4) gives us the plate-current change due to the application of a signal voltage between points B and D. The voltage variations of points A and C can now be found simply by multiplying the plate-current change as obtained in Eq. (15-4) with the values of the resistors R and R_c. It will be noted that an *increase* of plate current makes A more *negative*, while C becomes more *positive*. By making the two resistors in the plate and in the cathode circuit equal, the same voltage variation takes place on A as on C, but their phases will be opposite. This evidently presents a method to obtain two 180-deg out-of-phase signals, as are required for the operation of a push-pull stage. All that is necessary is to couple A and C by means of two capacitors to the grids of the two tubes used in the push-pull stage. The circuit accomplishing this is shown in Fig. 15-2. This figure also shows the conventional way of obtaining the required bias for the push-pull stage by means of a cathode resistor, shunted by a capacitor for the reason explained in the preceding section.

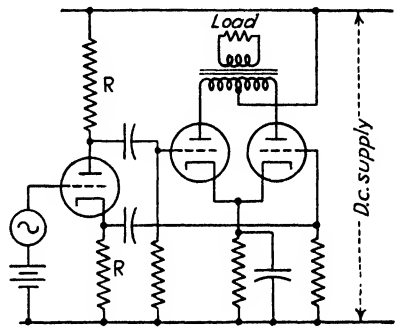

FIG. 15-2.—Two equal resistors R placed in the cathode and anode lead provide a push-pull signal. No voltage gain is obtained from such a stage, however.

15-4. Voltage Gain of a Phase-inversion Stage with Cathode Resistor.—In the arrangement just described, the voltage variations of A and C were alike since the resistors R and R_c were made alike. It is now of interest to determine the size of these voltage variations in relation to the voltage variation applied to the grid, *i.e.*, to B. Making R_c equal to R in Eq. (15-4) and multiplying this equation for the current change with R will give us the desired result. This will be

$$\Delta i_b R = \Delta e_s \frac{\mu R}{\mu R + r_p + 2R} = \Delta e_s \frac{1}{1 + \dfrac{r_p + 2R}{\mu R}} \tag{15-5}$$

The value of the fraction with which Δe_s is multiplied is always smaller than unity since the denominator is larger than the numerator. The output voltage appearing across the two resistors is therefore always smaller than the signal voltage. Plain reasoning will also show the truth of this statement. If point B in Fig. 15-1 swings more positive by 1 volt, we expect the current in the tube to increase; but it certainly cannot increase so much that point C would swing positive in excess of 1 volt because that

would mean that the grid would now be more negative with respect to the cathode than it was before the signal changed the potential point of B. With a higher negative grid voltage the current could not increase. The variation of potential of C can therefore never exceed that of the grid, which can also be expressed by saying that the cathode "follows" the grid. This explains the name "cathode follower" for this type of circuit.

15-5. True Cathode-follower Circuit.[1-3]—The true cathode follower is characterized by the absence of the load resistor in the plate lead, as shown in Fig. 15-3. To analyze the performance of such a circuit it is only necessary to put $R = 0$ in the general equation (15-4). This will give us the plate-current change resulting from a signal-voltage change. By multiplying with R_c we obtain the voltage variation of point C, *i.e.*, the cathode.

$$\Delta e_{R_c} = \Delta i_b R_c = \Delta e_s \frac{\mu R_c}{r_p + R_c(\mu + 1)}$$

$$= \Delta e_s \frac{\mu R_c}{\mu R_c + r_p + R_c} = \Delta e_s \frac{1}{1 + \dfrac{r_p + R_c}{\mu R_c}} \qquad (15\text{-}6)$$

This equation again discloses the fact that no amplification is obtained from a cathode-follower circuit. It is therefore justifiable to ask just what the arrangement is good for. It will be noted that the signal is applied to the grid of a tube, and, therefore, the source furnishing the signal does not furnish any current. A voltage almost equal to the signal in size now appears across the resistance R_c. It is for this reason that cathode-follower circuits are also referred to as "impedance-changing" tubes. A voltage may, for instance, appear across a device of extremely high impedance and prevent us from placing a potential divider across it if we wish to obtain only a fraction of this voltage or prevent us from sending it over a network with leakage resistance or stray capacity. If we apply this voltage to a cathode-follower stage, the signal can be made to reappear across a relatively low impedance and permit us more convenient manipulation of it. The cathode-follower circuit can handle very large signal voltages applied to its grid and reproduce them very faithfully across the cathode resistor. For further information, the reader is referred to the numerous articles that have appeared on this subject in the past years.

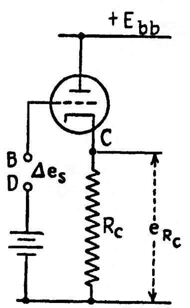

FIG. 15-3.—In a true cathode-follower circuit, the anode is connected directly to the plate-supply voltage and all the load is placed in the cathode lead.

15-6. Phase Inversion by Means of Two Tubes with Common Cathode Resistor.[3, 6-8, 15]—One arrangement deserving particular attention is shown

in Fig. 15-4. Two identical tubes with the same load resistors R in their plate circuits have a common cathode resistor that is not by-passed by a capacitor, as in an ordinary push-pull arrangement. The grid of the right-hand tube is held at a fixed potential by the bias battery, and the signal is applied to the grid of the left-hand tube. The circuit operates as follows. Assume that the signal voltage is driving the grid of tube 1 positive. The current in tube 1 will therefore increase. This means that point A will swing negative, and C will attempt to swing more positive. In so doing, however, the grid of tube 2 becomes more negative, and the current in tube 2 decreases. This means that B becomes more positive. It is seen that this circuit has phase-inverting properties. The mathematical analysis of the performance of the circuit is not very difficult, as long as we can assume that the tubes are operating in the linear part of their characteristics or, in other words, as long as the equivalent-plate-circuit theorem may be applied to the circuit. Let the signal make the grid of tube 1 more positive by a given amount, say, 1 volt. The following artifice will be found of great help in analyzing this circuit. Instead of letting grid 1 go positive by 1 volt, we let grids 1 and 2 at first go positive ½ volt. After having calculated what will take place under this condition, we let grid 1 go positive by an additional ½ volt, and grid 2 is made to go ½ volt negative. Clearly, the total effect of these two successive steps is that grid 1 will be positive by 1 volt while grid 2 will be back at its original level given by the bias voltage. This is the condition for which we wanted to determine the current change taking place.

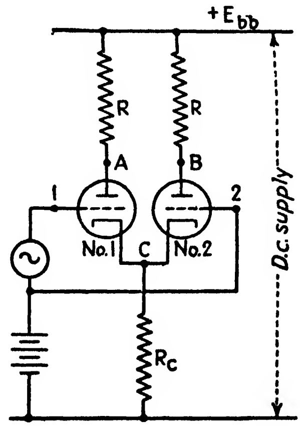

FIG. 15-4.—When a signal is applied to grid 1, while grid 2 remains at a fixed potential, the potentials of points A and B will swing in opposite directions, thus furnishing a push-pull signal. The signal is, however, slightly unbalanced.

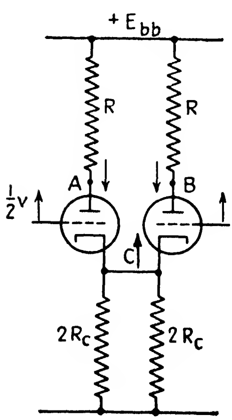

FIG. 15-5.—The performance of the phase-inversion circuit shown in Fig. 15-4 can be analyzed most conveniently with the aid of the circuit shown.

During the first step, when both grids are made ½ volt more positive, the two tubes are evidently in parallel. If we replace the cathode resistor R_c by two resistors, each with a value of $2R_c$, connecting them in parallel as shown in Fig. 15-5, there will obviously be no change in performance. Since, with identical grid-voltage changes applied to the two tubes, there will be no current in the conductor connecting the two cathodes, we can

cut this connection without affecting the performance of each tube. The performance taking place then can be calculated with the aid of Eq. (15–4). It is seen that during the first step the potential of point C will go up (it follows the grid), while A and B will come down together. The amount of the voltage variations of A and B can be obtained by multiplying the current change calculated for each tube with the aid of Eq. (15–4) (using a value of $2R_c$ as the cathode resistor) by the load resistance R.

During the second step, when grids 1 and 2 swing equal amounts in opposite directions, the potential of point C in Fig. 15–4 will not change if linear operation can be assumed. During this step A and B will move equal amounts in opposite directions, A coming downward, B going upward. Since during this step the potential of C does not change, one could consider the resistor R_c replaced by a battery, and the circuit then performs in true push-pull fashion. During the first step, when both grids were swinging positive simultaneously, the potential of A and B was decreasing simultaneously. If we consider the voltage existing between A and B as the output voltage, evidently, the output voltage during the first step will be zero. Output voltage is, therefore, produced only during the time when one grid swings in the opposite direction from the other. Since during this time, as just explained, the cathode resistor R_c can be considered as absent, the total voltage gain of such a stage, *i.e.*, the ratio of the voltage appearing between A and B to the voltage introduced to the grid of one of the tubes, is the same as if we were dealing with a single tube with a load resistor R in its plate circuit.

The calculations pertaining to such a case are then simply as follows. Suppose we wish to find the performance of the circuit when the grid of the first tube is made Δe_s volts positive with respect to the fixed bias voltage. At first we calculate what happens to the potential of A and B if both grids are made an amount $\Delta e_s/2$ positive. If we apply Eq. (15–4) to this step and consider the cathode resistor of each tube as $2R_c$, the potential variation of A and B will be

$$\Delta e_A' = \Delta e_B' = -\frac{\Delta e_s}{2} \times \frac{\mu R}{r_p + R + 2R_c(\mu + 1)} \tag{15–7}$$

During the second step we have seen that each tube performs as if no cathode resistor were present in its circuit, owing to the push-pull application of the signals. Under this condition, the change of potential of A and B will be given by

$$\Delta e_A'' = -\frac{\Delta e_s}{2} \times \frac{\mu R}{r_p + R} \tag{15–8}$$

$$\Delta e_B'' = +\frac{\Delta e_s}{2} \times \frac{\mu R}{r_p + R} \tag{15–9}$$

The total change of potential of A and B is given by the sum of the components of Eqs. (15–7) and (15–8), or (15–9), respectively. The final result is consequently

$$\Delta e_{A,B} = \frac{\Delta e_s}{2}\mu R\left(\mp\frac{1}{r_p+R} - \frac{1}{r_p+R+2R_c(\mu+1)}\right)$$

$$= \frac{\mu\,\Delta e_s}{2}\times\frac{R}{r_p+R}\left(\mp 1 - \frac{r_p+R}{r_p+R+2R_c(\mu+1)}\right) \quad (15\text{–}10)$$

The minus sign in the parentheses refers to Δe_A, the plus sign to Δe_B. The total output voltage is given by the difference of the expressions for Δe_A and Δe_B. It is seen that, when forming the difference, the second term within the parentheses cancels out, resulting in a total output voltage equal to what would be calculated for a single tube with a load resistance R and a signal Δe_s applied to this tube.

15–7. Regenerative and Degenerative Feedback.—In the cathode-follower circuit described in Secs. 15–1 and 15–5 we applied a signal to the grid of the tube, but the voltage appearing between cathode and grid was not equal to the signal, owing to the fact that the plate current flowing through the cathode resistor made the cathode "follow." In other words, when the grid was going up, the current increased and the cathode was, therefore, also going up in potential. One could also say that the voltage appearing on the grid of the tube consists of the sum of two voltages: one, the signal voltage itself; the other, the voltage appearing across the cathode resistor. Whenever an amplifier—whether it is a single-tube or a multistage amplifier—is arranged so that either its whole output (as in the case of the true cathode-follower circuit) or a part of the output is placed in series with the original signal, we say that feedback is employed. When the feedback voltage is introduced with such a polarity as to aid the original signal, the feedback is said to be "regenerative"; when the feedback voltage is opposing the original signal, it is said to be "degenerative." It is clear that in the case of the cathode-follower circuit the feedback is degenerative because the motion of the cathode is in such a direction as to reduce the voltage variation that would appear between cathode and grid if the cathode remained at a fixed level.

Amplifiers employing degenerative, also called "inverse," feedback have certain desirable characteristics, and although it is impossible to treat this subject in detail here, the reader should be familiar with the fundamentals of inverse feedback so that he will be in a position to make use of the voluminous literature on this subject.

15–8. Mathematical Analysis of Feedback.—Let us assume that the amplifier shown in Fig. 15–6 has an over-all voltage gain of A. This means that the output voltage appearing across the load will be A times as high

as the voltage applied to the input terminals of the amplifier. Let A be, for instance, equal to 1,000. Then in order to obtain a voltage of 100 across the load, it will be necessary to apply $\frac{1}{10}$ volt to the input terminals of the amplifier. Now let a network, which for simplicity's sake is shown as two resistors R_1 and R_2, be placed across the output terminals and let a fraction β of the output voltage e_o be placed in series with the input terminals as shown in Fig. 15-6. With the polarity marked as shown in this figure, the feedback voltage is aiding the signal voltage. This is not the condition in which we are interested, but since most articles written on feedback start their mathematical derivation under the assumption that

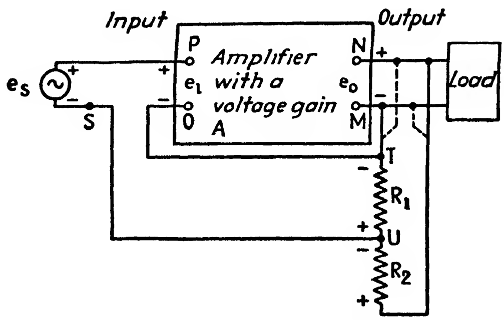

FIG. 15-6.—The principle of the feedback amplifier.

the feedback voltage is of the same polarity as the input voltage, this procedure will be followed here, which will make it easier for the reader to refer to the literature. Under this condition, the input voltage applied to the amplifier terminals themselves is given by

$$e_i = e_s + \beta e_o \tag{15-11}$$

The output voltage e_o is equal to A times the input voltage, and we therefore have

$$e_o = Ae_i \tag{15-12}$$

Solving Eq. (15-12) for e_i and introducing this value into Eq. (15-11), we have

$$\frac{e_o}{A} = e_s + \beta e_o \quad \text{and} \quad \frac{e_o}{e_s} = \frac{A}{1 - \beta A} = A\frac{1}{1 - \beta A} \tag{15-13}$$

The ratio of the output voltage to the signal voltage is no longer A, as it was when the signal was applied directly to the input terminals of the amplifier, but is now given by Eq. (15-13).

15-9. Effects of Negative, or Inverse Feedback.[9-14, 16-18]—Let us now turn our attention immediately to the case where the voltage fed to the input is of such a polarity as to oppose the signal voltage. Mathematically, this means that the fraction β must be taken as negative when sub-

stituted into Eq. (15–13). This will evidently make the denominator of the fraction in Eq. (15–13) positive, and since it is larger than the numerator, the ratio of the output voltage to the signal voltage is less than the voltage gain A of the amplifier itself. In the example stated in the beginning of Sec. 15–8, we assumed that the amplifier had a voltage gain A of 1,000. Let us assume that the two resistors R_1 and R_2 are in the ratio of 1:9, which will mean that one-tenth the output voltage is fed back to the input, or $\beta = -0.1$ (assuming negative feedback). When these values are substituted in Eq. (15–13), the ratio of output voltage to signal voltage becomes $1{,}000/101 = 9.9$. The over-all gain of the amplifier has been reduced from a value of 1,000 to approximately 10 by the introduction of inverse feedback! Would it not be simpler to build an amplifier with a gain of 10 to start out with instead of constructing one with a gain of 1,000 and then to knock it down to the value of 10? Before answering this very legitimate question, let us try to obtain a physical understanding of the relation expressed in Eq. (15–13). This can be done easily if we start with the voltage appearing across the load instead of using the signal voltage as a starting point. Let us, therefore, ask ourselves what signal voltage will be required to obtain a voltage of 100 volts across the load. The reasoning can be carried out best by making use of the instantaneous values. If the voltage gain of the amplifier is 1,000, then we know that in order to make terminal N instantaneously positive by 100 volts with respect to terminal M, it is necessary to make terminal P instantaneously positive by 0.1 volt with respect to terminal O. With negative feedback the resistors R_1 and R_2 will not be connected as with the full lines but as shown by the dotted lines; therefore, with the desired 100 volts across terminals N and M, point U will be negative with respect to point T by 10 volts. Point S will therefore be 10 volts negative with respect to point O. But we have seen that point P *must* be 0.1 volt positive with respect to point O in order to obtain the postulated voltage of 100 volts across the load. This condition can evidently be fulfilled only when the signal voltage furnishes a signal of 10.1 volts at this instant. The over-all voltage gain from signal to load is therefore the ratio 100/10.1, which is exactly the value found by the application of Eq. (15–13).

15–10. Advantages of Negative Feedback.—What are the advantages of this amplifier over a simple amplifier with a voltage gain of 10? Let us assume that owing to aging and deterioration of tubes the voltage gain of each amplifier drops to one-half its original value. The amplifier originally built with a voltage gain of 10 will therefore have a voltage gain of only 5; the voltage gain of the feedback amplifier will drop from 1,000 to 500. In order to obtain 100-volt output we must therefore have on the actual input terminals of the amplifier 0.2 instead of 0.1 volt. Since the feedback network R_1-R_2 is still making S 10 volts negative with respect to O, the signal must furnish a voltage of 10.2 instead of 10.1. The over-

all voltage gain of the amplifier will now be $100/10.2 = 9.8$. A deterioration of 50 per cent of the voltage gain of the actual amplifier has resulted in a reduction of only approximately 1 per cent in the over-all gain of the feedback amplifier. The amplification obtainable with the feedback amplifier is therefore practically independent of the tube characteristics. This feature is of particular value when precision measurements are to be made and an amplifier must be used between the input and the indicating instrument.

Equation (15–13) may be rewritten in the form

$$\frac{e_o}{e_s} = \frac{1}{(1/A) - \beta} \tag{15–13a}$$

If the voltage gain A of the amplifier is large, so that its reciprocal $1/A$ is small compared to the value of β, then in the case of negative feedback (where β is negative), the over-all gain of the amplifier will simply be $1/\beta$. This was borne out by our example where with $\beta = -0.1$ the over-all gain of the amplifier was very nearly 10, which is the reciprocal of 0.1. The voltage gain of the amplifier can therefore be conveniently controlled by changing the feedback ratio β.

15–11. Reduction of Output Impedance Due to Inverse Feedback.— The advantage of eliminating the effect of changes in the tube characteristics on the over-all voltage gain is not the only one arising from inverse feedback. In an ordinary amplifier a change of the value of the load resistance would necessarily also change the over-all voltage gain; with an inverse feedback amplifier the over-all voltage gain was seen to be essentially determined by the feedback network. Such an amplifier will therefore furnish the same voltage to the load, even if the value of the latter should be changed over a fairly wide range. But if the voltage delivered by a generator to a load does not change appreciably with a change of the value of the load, it means that the generator has a low internal impedance. This is exactly what inverse feedback does: it reduces the output impedance of the amplifier. This is quite often of even greater importance than the independence from the changes in tube characteristics.

15–12. Factors Limiting the Amount of Inverse Feedback.—The practical application of inverse feedback is not so easy as the simple treatment given in the preceding sections would indicate. If an amplifier had a completely flat characteristic as far as frequency is concerned, then the output voltage would be always equal to A times the input voltage, and the phase relationship would always be the same. Unfortunately, the voltage gain A is not a constant applying to all frequencies. Not only the value of A changes, but the phase relationship between the input voltage and output voltage does not remain constant. In an amplifier having three or more stages, the output voltage may be in phase with the input voltage for a

certain range of frequencies (which means that point N will be positive with respect to M when the input terminal P is positive with respect to terminal O). For higher frequencies this relationship may be reversed. A connection giving negative feedback at one frequency may therefore be found to give a positive feedback at another frequency. Positive feedback usually leads to oscillation of the system, and the amount of feedback that can be employed in the case of a high-gain amplifier is therefore usually limited to a value dictated by the appearance of unwanted oscillations. The reader wishing further information on this subject will find it exhaustively treated in many books and magazines.

15–13. Cathode Follower as an Inverse-feedback Amplifier.—It may be interesting and instructive to show that the relation developed in Sec. 15–5 for the cathode-follower circuit may be derived also on the basis of considering a cathode-follower circuit as a negative feedback amplifier. In a true cathode-follower circuit the only load of the tube is the resistor R_c included in the cathode lead. If this resistor were placed in the plate circuit of the tube or if the signal were applied between *cathode* and grid of the tube, then the voltage gain of such an arrangement would be that given by Eq. (10–2), with the value R_L replaced by the value R_c. In this case of a single-stage amplifier, the voltage gain v_g would then be equal to the amplification A referred to in the Eqs. (15–11) to (15–13). In the cathode-follower circuit the whole output voltage would now be placed in series with the signal and with such a polarity as to oppose the signal. This means that $\beta = -1$. Under this condition we obtain

$$\frac{e_o}{e_s} = \frac{A}{1 + A} \tag{15–14}$$

Introducing for A the voltage gain as given by Eq. (10–2), we find

$$\frac{e_o}{e_s} = \frac{\mu R_c/(r_p + R_c)}{1 + \mu \dfrac{R_c}{r_p + R_c}} = \frac{1}{1 + \dfrac{r_p + R_c}{\mu R_c}} \tag{15–15}$$

Comparison of this equation with Eq. (15–6) shows them to be identical.

PROBLEMS

15–1. A 6SF5 tube (for plate characteristics, see Fig. 10–24) is to be used as a cathode-follower or impedance-changing tube in a circuit as shown in Fig. 15–7.

a. Plot the relation between input voltage e_i and output voltage e_o.

b. Assuming that the grid begins to draw current when it is 0.5 volt negative with respect to the cathode, at what value of input voltage will the tube cease to be an infinite impedance device?

(This problem can be solved most conveniently by drawing a load line with 200,000 ohms, choosing the grid-voltage values as given in the plate characteristics, and then

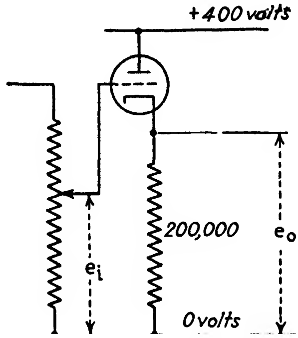

FIG. 15-7.—Circuit diagram for Prob. 15-1.

determining corresponding values of e_i and e_o from the intersection of the load line with the characteristics.)

15-2. A 6SC7 is to furnish a push-pull signal approximately balanced with respect to ground for the operation of a type 905 cathode-ray tube. This is a tube with a 5-in. screen, and the maximum deflection from the center of the screen is to be 50 mm, or approximately 2 in. The sensitivity of the tube is given as 0.46-mm deflection per volt applied to the deflection plates. The circuit proposed for the 6SC7 is shown in Fig. 15-8. The 6SC7 is a double triode, and the characteristics applying to each section are given in Fig. 9-10.

a. In order to obtain 1 ma in each tube section, with the input terminals shorted, how large must R be made?
b. To obtain 50-mm deflection on the screen, what input voltage will be needed?
c. How much is the contribution of each tube element to the total output voltage?

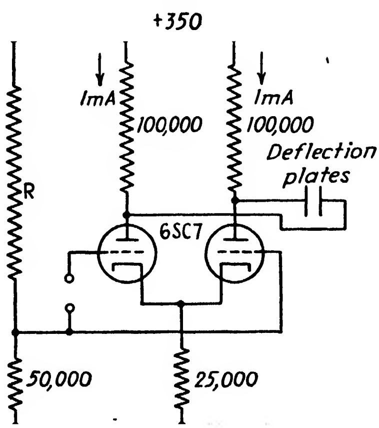

FIG. 15-8.—Circuit diagram for Prob. 15-2.

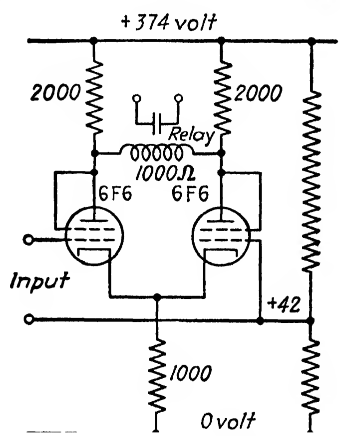

FIG. 15-9.—Circuit diagram for Prob. 15-3.

15-3. For a 6F6, triode-connected, the manufacturer gives the following data: plate voltage, 250 volts; grid voltage, −20 volts; plate current, 31 ma; r_p = 2,600 ohms;

$\mu = 6.8$; $g = 2{,}600$ microhms. A relay with 1,000-ohm coil resistance is connected in a balanced circuit, employing two 6F6's, as shown in Fig. 15-9.

a. Check whether a bias of +42 volts, as shown, results in operating conditions as recommended by the manufacturer.

b. What input voltage is needed if the relay requires 20 ma for operation?

15-4. A 6J5 operating in a circuit as shown in Fig. 15-10 will have a plate current of 8 ma and a plate voltage of about 240 volts. Under this condition the plate resistance is about 8,000 ohms, and the amplification factor 20. What will be the voltage gain of such a stage?

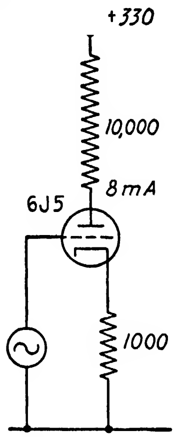

FIG. 15-10.—Circuit diagram for Prob. 15-4.

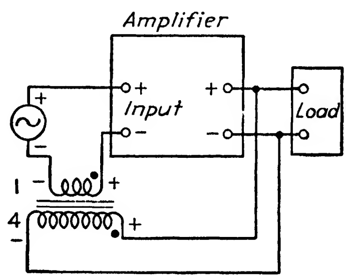

FIG. 15-11.—Diagram for feedback amplifier of Prob. 15-5.

15-5. An amplifier has an over-all gain from its input terminals to the terminals of the load of 20. A 4:1 transformer is connected, as shown in Fig. 15-11, so as to oppose the signal. What will be the over-all gain from signal to load?

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CHAPTER XVI

DETECTION; VACUUM-TUBE ALTERNATING-CURRENT VOLTMETER; VACUUM-TUBE WATTMETERS

16-1. Use of Nonlinearity of Characteristics.—Up to this point we have been using tubes as amplifiers only. In all the applications the nonlinearity of the tube characteristics was decidedly undesirable. The use of the equivalent-plate-circuit theorem, for instance, was limited to a small region because of the fact that the tube characteristics were not linear. There are, however, a number of applications of tubes that would not be possible if the tube characteristics were straight lines, in other words, if the tubes were perfect amplifiers. One of the most important tasks that a tube can perform, owing to the nonlinearity of the plate characteristics, is to produce a change of the average or direct plate current when an ac signal is applied to its grid. If the tube were a perfect amplifier, then the dc component of the plate circuit would not change with an application of an ac signal to the grid of the tube since the increases and the decreases of plate current occurring during positive and negative swings of the signal would be equal in amount. This means that the average or dc value of the plate current would remain unchanged. However, by properly selecting the value of the dc bias or by special circuit arrangement in the grid circuit of the tube, it is possible to obtain a change in the dc value of the plate current with the application of an ac signal to the grid of the tube. When a tube is operating under such a condition, it is said to act as a "detector" or "demodulator."

16-2. Need and Uses of Detection.—As the subject of detection is quite complicated, we cannot hope to cover more than the very fundamentals of it here. Fortunately, for industrial purposes it is not necessary to go into as much detail as is necessary for the radio and communication engineer. Why should the electronic engineer be interested in this subject at all? In the preceding chapters we have seen that the tube served the purpose of obtaining an indication from a direct voltage not permitting any current drain, or of obtaining relay operation from such a voltage. It is evident that there must be many cases where the voltage from which we wish to obtain relay operation or a meter indication will be an alternating voltage. The first thought occurring in connection with this problem is, of course, to simply amplify the alternating voltage in question until it is able to operate an ac voltmeter or ammeter, or until it is capable of operating an ac relay. Anyone familiar with electrical instruments and

relays is aware of the fact that dc instruments are considerably more sensitive, and dc relays considerably simpler in construction and more sensitive than their ac counterparts. A dc microampere meter is a standard and reasonably priced instrument. No such instrument is available for the measurement of small alternating currents, except possibly rectifier instruments, and these—as the name implies—are really dc instruments. To make things worse, most of the ac instruments are frequency dependent. To make an ac relay operate from an alternating voltage of several hundred thousand cycles per second would be utterly impractical if not impossible. In all these cases a device that will furnish a direct current or a dc change with the application of an ac signal will permit the use of dc instruments and relays that are much more sensitive and reliable.

The use of a plain rectifying element, electronic or otherwise, is therefore already a step in the right direction if we wish to operate a meter or a relay from a source of alternating voltage not capable of furnishing a large amount of current, or from a source of high frequency. With such an arrangement the source will still have to furnish, however, at least as much power as the dc instrument or relay requires. Although this power may be much smaller than a corresponding ac meter or relay would require, it might in a particular application be too heavy a drain on the source of voltage. If in such a case we can apply the voltage to the grid of a tube, which takes a negligible amount of current, and obtain a dc plate-current change, we shall obviously have a very desirable arrangement.

16–3. Use of Diodes for the Measurement of Alternating Voltage.—Before discussing the use of triodes for the indication of alternating voltages, it may be well to pause for a short discussion of how a diode or rectifier tube may be used to convert alternating voltages into direct currents, which are much easier to measure and to observe. The reader is referred to Chap. VII, in which rectifier circuits containing tubes were discussed in detail. In Secs. 7–11 and 7–12 it was shown how to determine the direct current that will flow through a meter connected in series with a diode to a source of alternating voltage. To refresh our memory on this subject, an example similar to the one treated there will be analyzed. Let it be desired to convert a 0–1 ma dc meter into an instrument capable of measuring a 100-volt alternating voltage of sinusoidal wave shape. We could connect the instrument with a diode in a half-wave rectifier circuit, as shown in Fig. 16–1*a*. How large would resistor *R* have to be to give us full-scale deflection of the 0–1 ma dc meter when a sinusoidal voltage with an rms value of 100 volts is applied as shown? Let the characteristic of the diode be as shown in Fig. 16–1*b*. This characteristic indicates that in the half cycle, during which conduction takes place, the voltage across the diode will be so small that the current flowing in the circuit will be nearly of sinusoidal wave shape. (If the voltage for which the instru-

ment is to be designed is much smaller, we will have to employ the construction outlined in Figs. 7–10 to 7–15.) As shown in Eq. (3–3), the average value of a full-wave rectified alternating voltage is equal to $2/\pi$ times the amplitude of the alternating voltage or also equal to 0.9 times the rms value. Since current flows in the meter only during alternate half waves, the average or dc value will be only one-half the above given expression. An alternating voltage of 100 volts rms will, therefore, produce in the instrument the same direct current as a direct voltage of 45 volts. If a current of 1 ma is desired, the total resistance must therefore be 45,000 ohms. This value can also be arrived at in the following manner, which may give a little clearer understanding of what happens in this circuit. The ratio of amplitude to average value in a half-wave rectifier circuit has been shown to be $\pi/1$. If we wish to have a dc meter indicate a current of 1 ma and we know that the current actually passing through the instrument consists of alternate half waves of sinusoidal shape, then the peak of the current must have a value of 3.14 ma. The peak value of an alternating voltage with an rms value of 100 volts is equal to 141.2 volts. The total resistance in the circuit necessary to produce a peak current of 3.14 ma when the voltage has a peak value of 141.2 volts is given by 141.2/0.00314, which in turn gives us a value of 45,000 ohms. This is the *total* resistance to be included in the circuit if the rectifying element is perfect, *i.e.*, if no voltage exists across it during the time of conduction. But as shown by the characteristics in Fig. 16–1*b*, it requires a certain amount of voltage to make a current pass through the diode. Strictly speaking, owing to the nonlinearity of the characteristic, it cannot be replaced by a resistance. In the present example, however, the voltage across the diode is always small compared to the total voltage acting in the circuit; for this reason we are not committing a serious error by considering it as a resistance during the time of conduction. The characteristic discloses that with a current of 3.14 ma flowing through the diode the voltage existing across it will be slightly less than 3 volts. At this instant, it consequently causes the same loss of voltage as a resistance of approximately 1,000 ohms. Subtracting this value from the total required 45,000 ohms gives us a value of 44,000 ohms for the series resistor R.

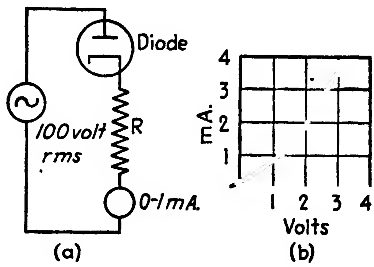

FIG. 16–1.—Conversion of a dc meter into an ac meter by means of a simple diode.

16–4. Protection of Diode from High Inverse Voltages.—In the circuit shown in Fig. 16–1*a*, the diode during the half cycle of nonconduction must be able to withstand the peak voltage of the alternating voltage in the opposite direction. If the manufacturer specified for the particular tube that

we intended to use a peak inverse voltage of 150 volts, for instance, we would be entirely safe with the circuit as shown since the amplitude of the alternating voltage is only 141 volts. We could not use this tube, however, in the circuit as shown if we wished to design a voltmeter to measure higher voltages. In this case, a circuit as shown in Fig. 16–2 would be required. During the half cycle when point A is positive with respect to G, current will flow through R_1, diode D_1, resistor R_2, and the dc meter; the conditions will be exactly the same as those prevailing in Fig. 16–1a except that R of the latter figure is now split up in the two resistors R_1 and R_2 for a reason to be seen presently. During the half cycle of opposite polarity, current will flow through diode D_2 and resistor R_3. The maximum inverse voltage to which D_1 is now subjected is evidently only the voltage produced across R_3 and the diode D_2 during the time the latter is conducting. But why add R_2 and R_3 at all? Would the circuit not work just as well with these two resistors short-circuited and the value of R_1 made correspondingly larger? If the circuit is set up without R_2 and R_3, it will be found that the dc meter will show an appreciable deflection before any alternating voltage is applied to A and G. The reason for this is that the two diodes form a series circuit with the instrument, and a current will be produced owing to the initial velocity of the electrons emitted by the cathodes. This current cannot be completely suppressed, but by including sufficient resistance in this series circuit, in our case resistors R_2 and R_3, it can be reduced to an insignificant value.

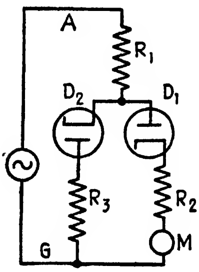

FIG. 16-2.—When the alternating voltage to be measured is high, the measuring diode may have to be protected against high inverse voltage by shunting it with another diode in the opposite direction.

16–5. Effect of Dc Component.—Under certain conditions the simple circuit shown in Fig. 16–1 must be replaced by the one shown in Fig. 16–2, even if the inverse voltage is within the safe limits of the tube. The circuit shown in Fig. 16–1 takes from the voltage that it measures alternate half waves of current; in other words, the source must be capable of furnishing a direct current. If the voltage to be measured with this circuit originates in a transformer winding or is due to the current flow through a resistor, then the circuit shown in Fig. 16–1 is satisfactory. But let us consider a case as shown in Fig. 16–3. Two capacitors, each 1 μf, are placed in series across a source of alternating voltage of 110 volts. We know that under such a condition there will be an alternating voltage of 55 volts across each capacitor; but if we were to connect an instrument as shown in Fig. 16–1 across one of the capacitors,

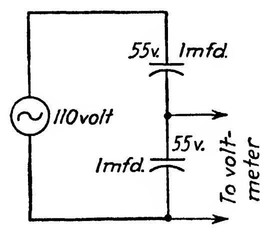

FIG. 16–3.—When the voltage across a capacitance is measured, the measuring instrument must pass current in both directions.

the direct current taken by it would quickly place a dc charge on the two capacitors, of such magnitude and polarity as to prevent the junction point between the two capacitors from going positive with respect to the lower terminal in Fig. 16–3; this will reduce the current through the instrument to zero. A measuring device built according to the circuit shown in Fig. 16–2, on the other hand, will not be subject to this error since both positive and negative half waves are permitted to flow; the positive half waves flow through the meter while the negative half waves are not suppressed but are shunted around the meter through the second diode.

16–6. Full-wave Bridge-type Circuits.—The circuits shown in Figs. 16–1 and 16–2 both required that the voltage to be measured is able to furnish a peak current equal to π times the direct current indicated by the meter. In some cases this may represent too much current drain. In such a case the full-wave circuit shown in Fig. 16–4 will be more suitable. During the half cycle when point A is positive with respect to point G, current will flow through the diodes D_1 and D_2, passing through the branch with the instrument from left to right. During the next half cycle, the current will flow through diodes D_3 and D_4, passing again through the instrument from left to right. The current through the instrument will now consist of successive half waves of sinusoidal shape, and the peak current that the voltage to be measured has to furnish is only $\pi/2$ of the dc value indicated by the instrument. In this circuit, too, it is necessary to include a resistance in series with the instrument as shown in Fig. 16–4 to minimize the current produced by the initial velocity of the emitted electrons. Considerations similar to those pertaining to Fig. 16–2 permit the proper choice of the value for this resistance.

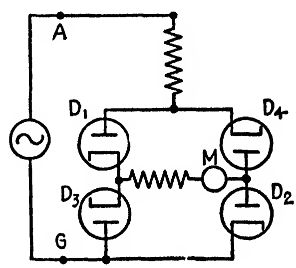

FIG. 16–4.—Four diodes may be connected in a bridge-type rectifier.

16–7. Precautions Required with Tube-type Voltmeters for High Alternating Voltages.—A word of caution is necessary in connection with instruments designed according to the diagrams shown in Figs. 16–2 and 16–4. As will be recalled, these circuits are suitable for measuring alternating voltages of any magnitude, regardless of the peak inverse voltage permissible by the tube. However, if the principles outlined in the preceding paragraphs are to come into play, it is necessary to make sure that the cathodes are emitting before the voltage to be measured is applied to the instrument. If this precaution is not taken, it is evident that the full peak voltage will appear across either diode, and a flashover is sure to result. Even if this should not take place, it is one of the worst beatings that can be administered to a tube to have a high voltage applied between cathode and anode while the cathode is heating up. Severe damage to

the cathode, especially if the latter is of the oxide-coated type, is certain to result.

16-8. Comparison of Dry-type Rectifier Instruments with Instruments Containing Tubes.—In the instruments described in the preceding paragraphs, the alternating voltage to be indicated had to furnish all the energy necessary for their operation. Its only advantage consisted in the fact that it permitted the use of a dc instrument, which requires considerably less energy for its operation than a corresponding ac instrument. The circuits shown in Figs. 16-2 and 16-4 are identical with those employed in connection with dry-type copper oxide or selenium rectifiers, except that the latter do not require suppression of an initial current. The advantage of this type of rectifier is, of course, that there is no need for heating any cathodes and the instruments may be built entirely self-contained. This type of instrument has become very popular for the measurement of alternating voltages of frequencies in the audio range. The capacitance effect of a dry-type rectifier is so much larger, however, than the capacitance existing between anode and cathode of an electronic rectifier that when attempting to measure alternating voltages at radio frequencies, the alternating current flows through this capacitance rather than through the dc instrument.

16-9. Combination of Rectifier and Dc Vacuum-tube Voltmeter.[1-5]—Suppose that the alternating voltage which we wish to measure, or which is to give us relay operation, is not capable of furnishing even the small amount of current demanded for the operation of a dc instrument, such as the approximately 3 ma or 1.5 ma peak value in the circuits of Figs. 16-2 and 16-4, respectively. Even then, the principle of the circuits shown in Figs. 16-1 and 16-2 may be used. All that we have to do is to take the instrument itself out of the circuit and replace it by a resistance. The direct voltage developed across this resistance can then be measured by means of a dc vacuum-tube voltmeter, such as described in Chaps. X and XI. With such a scheme, it is evidently possible to increase the value of the series resistance many hundredfold, if so desired, so that the alternating voltage that has to be measured will have to furnish a current correspondingly smaller than in the direct-reading arrangements of Figs. 16-1 to 16-4. The basic arrangement of such a circuit is shown in Fig. 16-5. (If the voltage to be measured has a peak not exceeding the peak inverse voltage of the diode, the circuit in Fig. 16-1*a* can, of course, be used in place of the one shown in Fig. 16-5.) The voltage applied to the dc vacuum-tube voltmeter in the circuit shown in Fig. 16-5 is, of course, not a pure direct voltage but consists of alternate cycles of positive half waves. If there should be any doubt that such a voltage may cause erroneous readings of the dc vacuum-tube voltmeter, the introduction of a resistance-capacitance filter between the vacuum-tube dc voltmeter and the voltage to be measured will remedy this situation. The circuit is shown in

Fig. 16–6. The direct voltage appearing across the capacitor will be equal to the dc component of the voltage that we wish to measure; the ac component will be the smaller, the smaller the capacitive reactance of

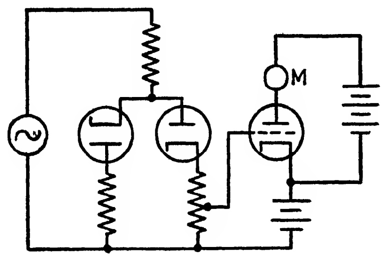

Fig. 16-5.—An alternating voltage may be observed with very small current drain by combining the rectifying action of the diode with a dc vacuum-tube voltmeter.

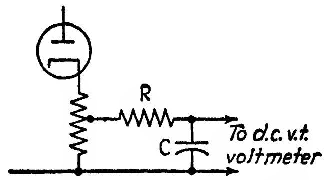

Fig. 16–6.—The ripple voltage appearing on the grid of the amplifier tube in Fig. 16–5 may be smoothed by a simple resistor-capacitor combination as shown here.

the capacitor is in comparison to the series resistor; in other words, the larger the time constant of the filter circuit.

16–10. Increase of Sensitivity by the Use of Capacitor Input.—Reference to Chap. VII discloses the fact that the circuit described in Sec. 16–9 is nothing more than the application of a rectifier circuit with resistive load to the measurement of an unknown alternating voltage. In Secs. 7–5 and 7–6 it was shown that, if the load of a rectifier consists of a capacitor, this capacitor will charge to the peak value of the positive half wave of the alternating voltage. This suggests immediately that, for the measurement of smaller voltages, an increase in sensitivity may be obtained by the use of this principle. The fundamental circuit of such an arrangement is shown in Fig. 16–7. The unknown voltage—in this case assumed so small that no by-pass for the negative half cycle is required—charges a capacitor to the peak value of the positive half cycle as outlined in Sec. 7–5. The direct voltage appearing across the capacitor is measured by means of a dc vacuum-tube voltmeter. A resistor is shown parallel to this capacitor in Fig. 16–7.

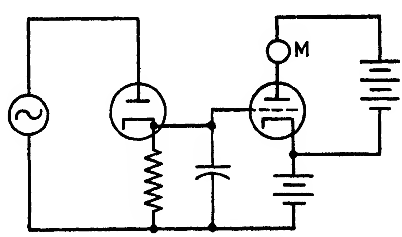

Fig. 16–7.—Increase of sensitivity may be obtained by using a capacitor across the output of the rectifier. This converts the circuit shown in Fig. 16–5 essentially into a peak voltmeter.

The reason for this is evident. If the dc vacuum-tube voltmeter is a truly zero current-drain instrument, the capacitor will remain charged to the peak value of the applied alternating voltage, even after the latter has been removed. A discharge path for this capacitor must therefore be

provided, but it is obvious that a very high resistance may be employed for this purpose. The current that the source will have to furnish for the purpose of measurement will therefore be extremely small. The circuit shown in Fig. 16–7 is the basis for many ac vacuum-tube voltmeters described recently in the literature. Incidentally, here, too, the influence of the initial speed of the emitted electrons in the diode is a disturbing and annoying factor; the capacitor may charge to as much as 1.5 volts with the input terminals of the instrument short-circuited. Ingenious schemes have been proposed for the cancellation of this spurious voltage. It may be canceled by the inclusion of a battery in the circuit or by the inclusion of an additional diode in the dc measuring circuit, furnishing a bias exactly equal to the one furnished by the measuring diode. The latter scheme has the advantage that, for an instrument operated from the ac line, variation of the initial speed of electrons caused by line-voltage variations will change both these voltages an equal amount and thus not disturb the balance of the instrument. Although the circuit shown in Fig. 16–7 is approximately three times as sensitive as if the capacitor were omitted (this statement holds essentially true only for sine-wave voltage), it must be kept in mind that the device shown in Fig. 16–7 is really a peak voltmeter. If the instrument is calibrated with a voltage of sinusoidal wave shape, its scale may, of course, be marked in terms of rms voltage. Its indication will then be correct only for voltages of sinusoidal wave shape. This subject will be discussed in more detail a little later.

16–11. Two Fundamental Methods of Using a Triode for Detection.— This chapter has dealt so far with the use of diodes only for the detection and indication of alternating voltages. The nonlinear part of the characteristics of a triode may also be used for this purpose. The triode can be used in two radically different ways to indicate the application of an alternating voltage to its grid by a change of plate current. The first method, called "plate detection," is based on the nonlinearity of the transfer characteristic; the second method, called "grid-leak detection," is based on the nonlinearity or, more correctly, on the rectifying action of the grid-cathode path of the tube.

16–12. Principle of Plate-circuit Detection.—The principle of plate-circuit detection is very easy to understand. In Fig. 16–8*a* let a triode with a transfer characteristic, as shown in Fig. 16–8*b*, be biased by means of a direct voltage to very nearly cutoff. A dc meter in the plate circuit of this tube will consequently register zero, or very nearly zero, current. Now let an alternating voltage be applied to the grid in series with the bias voltage. During the negative half cycle of the alternating voltage, the grid will become even more negative than the cutoff value. Since the current is already zero with the cutoff bias voltage, nothing is going to happen during this part of the cycle. But during the positive half cycle

of the applied alternating voltage, the actual grid voltage becomes less negative than E_{cc}, and the tube will therefore pass current during this part of the cycle. The plate current will consist of alternate half cycles, as shown in Fig. 16–8. If the transfer characteristic were a straight line down to the cutoff point, these half cycles would evidently be of sinusoidal wave shape and the dc meter in the plate circuit would indicate a value equal to the amplitude divided by π. This condition will be closely approached if the amplitude of the alternating voltage is large enough to bring the operating point up quite far into the linear part of the transfer characteristic. We can then make at least an approximate estimate of what

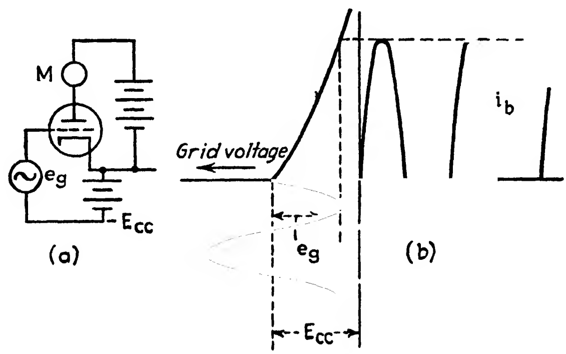

FIG. 16–8.—The principle of plate detection.

current can be expected in the dc instrument. The reader may check for himself the statement that the application of an alternating voltage with a sinusoidal wave shape and a certain rms value to the grid of the tube can at the very most produce a current change in the plate circuit equal to the transconductance of the tube multiplied by 0.45 times the rms value of the applied alternating voltage.

In the discussion of half-wave rectifier circuits, it was shown that the presence of an inductance in such a circuit leads to a decrease of the rectified current. In a similar way, if the circuit shown in Fig. 16–8 is to be used in connection with high frequencies, the inductance of the instrument may lead to an undesirable reduction in sensitivity. It is, therefore, always desirable to by-pass the instrument with a capacitor, the reactance of which is low at the frequencies at which measurements are to be made.

16–13. Range of Voltages for Plate-circuit Detection.—With a circuit as shown in Fig. 16–8, it appears that the amplitude of the alternating voltage applied to the grid must not exceed the cutoff bias if it is desired to keep the grid negative at all times. If voltages larger than this are to be measured, it would seem that a voltage divider will have to be placed across such voltages. A voltage divider will take current from the voltage under observation, however, and we are therefore giving up one of

the outstanding advantages of the vacuum tube. Although it is true that we have the choice of many tubes, differing greatly in their cutoff bias, this method of fitting the tube to the particular problem is at best only a makeshift.

The range of plate-circuit detection can be extended, however, very conveniently by the use of a cathode-follower circuit; by placing a resistor in the cathode lead, voltages with amplitudes in the order of 100 volts or more can be applied to the grid. At the same time, such a connection makes the characteristic much straighter. Detectors using this principle are sometimes called "linear" detectors. The connection is shown in Fig. 16–9. In some of the earlier literature vacuum-tube voltmeters using this principle were referred to as "reflex tube-voltmeters."

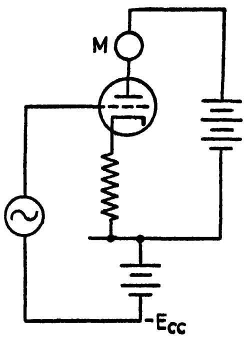

FIG. 16·9.—The inclusion of a cathode resistor extends the range of voltages for which plate detection may be used. Such a circuit was formerly called a "reflex" voltmeter.

It has been stated that as long as the grid remains negative with respect to the cathode, so that no grid current will flow, the alternating voltage under observation does not have to furnish any current. This statement is not correct for ac operation since even with the electrodes cold the grid will have a certain capacity to the cathode and anode, and with an alternating voltage applied to it, a capacitive current will flow from the source to it. This effect will ordinarily not be felt for power and audio frequencies but may become quite serious in the case of radio frequencies. Although it is true that such a current is a "wattless" current, being 90 deg out of phase with the applied voltage, it may upset the tuning of a radio-frequency circuit.

The lower limit of voltage that can be successfully detected by plate-circuit detection is obviously given by the curvature of the transfer characteristic. The more gradual the approach of this curve to the X axis, the less desirable will the tube be for plate-circuit detection. Operation on the curved part of the characteristic during the positive as well as the negative half cycle will be discussed in great detail later. For the detection of low voltages plate-circuit detection is not so desirable as the second method of detection, to be presented in the next section.

16–14. Combination of Diode and Amplifier Tube as Intermediate Step to Grid-leak Detection.—Grid-leak detection is based on a radically different principle. Experience indicates that it is considerably harder to understand than plate-circuit detection, although there is really no reason for this. In Chaps. VII and VIII, dealing with rectifiers and rectifier circuits, we usually placed the load, whether it was a resistor or a capacitor, between the cathode and one side of the source of alternating voltage (see, for example, Figs. 7–3 to 7–6). When two devices, whatever they may

be, are placed in series across a source of voltage, the current that will flow and the voltages that will exist across each one of the two devices will obviously not change if the two are interchanged. (Observe that "interchange" does not mean "reverse the polarity" of one or both of them.) Consider now the simple rectifier circuit shown in Fig. 16–10*a*. It will be noted that this is identical with Fig. 7–2 except for the interchange of the diode and its load resistor. Let us plot the potential of point H with respect to ground during one full cycle of the signal voltage. As in Chaps. VII and VIII, we shall assume the rectifier to be perfect. During the half cycle when point B becomes positive with respect to point A, current flows

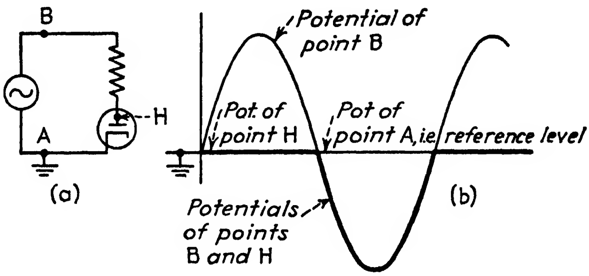

FIG. 16-10.—Point H will accompany B during the negative half cycle of the potential of B, but it will stay at the level of point A during the time that B swings positive. As a result, H has an average negative potential with respect to A.

through the resistor; during this time the voltage across the diode is zero, which means that H is at the same potential as A. During the half cycle when B becomes negative with respect to ground, no current will flow through the resistor and H will therefore be at the same potential as B. The graph showing the potential of H with respect to ground therefore consists of alternate negative loops, each consisting of a half sine wave. This condition is shown in Fig. 16–10*b*. There is another way of looking at this situation. The voltage of H with respect to ground consists of the sum of the voltage existing between points B and A and between H and B. The voltage between B and A is a pure alternating voltage and has no dc component. Across the load resistor, on the other hand, there exists a direct voltage due to the rectifier action of the diode. The voltage of H with respect to A must, therefore, have a dc component equal to the dc component of the voltage existing across the load resistor. Since we are dealing with a half-wave rectifier circuit, the dc component will be equal to the amplitude of the alternating voltage applied to this combination divided by π.

Now let us connect the voltage appearing between A and H (with the diode remaining in the circuit) to a triode operating in a reasonably linear part of its characteristic. Let H be connected to the grid of the amplifier tube and A connected either directly or in series with an appropriate bias

voltage to the cathode. With no alternating voltage across *A* and *B*, points *H* and *A* will be at the same potential, and the amplifier tube will show a plate current determined only by the bias voltage (if such a voltage is provided). With the appearance of an alternating signal voltage, however, *H* will swing negative during the negative half cycle of the signal, as shown in Fig. 16–10*b*. The plate current of the amplifier tube will therefore be *decreased* during this time. No corresponding increase will occur during the positive half cycle of the signal. A dc meter in the plate circuit of the amplifier tube will therefore show a decrease of current.

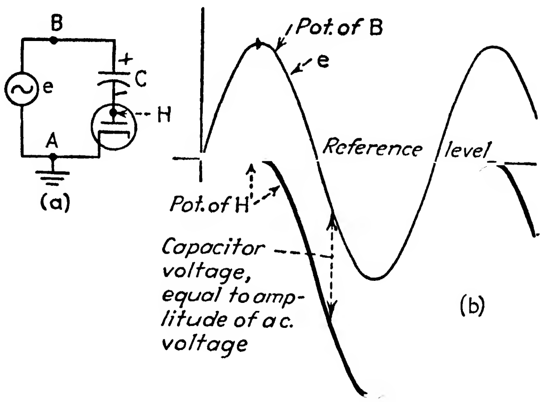

FIG. 16–11.—With the resistor of Fig. 16–10 replaced by a capacitor, point *H* will have an average negative potential with respect to point *A* equal to the peak of the alternating voltage.

If the resistor shown in Fig. 16–10*a* is replaced by a capacitor, we obtain a circuit as shown in Fig. 16–11*a*. How would the graph of the potential of *H* with respect to ground look now? Since this circuit is the same as the one shown in Fig. 7–3*a*, it is clear that the capacitor will charge to the peak value of the alternating voltage existing between *B* and *A*. In other words, during the first quarter cycle when *B* becomes positive with respect to *A*, or ground, current will flow through the capacitor; *H* will remain at ground potential. Since there is no discharge path provided for the capacitor, it will retain its charge from then on, and its action will be simply as if a battery with a voltage equal to the peak value of the alternating voltage had been placed in series with the latter. The potential of *H* with respect to ground is shown in Fig. 16–11*b*. The average or dc potential of *H* with respect to ground is now equal to the amplitude of the alternating voltage existing between *A* and *B*. It will be noted that neither in Fig. 16–10 nor in Fig. 16–11 does *H* ever become positive with respect to the cathode; in Fig. 16–10*a* this is prevented by the presence of a resistor between *B* and *H*. In Fig. 16–11*a* a capacitor serves the same purpose.

As in the circuit shown in Fig. 16–10, the potential of H with respect to ground could now be applied to the grid of a triode. Since the average potential of this point with an application of an alternating voltage at A and B will be equal to the peak of the applied alternating voltage, the plate current of the tube would decrease the same amount as it would if a direct voltage equal to the peak value of the alternating voltage were applied to the grid of the tube. The plate-current change obtainable with this circuit will evidently be π times as much as that obtainable with the circuit shown in Fig. 16–10*a* (linear operation of the amplifier tube assumed) because the direct voltages developed between B and H in Figs. 16–10*a* and 16–11*a* are in this ratio. If the reader has any doubt about this, he may reflect again on the circuits shown in Figs. 7–2 and 7–3, discussed in Secs. 7–4 and 7–5.

16–15. Principle of Grid-leak Detection.[6]—The potential of point H in the circuits shown in Figs. 16–10 and 16–11 was applied to the grid of an amplifier tube. This simply placed the anode of the diode parallel to the grid of the amplifier tube, and if zero grid bias were used in the connection described in Sec. 16–14, the two cathodes would also be in parallel. Now, the two electrodes grid and cathode in an ordinary amplifier tube form a rectifying path just as much as any two electrodes, one of which is emitting electrons. Consequently, we may just as well throw the diode out and let the grid-cathode path of the amplifier tube (triode or pentode) serve in place of the diode shown between H and A in Figs. 16–10 and 16–11. This gives us the circuit shown in Fig. 16–12, which is the fundamental one pertaining to grid-leak detection. With no alternating voltage between A and B, the grid of the tube will be essentially at cathode potential (or slightly negative, owing to the current produced by the electrons having sufficient initial speed); with the appearance of an alternating voltage between A and B, rectification will take place, as outlined in the preceding paragraph, and H will acquire an average potential more negative with respect to the cathode. It is a fortunate fact that a tube with zero grid bias is operating in one of the most linear portions of the transfer characteristic. Consequently, if the average potential of H becomes negative with respect to ground with an application of an alternating voltage between A and B, the plate current of the tube will be decreased. The fundamentals of grid-leak detection will be grasped at the very instant when it is realized that the grid of the tube is playing a dual role: (1) the grid-cathode path of the tube is used as a rectifier in exactly the same manner as the simple diode shown in Fig. 16–10*a* or 16–11*a*, and (2) the tube as a whole is used

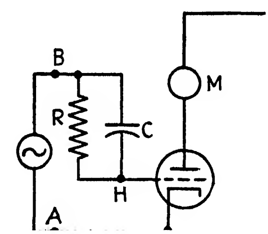

FIG. 16-12.—In grid-leak detection the grid-to-cathode path serves as a rectifier in exactly the same manner as the diode shown in Figs. 16-10 and 16-11. The tube then acts as an amplifier.

as an amplifier, amplifying whatever voltage variations appear on the grid, including those due to its rectifying action. In Fig. 16–12 a resistor as well as a capacitor is shown in series with the grid of the tube. With a resistor alone in series with the grid, the grid would show a potential with respect to the cathode exactly the same as H in Figs. 16–10a and b. If

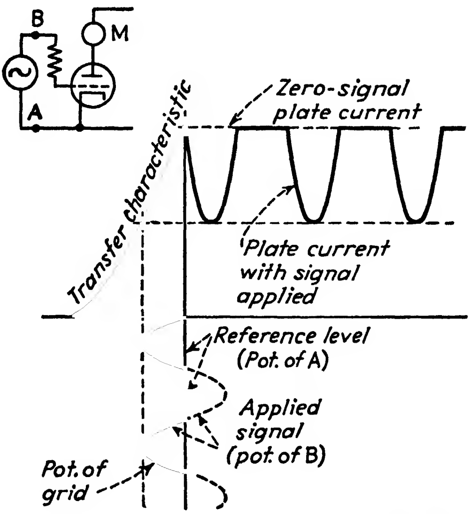

FIG. 16–13.—Grid voltage and plate current when only a grid resistor is used.

such a voltage, consisting of alternate negative half cycles of sinusoidal wave shape, is applied to the grid of an amplifier tube, the plate current will show variations as shown in Fig. 16–13. It will be noted that in this case the variation of the plate current from its steady zero bias value will be essentially proportional to the average value of the applied alternating voltage, provided that the tube is operating in the fairly linear part of the transfer characteristic. If, on the other hand, a capacitor only is used in the circuit shown in Fig. 16–12, it will charge to the peak value of the positive half wave of the applied alternating voltage. From then on the actual grid voltage simply consists of the alternating voltage developed between A and B and, in series with it, the direct voltage of the charged capacitor. The potential of the grid will be given by the same curve as shown for H in Figs. 16–11a and b. The resulting plate-current variations are shown in Fig. 16–14. The change of the dc component of the plate current is seen to be much larger in Fig. 16–14 than in Fig. 16–13, both figures having been drawn for the same amount of alternating voltage applied to the grid of the tube. This result is, of course, not at all surprising. When a half-wave rectifier works into a resistive load, the voltage appearing across this load will be proportional to one-half of the average value of the alternating voltage; when the same rectifier works into a capacitive load, the

voltage appearing across the capacitance will be equal to the peak value of the positive half wave of the alternating voltage. With a sinusoidal input voltage a grid-leak detector using a capacitance in the grid circuit will, therefore, give approximately three times as much plate-current change as with a resistor.

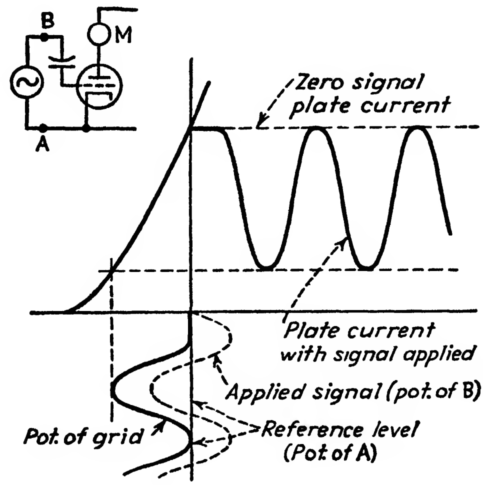

FIG. 16–14.—Grid voltage and plate current if only a capacitor is used in the grid circuit.

16–16. Importance of the Time Constant of the Capacitor-resistor Combination in Grid Circuit.—A pure capacity, not shunted by a resistor, cannot be used in the grid circuit, however, for the following very obvious reason. When the load of a half-wave rectifier—or full-wave circuit for that matter—is a capacitor, the direct voltage appearing across the capacitor will remain even after we cease to apply the alternating voltage to the circuit. Applying this to a grid-leak detector means that the reduction of plate current resulting from the application of an alternating voltage to the grid circuit will remain even after the removal of the alternating voltage. It is for this reason that the capacitor is always by-passed with a resistor, usually called the "grid leak." The direct voltage appearing across a capacitor shunted by a resistor will then follow exactly the same laws as discussed in connection with Figs. 7–4*a* and *b*. The time constant in the grid circuit of the tube is of the utmost importance to the radio engineer and deserves the attention also of the industrial electronic engineer. Let us assume that, tempted by the higher sensitivity of the large capacitor arrangement, he uses a capacitor of ½ μf and a resistance of 2 megohms parallel to it. The time constant of this combination is, therefore, 1 sec. With the application of a 60-cycle alternating voltage to the grid of the tube in series with this combination, a reduction of plate current will take place of the same size as if the bias of the tube had been made more negative by an amount equal to the peak voltage of the ap-

plied alternating voltage. Let us assume that the decrease in plate current amounted to 5 ma in this particular case. When the alternating voltage is suddenly reduced to zero, the plate current will not return immediately to the original value since the capacitor charge will have to leak off through the high resistance of the grid leak. In this particular example, approximately 37 per cent of the 5-ma reduction in plate current will still persist at the end of the first second after removal of the alternating voltage. If the device is used only for the *measurement* of an alternating voltage, the resulting sluggishness of the instrument might not be of too much consequence. For instantaneous *relay operation*, on the other hand, such a behavior might be decidedly undesirable.

This sluggishness may be put to use, however. We obviously have here an opportunity to obtain in a very simple manner a quick-acting but slow-releasing relay.

If the reader has clearly understood the discussion just presented, he should have no difficulty in realizing that in order to obtain the increased sensitivity afforded by the capacitor the time constant of the resistor-capacitor combination in the grid circuit must be large compared to one cycle of the alternating voltage to be applied to the instrument. In other words, the "ripple voltage" across the capacitor should be small, as discussed in detail in Secs. 8–8 to 8–10. When the time constant is chosen in line with this reasoning, then upon a sudden removal of the applied alternating voltage the plate current will not be able to return to its original value very fast. In an ordinary radio receiver we must "detect" an alternating voltage of very high frequency, which increases and decreases in magnitude at a frequency in the audio range. The time constant of the resistor-capacitor combination used in such circuits should, for maximum detection efficiency, therefore, be large compared to one cycle of broadcasting frequency, but it should be small compared to one cycle of the audio frequency. To go into greater detail on this subject is beyond the scope of this book.

16–17. Detection of Small Signal Voltages.—Our discussion of grid-leak detection so far has been based on the assumption that the grid-cathode path of the tube is a perfect rectifier; in other words, we assume that no current is flowing as long as the grid is negative with respect to the cathode, no matter how small an amount, and that as soon as it does become even the slightest amount positive with respect to the cathode, a current will flow. Actually, the grid-cathode path of a tube has a volt-ampere characteristic similar to that of any vacuum-type diode, such as shown in Figs. 7–9 and 7–13. The considerations arrived at in the preceding paragraphs will therefore apply only in those cases where the alternating voltage has an amplitude considerably larger than the voltage occurring across the rectifying element. The reader should refer to the discussion presented in Sec. 7–11 to understand the statement just made. When the

applied signal is small, on the other hand, it is necessary to make use of the grid-current–grid-voltage characteristic. This is not too difficult for a resistive load, in which case the construction shown in Fig. 7–13 applies, using the grid-voltage–grid-current characteristic and a load line corresponding to the grid-leak resistor; but if the load is capacitive, an exact solution, taking into account the grid-current–grid-voltage characteristic, is rather difficult and beyond the scope of this book. Furthermore, the detection of small alternating voltages is no longer of so much practical interest as it was in the early days of radio; if it is desired to convert a small alternating voltage into a direct current, the usual practice at present is to amplify the voltage first until it amounts to a few volts. Detection accomplished after sufficient amplification of the signal will be much more satisfactory because the output voltage, *i.e.*, the dc change, will obviously be related more linearly to the applied alternating voltage.

16–18. Alternate Connections to Obtain Grid-leak Detection.—The connection of the capacitor and the resistor in the grid circuit, as shown in Fig. 16–12, is not the only one that will produce the effect described in the preceding sections. In Fig. 16–15 the resistor is placed from grid to cathode instead of across the capacitor itself. When it is remembered that

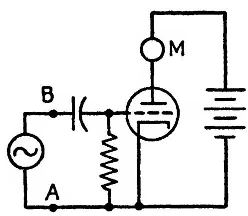

FIG. 16-15.—An alternate method of providing a discharge path for the capacitor so that the charge collected across it can leak off when the applied alternating voltage is reduced to zero.

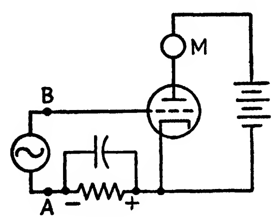

FIG. 16-16.—If none of the terminals of the alternating voltage is grounded, the grid capacitor and resistor may be connected as shown here.

the purpose of the resistor is only to permit the capacitor to discharge, after having been charged to the peak value of the alternating voltage applied to the grid, it is evident that the same action is taking place in the circuit shown in Fig. 16–15, provided that the dc resistance of the source is small compared to the resistance of the grid-leak resistor. The capacitor will simply discharge through the series combination of the source and the grid-leak resistor.

In the two circuits shown in Figs. 16–12 and 16–15, terminal A of the source was connected to the cathode of the tube; if the latter happened to be grounded, then A would also be grounded. In some circuits the voltage to be detected originates in a coil by mutual induction; in such cases the

designer does not necessarily have to ground one of the terminals, and a connection of the grid capacitor and the grid-leak resistor, as shown in Fig. 16–16, is usually more desirable. The exchange of the position of the source and the resistor-capacitor combination will of course not change the performance of the circuit in the slightest; as a matter of fact, we now are back to the identical connection of a diode and a load shown in Fig. 7–4*a*. The reader is urged to study the two diagrams carefully with this in mind. The only difference is the choice of the reference point. In Fig. 7–4*a* we chose as reference point the lower terminal of the source; in the detector circuits the cathode of the tube was chosen as the reference point. But the choice of a reference point does not, of course, affect the operation of a circuit.

16–19. Summary of Salient Features of Plate-circuit and Grid-leak Detection.—To summarize the salient features of plate-circuit detection and grid-leak detection, the following fundamental facts about these two types should be firmly fixed in the reader's mind. Plate-circuit detection makes use of the nonlinearity of the transfer characteristic; the voltage to be detected does not have to furnish any power (the charging current taken by the electrode capacitances is "wattless"); detection manifests itself in an *increase* of the plate current with the application of the ac signal to the grid of the tube. The increase in plate current occurring with the application of the ac signal does not bear a simple relation to the applied signal voltage. If the tube is biased to cutoff and the applied ac signal is fairly large so that the operating point swings well up into the linear part of the characteristic, then the increase in plate current is approximately proportional to the average value of the applied voltage. An instrument calibrated with a known wave shape and the scale of which is marked in terms of rms voltage will indicate correctly only if the unknown voltage is of the same wave shape as the calibrating voltage.

In grid-leak detection, a tube is operating in the linear part of the transfer characteristic; in other words, it is truly operating as an amplifier. Owing to the inclusion of a resistor or a capacitor in the grid circuit, rectification takes place in this part of the circuit exactly the same as if the grid-cathode path of the tube in question were a separate diode. The tube then merely amplifies whatever happens in the grid circuit. With grid-leak detection, the voltage to be detected must furnish a small amount of power, the load being represented by the resistor or capacitor, or both, in the grid circuit. The voltage rectified in the grid circuit is of such a polarity as to increase the negative grid bias; therefore, in contrast to plate-circuit detection, the plate current will *decrease* with an application of the alternating voltage to the grid. The decrease in plate current due to the application of an alternating voltage will be approximately proportional to the average value of the latter if only a resistor, not a capacitor, is used in series with the grid. When a capacitor-resistor combination with a time

constant well in excess of 1 cycle of the alternating voltage is employed, the decrease in plate current is proportional to the peak of the positive half wave of the applied alternating voltage. In other words, in the case of a sinusoidal voltage, the sensitivity of the circuit will be increased approximately threefold by using a capacitor-resistor combination in place of a resistor only. This advantage is purchased, however, at the price of now having essentially a peak voltmeter. Grid-leak detection is especially suitable if it is desired to detect small alternating voltages.

16–20. Effect of the Type of Nonlinearity on the Indications of an Ac Vacuum-tube Voltmeter.—All the instruments described in the preceding chapters produced a plate-current change, or a rectified current in the case of simple rectifier instruments, that was either essentially proportional to the average value of the alternating voltage, or proportional to the peak value; as a matter of fact, for small alternating voltages even this proportionality ceased to exist. So far, the nonlinearity of the transfer characteristics has been nothing but a nuisance. Even when the tube is used as a plate-circuit detector, we see that things would be much simpler if the transfer characteristic were a straight line clear down to the cutoff grid voltage. For if the transfer characteristic shown in Fig. 16–8*b* were a straight line and if the grid bias were placed exactly at the intersection point with the X axis, then the plate-current change would be proportional to the average value of the applied alternating voltage, no matter how small the latter might be.

The question naturally arises whether it is possible to arrange a vacuum tube in such a manner that the plate-current change will be an indication of the rms value of the applied alternating voltage regardless of the wave shape of the applied alternating voltage. It is for the solution of this problem that the curvature of the transfer characteristic is not without its blessings; if it is of the right kind, then it may be used to convert the tube into a device giving a plate-current change proportional to the square of the rms value of the voltage applied to the grid.

16–21. Conditions under Which a Tube Will Operate as an Rms Voltmeter.[7]—In Fig. 16–17 is shown the transfer characteristic of a hypothetical triode. In order to make the following discussion more clear, it was drawn with more curvature than would be encountered in an actual tube. Let us assume that the tube is biased to -3 volts, under which condition the plate current is 4 ma, as indicated by the transfer characteristic. Now let an alternating voltage with an amplitude of 2 volts be applied in series with this bias. The actual grid voltage will therefore vary between -1 and -5 volts. During the positive half cycle of the alternating voltage the plate current will increase to a maximum of 10 ma; during the negative half cycle it will decrease to 2 ma. The increase of plate current during the positive half cycle is therefore considerably larger than the decrease during the negative half cycle, and the meter in the plate circuit conse-

quently registers an increase of current. We now draw a tangent to the transfer characteristic at point P. If the transfer characteristic of the tube were this straight line ST instead of the actual curve, the application of an alternating voltage in series with the grid bias would simply produce an alternating current superimposed on the direct current of 4 ma, and the dc meter would keep on reading exactly 4 ma. We could consider the actual tube as consisting of two hypothetical tubes or devices, one of which has a characteristic as given by the tangent ST. The other tube or device (which would certainly have to be the product of a vivid imagination)

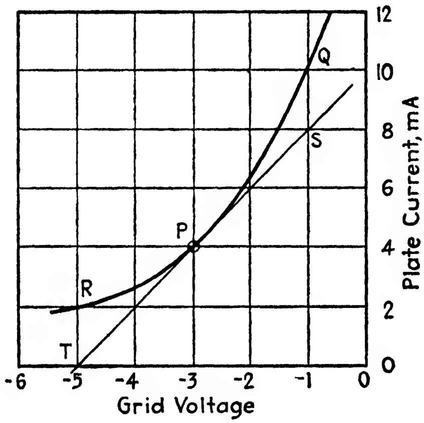

FIG. 16-17.—The curvature of the characteristic of a tube may be considered as the result of superimposing the characteristics of two fictitious tubes.

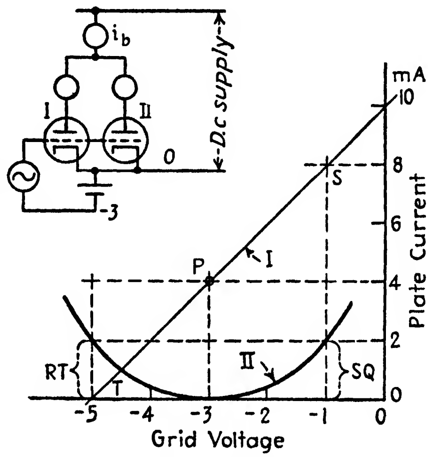

FIG. 16-18.—The two fictitious tubes or devices I and II, each having a characteristic as indicated in the graph shown above, will have a combined current exactly like the actual tube.

has a characteristic making up for the difference of the actual transfer characteristic and the tangent. This substitution of the tube by two separate devices with transfer characteristics as outlined above is shown in Fig. 16-18. The reader may convince himself easily that the total current taken by the two devices in parallel is exactly equal to the current taken by the actual tube. He should note that with a grid bias of -3 volts tube I takes 4 ma, and tube II takes 0 current. The meter measuring both currents together will therefore also show 4 ma. When the alternating voltage with 2 volts amplitude is now applied to the two tubes, the direct current in tube I is not going to change at all since its characteristic, given by the line ST, is entirely linear. In device II, however, which had zero current as long as only the bias voltage of -3 volts was applied, there will now be positive-current pulses during the positive as well as during the negative half cycle of the alternating voltage. The change of current indicated by the common meter is therefore due only to

the presence of device II. Let us assume that the transfer characteristic of the hypothetical device II, which was found by plotting the difference between the ordinate of the actual transfer characteristic and the ordinate of the tangent ST of Fig. 16–17, turns out as a parabola of the second order; in other words, that its ordinates are proportional to the square of the abscissas measured from the -3 point as the zero point. If, and only if, this is the case, the instantaneous value of current flowing in device II is proportional to the square of the instantaneous value of the alternating voltage applied to the grid. The average or direct current flowing in device II, which at the same time is the change of direct current taking place in the actual tube circuit, will therefore be proportional to the average of the square of the instantaneous values of the applied alternating voltage. The square root of the average of the square of an alternating voltage or current is known as its rms value; consequently, the dc plate-current change occurring in Fig. 16-18 will be proportional to the square of the rms value of the applied alternating voltage, regardless of the wave shape of the latter.

16–22. Determination of the Square-law Region.—In order to apply the principles outlined in the preceding paragraphs, it is necessary to find a region of the transfer characteristic where the drawing of a tangent and the plotting of the differences between the ordinates of the transfer characteristic and this tangent will result in a parabola as shown in Fig. 16–18. The mathematician would say that it is necessary to find that part of the transfer characteristic where the plate current follows a square law with respect to the grid voltage. A very convenient method of finding the region over which this condition holds true consists of plotting the transconductance of the tube against the grid voltage; where this relation is linear, the desired relation between plate current and grid voltage holds true. We shall now proceed to investigate this statement further.

In Sec. 9–12 it was shown that the transconductance is defined as the rate of change of plate current with grid voltage. Equation (9–4) reads as follows:

$$g_m = \frac{\Delta i_b}{\Delta e_c} \qquad (e_b = \text{constant}) \qquad (9\text{–}4)$$

From this definition it follows that the transconductance is simply the slope of the transfer or mutual characteristic of a tube. Examination of the transfer characteristic in Fig. 16–17 shows that the transconductance of this tube will be smaller at point R than it is at point P; at point Q it is larger than at P. At P the transconductance is evidently equal to the slope of the tangent ST. This happens to be 2 ma plate-current change for every volt of grid-voltage change, which means that the transconductance at P is 2,000 micromhos. If the transfer characteristic of the actual tube can be resolved over a certain region into the two compo-

nents I and II shown in Fig. 16–18, then the relation between instantaneous values of plate current and signal voltages e_s can be obtained by expressing the ordinates of the two characteristics I and II in Fig. 16–18 as a function of the signal voltage. (Note that the values of the signal voltage are measured from the grid-bias value of -3 volts, which is consequently the zero point for the signal voltage.) The graph of the plate current flowing in hypothetical tube I is a straight line and its equation is therefore given by

$$i_{\mathrm{I}} = a + be_s \tag{16–1}$$

The values of the coefficients a and b in Eq. (16–1) are easily found. For a signal voltage of zero the current must come out as 4 ma; the increase of current with every volt increase in grid voltage must be 2 ma. For the particular tube discussed in Figs. 16–17 and 16–18 we therefore have $a = 4$, $b = 2$.

We now turn our attention to hypothetical device II. If its graph is a parabola with the vortex at the grid voltage of -3, then its ordinates may be expressed by

$$i_{\mathrm{II}} = ce_s^2 \tag{16–2}$$

Since, for the case shown in Fig. 16–18, a signal voltage of $+2$ or -2 is seen to result in an ordinate of 2 ma on characteristic II, the value of coefficient c in Eq. (16–2) must consequently be 0.5. The plate current of the actual tube—if it does follow a square law for a certain region to the right and left of the selected operating point P—can therefore be expressed by

$$i_b = a + be_s + ce_s^2 = 4 + 2e_s + 0.5e_s^2 \tag{16–3}$$

The reader should convince himself that with signal voltages of 0, $+2$, and -2, this equation will yield 4, 10, and 2 ma for the plate current, which agrees with the ordinates of P, Q, and R of the actual characteristics.

The rate of change of plate current per volt *signal-voltage* change (which is, of course, the same as the rate of change per volt *grid-voltage* change since a 1-volt change of signal voltage means also a 1-volt change of actual grid voltage, measured from the cathode instead of the bias point) is given by the slope, or differential quotient, of Eq. (16–3); this is

$$g_m = \frac{di_b}{de_c} = b + 2ce_s \tag{16–4}$$

Equation (16–4) represents a straight line. Consequently, if for any given tube the transconductance is plotted as a function of the grid voltage and if this graph is reasonably linear over a certain region, then the plate current of the tube follows a square law over this region of grid voltages.

Not only does the reasonably linear part of this graph tell us the region

over which the tube follows a square law, but examination of Eq. (16–4) indicates that the slope of this linear part must be equal to $2c$. Figure 16–19 shows a graph of the transconductance of the hypothetical tube shown in Fig. 16–17. The reader will do well to study the relation between Figs. 16–19 and 16–17. At point Q of the original characteristic the plate current is changing at the rate of 2 ma for a grid-voltage change of ½ volt; the transconductance at this point, *i.e.*, at −1 volt grid voltage, is therefore 4,000 micromhos, or 4 millimhos. The reader should take additional points on the transfer characteristic shown in Fig. 16–17 and determine the slope of the curve at these selected points. Comparison with Fig. 16–19 will then disclose that the latter is actually a graph of the transconductance as a function of the grid voltage. The slope of the straight line shown in Fig. 16–19 is seen to be 1,000 micromhos, or 1 millimho per volt of grid-voltage change. According to Eq. (16–4) the coefficient c is therefore one-half this value, or 0.5 ma/volt². This agrees with the figure arrived at on the basis of Fig. 16–18.

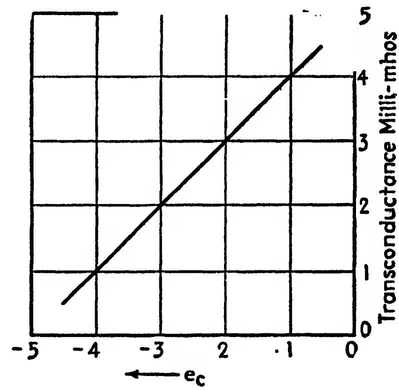

FIG. 16-19.—If the transconductance of a tube plotted as a function of the grid voltage turns out as a straight line over a part of the graph, the plate current will follow as square law for this region of grid voltages.

It must be remembered, however, that in Figs. 16–17 and 16–18 somebody, with a lot more intuition than we have, suggested the use of −3 volts as the operating bias of the tube, which turned out to be the correct choice. In actual practice, we usually have only the transfer characteristics of the tube at our disposal. By mere inspection of these characteristics, we are certainly not able to designate the region over which the tube current follows a square law, especially since in actual tubes this region is usually rather limited. But we can determine the slope of this curve, *i.e.*, the transconductance, for various values of grid voltage and plot it against the grid voltage. Better yet, if the actual tube and the necessary equipment are available, we can measure the actual transconductance by methods described in various books under measurement of tube coefficients and plot these figures of the transconductance against grid voltage. Examination of this plot usually shows that parts of it are reasonably linear; this will be the region over which the tube must be operated. The operating bias is chosen in the center of this region, and the amplitude of the alternating voltage to be applied in series with the bias voltage must not exceed a value that will bring the instantaneous grid voltage outside of the region just determined. The constant c in Eqs. (16–2) and (16–3)

can be obtained by determining the slope of the linear parts of the g_m vs. e_c graph, and dividing this value by 2.

16–23. Calculation of Performance of the Rms Tube Voltmeter.—Why should we wish to determine the coefficient c? In Figs. 16–17 and 16–18 we saw that the plate-current change occurring with the application of the alternating voltage was due only to the fact that hypothetical device II was beginning to show a direct current with the application of the alternating voltage. The instantaneous current in this device was given by Eq. (16–2) and is therefore determined only by the value of the coefficient c. The average current taken over one whole cycle of the alternating voltage is given by the average of the squares of the instantaneous values, multiplied with the coefficient c. But by definition, the average of the square of the instantaneous value of an alternating voltage or current is equal to the square of its rms value. Consequently, the plate-current change taking place with the application of an alternating voltage to the grid of a tube working in a region where the plate current follows a square law with respect to the grid voltage is proportional to the square of the rms value of the applied alternating voltage, and the proportionality factor between the two is equal to the coefficient c of Eq. (16–3). In the example treated in Figs. 16–17 to 16–19, the application of an alternating voltage with an rms value of 1 volt will produce a plate-current change of $\frac{1}{2}$ ma; if the alternating voltage is reduced to $\frac{1}{2}$ volt, the plate-current change will drop to $\frac{1}{8}$ ma, which is one-quarter of the plate-current change caused by the alternating voltage with 1 volt rms value.

Since the plate current before the application of the signal was 4 ma, it is evident that the changes of plate current that can be expected with such an arrangement are rather small compared to the steady value. Under such a condition, it is desirable to use a more sensitive meter and to buck out the steady component of the plate current by any one of the bucking or compensating circuits discussed in Chap. XI.

16–24. Analysis of Performance of Rms Vacuum-tube Meter for Sinusoidal Voltages.—It has been stressed repeatedly that this circuit will give a plate-current change proportional to the square of the rms value of the applied alternating voltage, regardless of the wave shape of the latter. If we are satisfied to study the performance of the circuit shown in Fig. 16–17 and its substitute shown in Fig. 16–18, under the condition that the applied voltage is of sinusoidal wave shape (as it is done in many books), then the analysis becomes even simpler. Consider the two fictitious tubes I and II and their corresponding transfer characteristics in Fig. 16–18. Let an alternating voltage of sinusoidal wave shape be applied to the grid of both of these devices. The current flowing in tube I will have a steady component equal to 4 ma and superimposed on it an alternating current of sinusoidal wave shape with the same frequency as the alternating voltage applied to the grid of the tube. In special tube II an alternating cur-

rent will flow, the instantaneous values of which are proportional to the square of the instantaneous values of the applied alternating voltage. It is well known that if we square the ordinates of a sine wave, a sine wave of double frequency will result; this sine wave is displaced upward, however, by an amount equal to one-half of its amplitude, which is therefore also its average value. When an alternating voltage of sinusoidal wave shape with an amplitude of 2 volts is applied to fictitious tube II, the current flowing in it will be given by Eq. (16–2), with $c = 0.5$. In Fig. 16–20 the instantaneous values of signal voltage and plate current, as given by Eq. (16–2), are plotted. The average current is seen to be 1 ma, or one-half of the length SQ or RT in Fig. 16–17 or 16–18. Since the rms value of a sinusoidal alternating voltage with an amplitude of 2 volts is equal to $\sqrt{2}$, the plate-current change calculated by squaring the rms value and multiplying it with the coefficient c gives the same result, namely, 1 ma.

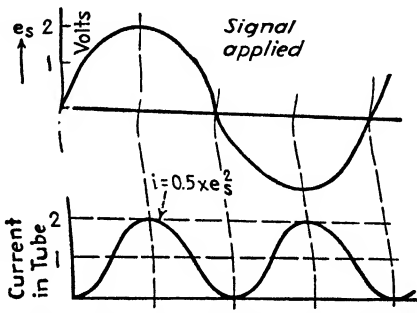

FIG. 16-20.—An alternating voltage applied to the grid of tubes I and II of Fig. 16-18 will result in a dc component in tube II as shown here.

16–25. Practical Need for an Rms Vacuum-tube Voltmeter.—In practice, the call for an instrument giving true rms voltage values is rather limited. The overwhelming majority of problems deal with voltages of sinusoidal wave shape, and since the relations among average value, peak value, and rms value of such a wave are known, an instrument giving any one of these three values will be satisfactory. In the few cases where the wave shape differs materially from a sine wave, the rms value is often not of much interest. Consequently, the attention that we have given to this subject seems to be greatly in excess of what it deserves. In fact, there is only one reason why so much trouble has been taken to show the procedure of designing a vacuum-tube voltmeter responsive to the true rms value of an alternating voltage: namely, the vacuum-tube wattmeter. The vacuum-tube wattmeter is at present hardly anything but a laboratory instrument, but since it offers the only method of measuring directly the alternating wattage in very low-power circuits, or when high frequencies are involved, it deserves to be discussed here. And, since fundamentally it is nothing but the combination of two rms voltmeters, the latter was discussed first.

16–26. Principle of the Vacuum-tube Wattmeter.[8–10]—In Fig. 16–21 is shown a source of alternating voltage connected to a load. Between the two is introduced a network of resistors consisting of the two equal resistors R and the resistors R_1 and R_2. These resistors should satisfy the

following condition: the load current i is passing through the equal resistors R, and these should act essentially like shunts. In other words, the voltage caused by the load current should be as low as possible and, in any case, should be small compared to the voltage of the source, or of the load. If this condition is fulfilled, the voltage between points D and G can be considered without much error as equal to the voltage existing across the load. The series combination of R_1 and R_2 should be of such high value that the current flowing through them is small compared to the load current itself. If this condition is satisfied, the voltages existing from A to D and from D to B will each be equal to iR. Let the ratio $R_1/(R_1 + R_2)$ be designated as p. The voltage existing between D and C will then be given by pe. In Fig. 16-21 the voltage of A with respect to C is designated as e_1, and the voltage of B with respect to the same point is designated as e_2. Each of these two voltages consists of two components, one of which is proportional to the *current* flowing through the load, while the other is proportional to the *voltage* existing across the load. Consider the half cycle of the alternating voltage when the source has a polarity as indicated in Fig. 16-21. If we assume for a moment that the load is resistive, then the current will certainly have a direction as indicated in this figure, and the polarity of the various voltages appearing across the different resistors will be as shown in this figure. The voltages e_1 and e_2 will be given by the following equations:

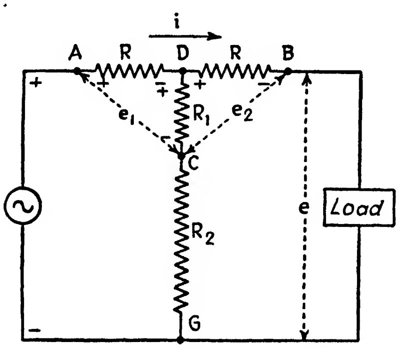

FIG. 16-21.—The fundamental relation pertaining to a vacuum-tube wattmeter.

$$e_{AC} = pe + iR = e_1 \tag{16-5a}$$

$$e_{BC} = pe - iR = e_2 \tag{16-5b}$$

Suppose that the two voltages e_1 and e_2 are applied to two identical vacuum-tube voltmeters operating in the region where the plate current follows a square law. The plate-current changes occurring in the two tubes will then be given by

$$\Delta i_{b_1} = c{e_{1\text{rms}}}^2 = c \times \text{average of } (p^2e^2 + i^2R^2 + 2pRei) \tag{16-6a}$$

$$\Delta i_{b_2} = c{e_{2\text{rms}}}^2 = c \times \text{average of } (p^2e^2 + i^2R^2 - 2pRei) \tag{16-6b}$$

The difference in the two plate-current changes will be given by

$$\begin{aligned} \Delta i_{b_1} - \Delta i_{b_2} &= c \times \text{average of } 4pRei \\ &= 4cpR \times \text{wattage} \end{aligned} \tag{16-7}$$

Equation (16–7) reveals the astonishing fact that the difference of the two plate-current changes is proportional to the wattage consumed in the load.

It is not necessary to employ two instruments to make use of this circuit. If it is possible to connect a single instrument in such a way that it records the difference of the two plate currents or a value proportional to it, then its indication will evidently be proportional to the wattage. Figure 16–22 shows the circuit incorporating these principles. It will be noted that the two tubes are arranged in a bridge circuit very similar to the Wold-Williams circuit shown in Fig. 11–12 and discussed in Sec. 11–11. When the circuit shown in Fig. 16–22 is analyzed in more detail with respect to the current that will flow through the meter, it will be found that, similarly to the circuit shown in Fig. 11–12*a*, the current through the meter cannot possibly exceed more than half of the difference between the two plate-current changes. Even this limiting value can be obtained only if the resistance of the instrument is zero.

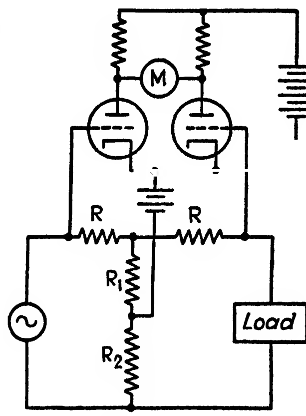

Fig. 16–22.—A vacuum-tube wattmeter circuit incorporating the principle outlined in Fig. 16–21.

16–27. Design Considerations of Vacuum-tube Wattmeter.—In designing the resistor network needed in connection with a vacuum-tube wattmeter, it is necessary to watch carefully that the two tubes are kept operating in the region where the square law holds true. This means that the amplitudes of the two voltages e_1 and e_2 must be kept below a value that would swing the grid of either tube out of the desired region. Suppose, for instance, that investigation of a certain type tube has revealed that its characteristic follows the square law between grid voltages of -2 and -5 volts. The proper grid bias would then be -3.5 volts, and the amplitude of the voltage applied to either tube must not exceed 1.5 volts. If we can assume that the instrument is to be used predominantly on currents and voltages with a nearly sinusoidal wave shape, we shall be on the safe side if we limit the rms value of the two voltages e_1 and e_2 to 1 volt (which makes the amplitude 1.4 volts). By methods of differential calculus, it can be shown that the maximum sensitivity of the instrument will be obtained when the voltages existing between points A and D and between D and C are alike. For instance, if it is required to design the instrument for a maximum load current of 50 ma and a voltage of 20 volts, we shall make the two shunt resistors r each 10 ohms, which will result in a voltage of $\frac{1}{2}$ volt from A to D and from D to B. The voltage divider R_1R_2 will also have to give us a voltage of $\frac{1}{2}$ volt across R_1; the value of p will therefore have to be $1/40 = 0.025$. The actual values of the two

resistors are not of much importance as long as the current flowing through this voltage divider is small compared to the load current. For instance, if we decide on a current of 0.1 ma through this branch (certainly small compared to the 50 ma load current), R_1 would have to be 5,000 ohms, and R_2 would have to be 195,000 ohms.

The double triodes, such as the 6SN7, are particularly suited for the construction of a vacuum-tube wattmeter. It is necessary, however, to take the characteristics of several tubes and select one, the two sections of which are closely matched. Even then, it is usually not possible to obtain perfect balance, and provision must be made to bring the meter reading to exactly zero. If the unbalance is not too large, it can sometimes be taken care of by the mechanical zero adjustment of the indicating meter. Otherwise, a flashlight cell with an adjustable resistor of 100,000 ohms or more may be placed across the meter with such polarity as to buck out the remaining unbalanced current.

Although these instruments are evidently suitable only for laboratory use since they require carefully selected and matched tubes and since their balance point may drift considerably, they represent the only means of measuring low wattages in the milliwatt and microwatt range. When they are properly designed, their accuracy is well within 5 per cent, which is certainly better than no measurement at all, or just guessing.

PROBLEMS

16–1. Three generators connected in series feed a load consisting of a resistor of 150 ohms and a capacitor of 10 μf, as shown in Fig. 16–23. What will the various meters

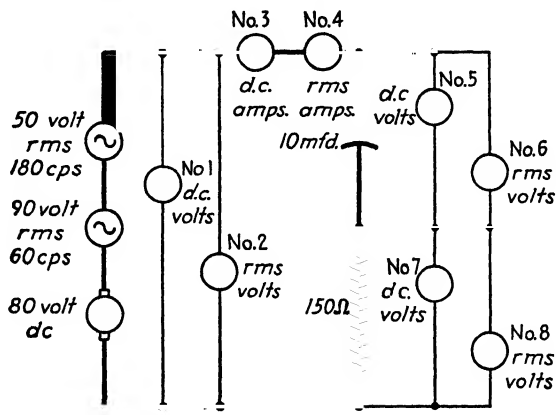

Fig. 16–23.—Circuit diagram for Prob. 16–1.

read? (It is assumed that the meters are perfect, *i.e.*, that the ammeters have no internal resistance, and the voltmeters have infinite resistance.)

16–2. A type 2A3 tube is operated from a supply voltage of 310 volts with a 1,000-ohm resistor in series with it. Included in the plate circuit are a direct current and a thermo-

couple milliammeter. The manufacturer gives the following values: plate voltage, 250; grid voltage, −45 volts; plate current, 60 ma. It is seen that with a bias of −45 volts the tube will operate in the above circuit with the recommended values of plate current and voltage. An ac signal of 15 volts rms and sinusoidal wave shape is applied to the grid.

What will the two meters read

a. Before the signal is applied to the grid?
b. While the signal is applied to the grid?

16–3. A storage battery needs a charge of 80 amp-hr. It is connected in series with a resistor and a half-wave rectifier to a sinusoidal alternating voltage, large compared to the battery voltage, so that the charging current can be considered without serious error as complete half waves of sinusoidal wave shape. Available is only an ac rms meter. Upon being connected into the circuit, it reads 7.2 amp. How long will it take to give the battery the required charge?

16–4. A dc instrument with a scale 0–100 has been combined with a tube acting as a half-wave rectifier and a series resistor to make an ac voltmeter. The series resistor has been adjusted to such a value that the application of 500 volts rms of sinusoidal wave shape results in exactly full-scale deflection. By mistake the instrument is used on a pure direct voltage. If it reads 80 on the scale, what is the actual direct voltage?

16–5. Available is a 2-ma dc instrument. It is desired to measure an alternating voltage of 500 volts rms and sinusoidal wave shape. Design a circuit accomplishing this, using a 6H6 double diode. The manufacturer states that the maximum voltage for one section should not exceed 117 volts rms.

16–6. A 110-volt dc relay with a coil resistance of 2,500 ohms is to be operated in the plate circuit of a triode-connected 6L6 tube (characteristics given in Fig. 9–16). Operation is to take place from an alternating voltage, and the circuit proposed for this purpose is shown in Fig. 16–24. The supply voltage for the tube circuit is 220 volts dc.

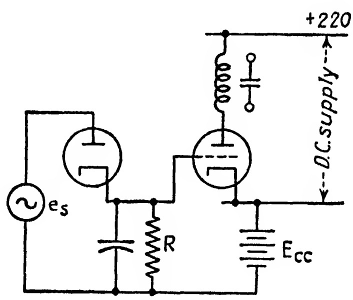

FIG. 16–24.—Circuit diagram for Prob. 16–6.

a. What bias voltage E_{cc} is required?
b. What rms value must the alternating voltage have to operate the relay? (Sinusoidal wave shape assumed.)
c. If the alternating voltage cannot furnish an average current in excess of 200 μa, how large must R be? What would be an appropriate value for the capacitor if the frequency of the alternating voltage has a minimum value of 60 cps?

16–7. A 6C5 tube (see Fig. 9–5) is operated from a supply voltage of 200 volts with a resistance of 10,000 ohms in series with it. The grid bias is −16 volts. A sinusoidal voltage of 10 volts rms is applied to the grid as shown in Fig. 16–25.

a. Plot the plate current for one cycle.
b. What will a dc meter in the plate circuit read?
c. Would a thermocouple milliampere meter, indicating rms value of current, read higher, equal, or lower than the dc instrument?

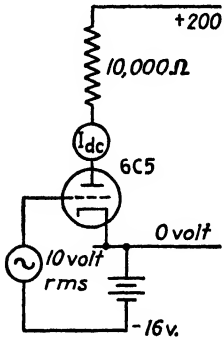

Fig. 16–25.—Circuit diagram for Prob. 16–7.

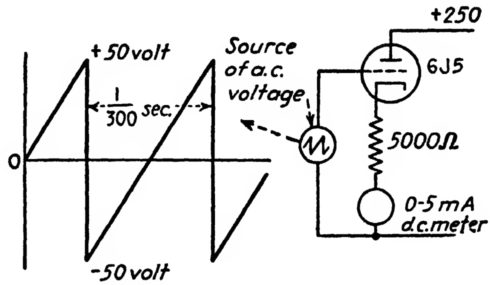

Fig. 16–26.—Circuit diagram for Prob. 16–8.

16–8. A saw-tooth voltage with a frequency of 300 cps and with a peak of 50 volts, as shown in Fig. 16–26, is applied to the grid of a 6J5 connected as shown in the same figure. (Plate characteristics given in Fig. 14–15.)

a. What will the dc meter in series with the cathode resistor read?
b. What would the same meter read if the voltage applied to the grid was a sine wave with a 50-volt amplitude?
c. What does the meter read with zero input voltage?

16–9. In the circuit shown in Fig. 16–27 the plate-supply voltage is an alternating voltage of sinusoidal wave shape with an amplitude of 120 volts. A sinusoidal alter-

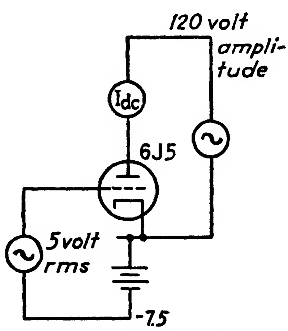

Fig. 16–27.—Circuit diagram for Prob. 16–9.

nating voltage of 5 volts rms is applied to the grid in series with the bias of −7.5 volts. What will the dc meter read, if the signal

a. Is in phase with the anode voltage?
b. Is 30 deg behind the anode voltage?

(By "in phase" is meant that the grid voltage reaches its positive maximum at the same instant as the anode voltage.)

16–10.

e_c	i_g μa
−1.0	0
−0.9	0
−0.8	0
−0.7	0.5
−0.6	1.6
−0.5	3.4
−0.4	7.5
−0.3	15.5
−0.2	28.0
−0.1	43.0
0	75
+0.1	102

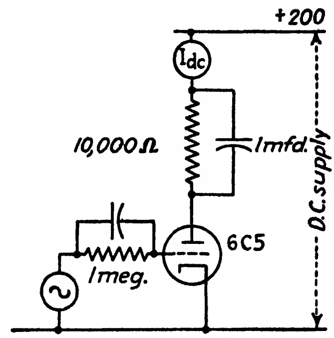

FIG. 16–28.—Circuit diagram for Prob. 16–10.

The grid-current–grid-voltage characteristics of a type 6C5 tube are shown in the accompanying table. The tube is connected in a circuit as shown in Fig. 16–28. Plate characteristics of the tube are given in Fig. 9–5. Assume the plate circuit to operate in the linear region, with r_p = 10,000 and μ = 20, as given by the manufacturer.

What will the plate current be before the application of any ac signal? What approximate plate-current change will take place with the application of 2.5 volts rms to the grid (sinusoidal wave shape), if the value of the capacitor in the grid circuit is such that its reactance is negligible compared to the 1-megohm resistor at the frequency of the signal applied to the grid?

What would the plate-current change be if the capacitor in the grid circuit was omitted?

16–11. On one section of a 6J6 the following values of plate current as a function of grid voltage were measured; the plate voltage was 105 volts.

e_c	−0.5	−1.0	−1.5	−2.0	−2.5	−3.0	−3.5	−4.0	−4.5	−5.0
i_b	13.5	10.5	7.7	5.6	3.88	2 5	1.48	0.81	0.5	0.25

The other section of the tube gave slightly higher values, but the curves coincided when the characteristic of the second section was shifted horizontally an amount equivalent to 0 25 volt. For this and the following problem, consider the two sections as identical. Design a vacuum-tube rms voltmeter, using both sections of the tube in parallel.

This problem can be considered as solved when the following questions are answered:

a. What is the peak of the alternating voltage that may be applied?
b. What grid bias must be used?
c. What range of dc meter would you suggest?
d. What will the calibration of the meter be?
e. What would be a suitable bucking arrangement?

16–12. Design a vacuum-tube wattmeter with the above type of tube, utilizing the two sections of the tube. The instrument is to show full-scale deflection when measuring the wattage consumed in a resistive load taking a current of 10 ma when connected to a voltage of 50 volts. (Current and voltage values are rms.)

A 0–200 μa dc meter with a resistance of 300 ohms is available. The instrument should be designed so that full advantage of the maximum permissible grid-voltage swing is taken, and that the balancing resistors in the plate circuits are of minimum value.

SUGGESTED ADDITIONAL READING

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CHAPTER XVII

PROPERTIES OF TUNED CIRCUITS

17-1. Range and Applications of Vacuum-tube Oscillators.—One of the most important uses of the vacuum tube is for the generation of alternating currents of any desired frequency. Vacuum-tube generators are usually referred to as "oscillators." We have seen that the vacuum tube as an amplifier was capable of controlling the flow of current in the plate circuit with a negligible expenditure of energy in its grid circuit. The basic principle of all vacuum-tube oscillators is to have a tuned circuit in one of the tube circuits, such as the plate circuit, and take the signal voltage required for the excitation of the grid circuit from the tuned circuit in the plate lead. The range of frequencies over which this principle can be applied is truly tremendous. Vacuum-tube oscillators have been constructed with frequencies as low as a few cycles per minute to as high as many million cycles per second.

Before presenting a chapter on vacuum-tube oscillators in this book, it may be desirable to show why the knowledge of this subject is of importance to the industrial electronic engineer. The use of high-frequency power was until recently essentially restricted to the broadcasting industry. In the past few years, however, high-frequency power has begun to play an increasingly important role in industry. Induction heating of metal parts and dielectric heating of nonconducting materials are the two principal applications requiring alternating currents of high frequency. Induction heating of metals can be accomplished with frequencies ranging from 60 to 500,000 cps, depending on whether the object is to be heated all the way through or whether it is desired to confine the heat to the outer layers, such as is required in surface hardening of gears and shafts. Dielectric heating of nonconducting materials, on the other hand, always requires frequencies of several million cycles per second.

But the industrial engineer is not justified in dismissing this subject if he does not happen to be concerned with any of these heating problems. Oscillators are often of tremendous importance, even if we are not at all interested in the ac power produced by them as such. In the first place, many measurements that the electrical engineer is called upon to make, such as the determination of inductance and capacitance, require a source of alternating current with a frequency most suitable for the particular problem. The ease with which a vacuum-tube oscillator can be constructed for various frequencies, and the relatively pure wave shape of the output,

make it well worth while to become familiar with the principles underlying vacuum-tube oscillators. Another important field of application arises from the fact that the frequency of the produced alternating current usually depends on the value of an inductance or a capacitor, or both; quite often it is possible to convert changes in mechanical quantities, such as displacement or pressure, into changes of capacitance or inductance. If a capacitance thus affected by a mechanical quantity is included in an oscillating circuit, the frequency of the generated oscillations will change with a change of the mechanical quantity; by measuring these frequency changes, information may be obtained on the mechanical quantity.

17-2. Classes of Vacuum-tube Oscillators.—There are many different methods by means of which a vacuum tube may be made to produce oscillations. An attempt is usually made to subdivide this field into several sections. Sometimes the subdivision is made on the basis of the circuit elements comprising the frequency determining network; thus there are oscillators using a combination of inductance and capacitance as the frequency-determining element, while others make use of a resistance-capacitance network. Another method of subdivision is based on the wave shape of the generated oscillations. When the wave shape is triangular or square, oscillators are sometimes referred to as "relaxation oscillators." Owing to the fact that regardless of the method of classification there is a continuous crossover from one category to the other, no attempt is made in this book to put every type of oscillator into its proper cubbyhole.

17-3. Comparison of Electrical and Mechanical Vibratory Systems.—The oldest and most important type of oscillator makes use of an inductance and a capacitance as the frequency-determining network. The performance of the combination of an inductance with a capacitance is so closely analogous to that of a combination of a spring and a mass that the reader is invited to study the following comparison of a mechanical and an electrical system very carefully. In order to obtain the maximum benefit from this presentation, only one sentence should be read in the left-hand column, and then immediately compared with the corresponding sentence in the right-hand column.

Let a capacitor with the capacitance C be traversed by the current i amp, or coulombs per sec, as shown in Fig. 17-1*a*.

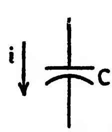

FIG. 17-1*a*. A current i of constant magnitude flowing through a capacitor will cause the voltage across the capacitor to rise at a uniform rate.

Let a spring with the spring constant k be stretched with the speed v ft per sec, as shown in Fig. 17-1*b*.

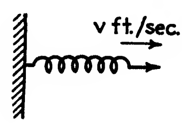

FIG. 17-1*b*. The stretching of a spring with uniform speed will cause the force exerted by the spring to build up at a uniform rate.

The voltage across the capacitor will rise at the rate

$$\text{roc } e = i\frac{1}{C} \qquad (17\text{-}1)$$

Let the current flow persist for t sec The voltage across the capacitor will then have built up to E_{max}, given by

$$E_{max} = t \text{ roc } e$$

$$= \frac{ti}{C} = \frac{Q_{max}}{C} \qquad (17\text{-}2)$$

because $ti = Q_{max}$ is the total charge

Electrical energy, or work, is now stored in the capacitor, which must be equal to the energy spent during the time that current was flowing and charging the capacitor

At the beginning of the period of current flow, the voltage across the capacitor was zero; at the end of the time t it had risen linearly to E_{max} The average voltage during the charging process is therefore $E_{max}/2$ and, since the current had the constant value i, the average wattage during the charging process was

$$P_{av} = i\frac{E_{max}}{2} \text{ (av watts)} \qquad (17\text{-}3)$$

The total amount of energy, or work, stored in the capacitor, is found by multiplying the average wattage by the time t; this results in

$$W = \frac{itE_{max}}{2} \text{ (watt-sec)}$$

$$= \frac{Q_{max}E_{max}}{2} \qquad (17\text{-}4)$$

Substituting into this equation the expression for E_{max} from Eq (17–2), we obtain

$$W = \frac{1}{2} \times \frac{1}{C} Q_{max}^2 \text{ (watt-sec)} \qquad (17\text{-}5)$$

The force exerted by the spring will increase every second by the amount vk, or

$$\text{roc } f = vk \qquad (17\text{-}1a)$$

Let the process of stretching last for t sec The force exerted by the spring will then have built up to F_{max}, given by

$$F_{max} = t \text{ roc } f$$

$$= tvk = s_{max}k \qquad (17\text{-}2a)$$

because $vt = s_{max}$ is the total amount that the spring has been stretched.

Mechanical energy, or work, is now stored in the spring, which must be equal to the energy spent during the time that the spring was being stretched

At the beginning of the period of stretching, the force exerted by the spring was zero; at the end of the time t it had risen linearly to F_{max} The average force during the process of stretching is therefore $F_{max}/2$ and, since the speed had the constant value v, the average rate of doing work during the stretching process was

$$P_{av} = v\frac{F_{max}}{2} \text{ (av ft-lb per sec)} \qquad (17\text{-}3a)$$

The total amount of energy, or work, stored in the spring, is found by multiplying the average rate of doing work by the time t; this results in

$$W = \frac{vtF_{max}}{2} \text{ (ft-lb)}$$

$$= \frac{s_{max}F_{max}}{2} \qquad (17\text{-}4a)$$

Substituting into this equation the expression for F_{max} from Eq. (17–2a), we obtain

$$W = \frac{1}{2} k s_{max}^2 \qquad (17\text{-}5a)$$

Suppose we connect an inductance L to the charged capacitor, as shown in Fig. 17–2*a* and then close the switch S.

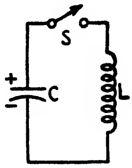

FIG. 17–2*a*.—When a charged capacitor is connected to an inductance, the current through the inductance will keep on increasing until the capacitor voltage has dropped to zero.

A current will build up through the inductance, starting out with zero and reaching a maximum value when passing through the zero charge (or voltage) condition on the capacitor.

While the current builds up, the charge (and voltage) goes down.

Suppose we connect a mass m, able to move freely, to the stretched spring, as shown in Fig. 17–2*b*, and then release the spring.

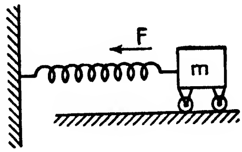

FIG. 17–2*b*.—When a stretched spring is attached to a mass capable of moving freely, the speed of the mass will increase until the spring no longer exerts any force.

The mass will accelerate toward the neutral position, starting out with zero speed and reaching a maximum speed when passing through the zero displacement (or force) position of the spring.

While the speed builds up, the displacement (and force) goes down.

A mathematical treatment, using differential equations, will show that the quantities involved rise and fall along sine waves, as shown in Figs. 17–3*a* and 17–3*b*.

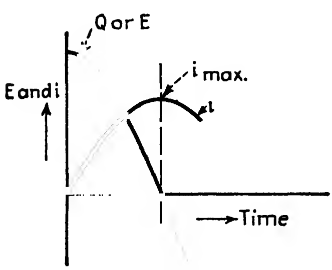

FIG. 17–3*a*.—Voltage across the capacitor and current through the inductance after switch S of Fig. 17–2 has been closed.

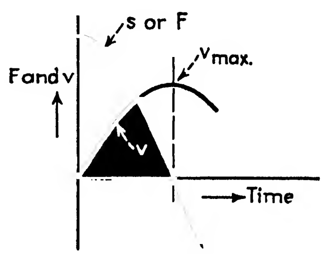

FIG. 17–3*b*.—Force exerted by the spring and speed of the mass after the latter has been released in Fig. 17–2.

At the instant when the capacitor is discharged, *i.e.*, when no voltage exists across it any more, all the energy originally stored in the capacitor must appear as magnetic energy in the inductance, carrying the current I_{max}.

At the instant when the mass passes through the neutral position, *i.e.*, when the spring is not exerting any force, all the energy originally stored in the spring must appear as kinetic energy of the mass m, moving with the speed V_{max}.

Any text on electricity gives the magnetic energy in an inductance as

$$W = \frac{1}{2} LI_{\max}^2 \text{ (watt-sec, or joules)} \tag{17-6}$$

By equating the energy originally stored in the capacitor, as given by Eq. (17-5), to the magnetic energy stored in the inductance, we obtain

$$\frac{1}{2} LI_{\max}^2 = \frac{1}{2} \times \frac{1}{C} Q_{\max}^2 \tag{17-7}$$

Solving for $I_{\max}$,

$$I_{\max} = Q_{\max} \sqrt{\frac{1}{LC}} \tag{17-8}$$

But if i follows a sine law, then the average current is

$$I_{av} = \frac{2}{\pi} I_{\max} = \frac{2}{\pi} Q_{\max} \sqrt{\frac{1}{LC}} \tag{17-9}$$

If we know the average current, we can find the time it takes to discharge the capacitor to zero.

Any text on mechanics gives the kinetic energy of a moving mass as

$$W = \frac{1}{2} mV_{\max}^2 \text{ (ft-lb)} \tag{17-6a}$$

By equating the energy originally stored in the spring, as given by Eq. (17-5a) to the kinetic energy stored in the moving mass, we obtain

$$\frac{1}{2} mV_{\max}^2 = \frac{1}{2} ks_{\max}^2 \tag{17-7a}$$

Solving for $V_{\max}$,

$$V_{\max} = s_{\max} \sqrt{\frac{k}{m}} \tag{17-8a}$$

But if v follows a sine law, then the average speed is

$$V_{av} = \frac{2}{\pi} V_{\max} = \frac{2}{\pi} s_{\max} \sqrt{\frac{k}{m}} \tag{17-9a}$$

If we know the average speed, we can find the time it takes for the spring to return to the neutral position.

This time is obviously one-quarter of the time of the whole sine wave. If T is the time for a whole cycle of the sine wave, we shall therefore have

$$\frac{T}{4} = \frac{Q_{\max}}{I_{av}} = \frac{Q_{\max}}{\frac{2}{\pi} Q_{\max} \sqrt{\frac{1}{LC}}}$$

or

$$T = 2\pi \sqrt{LC} \tag{17-10}$$

$$\frac{T}{4} = \frac{s_{\max}}{V_{av}} = \frac{s_{\max}}{\frac{2}{\pi} s_{\max} \sqrt{\frac{k}{m}}}$$

or

$$T = 2\pi \sqrt{\frac{m}{k}} \tag{17-10a}$$

The frequency of the oscillations will be

$$f = \frac{1}{T} = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \tag{17-11}$$

$$\omega = 2\pi f = \frac{1}{\sqrt{LC}} \tag{17-12}$$

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \tag{17-11a}$$

$$\omega = 2\pi f = \sqrt{\frac{k}{m}} \tag{17-12a}$$

17-4. Purpose and Use of the Comparison between Mechanical and Electrical Vibrations.—The foregoing comparison of mechanical and electrical vibrations was not presented for the purpose of entertainment or amusement. The displacement of a spring, the force exerted by it when stretched, the speed with which something moves, are quantities that most of us can visualize more easily than voltages, charges, currents, capaci-

tances, etc. If we can, with the aid of a comparison, fix in our minds the complete equivalence of certain electrical and mechanical quantities, then we shall have a powerful tool to analyze the electrical oscillation. From the analysis presented in Sec. 17–3, we see that

Current i	is equivalent to speed v
Voltage e	is equivalent to force f
Charge $q = it$	is equivalent to displacement $s = vt$
Inductance L	is equivalent to mass m
Capacitance C	is equivalent to reciprocal of the spring constant k
(A large capacitor)	is equivalent to (soft spring)

The reader should use the accompanying table to analyze in his own mind such phenomena as, for instance, the reversal of the voltage on the capacitor, after having passed through zero voltage (the force exerted by the spring reverses its direction after having passed through the zero or neutral position). Whenever any doubt exists about the electrical conditions, contemplation of the mechanical system will usually clear up such questions.

17–5. Effect of Friction and Resistance.—If no friction were present in the mechanical case or if no resistance were present in the electrical case, the oscillations would keep on forever. Since actually these two factors are always present, the amplitude of the oscillations will decrease with each cycle and the mechanical or electrical system will finally come to rest. If we wish to maintain the oscillations, it will be necessary to supply the oscillating system with an amount of energy during each oscillation equal to the losses caused by friction or other phenomena during one cycle or oscillation. A pendulum clock or a watch is an excellent example for this statement. In both cases a mechanism, usually called the "escapement" and operated by the pendulum, or the balance wheel itself, causes the mainspring to furnish a tiny amount of energy to the oscillating system, thus making up for the losses incurred during one oscillation. It is important that this energy is delivered to the oscillating system at the proper instant, or in the proper phase relationship. Everyone is familiar with the fact that a heavy church bell cannot be put into oscillation, nor kept swinging, by pulling indiscriminately at the rope, but that it is necessary to pull in synchronism and at the proper instant in each cycle. The escapement mechanism in a clock or in a watch automatically does this. In a vacuum-tube oscillator the dc supply source is the equivalent of the mainspring in a clock and furnishes a steady force in one direction. The pendulum or balance wheel finds its equivalent in the tuned circuit in a vacuum-tube oscillator, while the escapement mechanism is simulated by an electrical network connecting the oscillating circuit to the grid of the tube and releasing plate current flow at the proper instant in such a way as to maintain the oscillating circuit in this condition.

17-6. Inductance-capacitance Circuit Excited by an Alternating Voltage.[3] Before starting with the analysis of vacuum-tube oscillators, it may be found profitable to examine tuned circuits from the point of view of ac circuits. In the comparison between mechanical and electrical oscillations presented in Sec. 17-3 it was shown that, when a charged capacitor is connected to an inductance, an alternating current of sinusoidal wave shape will flow in this circuit, which means, of course, that the voltage across the inductance and the capacitance will also be of sinusoidal wave shape. It should be noted that there was no outside source of alternating voltage required to produce or maintain this alternating current. The frequency of these oscillations was given by the following:

$$\omega = \frac{1}{\sqrt{LC}} \tag{17-12}$$

By squaring Eq. (17-12), we obtain

$$\omega^2 = \frac{1}{LC} \tag{17-13}$$

which can also be written in the form

$$\omega L = \frac{1}{C\omega} \tag{17-14}$$

It will be remembered that ωL and $1/C\omega$ are equal to the inductive and capacitive reactance, respectively, of L and C if these two components were incorporated in a network in which alternating currents of the frequency $f = \omega/2\pi$ were flowing. We therefore see that the oscillation occurring when a charged capacitor is connected to an inductance takes place with a frequency at which the inductive reactance of the coil is equal to the capacitive reactance of the capacitor.

We have become so used to looking at inductances and capacitances from the point of view of ac circuits that the latter result seems perfectly natural. It should again be pointed out that nowhere was an ac generator in sight when we connected the charged capacitor, across which existed a *pure direct voltage,* to the inductance. It will be shown now, however, that we can arrive at the identical result by starting with an ac generator. In Fig. 17-4*a* let an ac generator be connected to a series combination of resistance, inductance, and capacitance. In Chap. IV it was shown that an alternating current will then flow in this circuit. The voltage across the resistance will be in phase with the alternating current, the voltage across the inductance will lead the current by 90 deg, while the voltage across the capacitance will lag the current by 90 deg. The voltages across the inductance and the capacitor are therefore 180 deg out of phase with each other, which means that the polarities of these two voltages are at any instant opposite to each other. This is true for any frequency. For

a frequency for which the inductive reactance $x_L = 2\pi fL$ is equal to the capacitive reactance $x_C = 1/2\pi fC$ these two voltages are not only opposing each other but are of equal size at any given instant. In Fig. 17-4*a* the polarities of the voltages are shown for an instant when the current has a direction as shown in this diagram and when it is increasing. If the generator operates at a frequency for which x_L is equal to x_C, the voltages between points *CD* and between *DA* are equal and opposite to each other. No voltage therefore exists between *A* and *C* and, as far as the generator is concerned, these two points could be connected by a jumper. It is evident that the generator has to furnish only a voltage sufficient to drive the

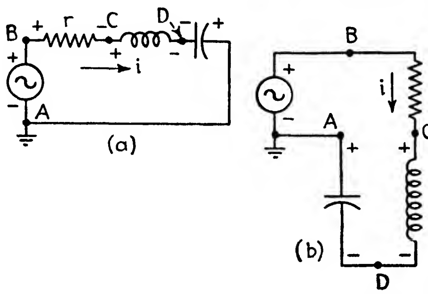

FIG. 17-4.—When an alternating current flows through a series combination of resistance, inductance, and capacitance, the polarity of the voltages existing across the inductance and capacitance are at any given instant of opposite sign. If they are of equal magnitude also, they will cancel each other completely, and the voltage required to drive the alternating current through the circuit is equal only to that required to drive it through the resistance.

alternating current *i* through the resistance *r*. The voltage that an ac generator will have to furnish in order to maintain a specified current in the circuit will therefore become less and less if the resistance of the circuit is decreased further and further. If we could reduce it to zero, the generator voltage could also be reduced to zero and the current would still maintain itself. In Fig. 17-4*b* the circuit of Fig. 17-4*a* is redrawn in our usual way to indicate polarities by heights. *A* and *C* are seen to be at the same level or potential. For a perfect inductance and capacitance and with the generator operating at the resonant frequency, *i.e.*, at the frequency for which inductive and capacitive reactance are equal to each other, *A* and *C* would always be at the same potential. We could connect them therefore by a jumper, which would divide the circuit in an upper and a lower part, the two being completely independent of each other. Under this condition it becomes still more evident that the generator has only to furnish the voltage necessary to drive the current through the resistance. Whatever current has been established in the lower part, however, would keep on flowing, even if we should remove the generator and the resistance above the jumper.

In practice it is of course impossible to construct a perfect inductance, *i.e.*, one without any resistance whatever. The resistance *r* indicated in

Figs. 17–4*a* and *b* is therefore usually a part of the inductance itself. Point *C* is therefore not accessible in an actual circuit, since the resistance *r* has moved into the inductance *L*, so to speak. We consequently cannot put a jumper from *A* to *C* and do away with the generator but shall need this generator to drive or maintain the current through the resistive part of the inductance.

17–7. The *Q* of a Coil.[1]—Let us assume that the inductance in Figs. 17–4*a* and *b* was 1 mh and the frequency of the alternating current such that the inductive reactance becomes 100 ohms. This would be the case with a frequency of approximately 16,000 cps. The capacitance must have a value of 0.1 μf in order for the capacitive reactance to become equal to 100 ohms also. Suppose that the resistance of the coil equals 1 ohm. Now, if we wish to maintain a current of 1 amp through the combination, the generator will have to furnish a voltage of 1 volt only, while the voltages across the inductance and the capacitance will be 100 volts. Our ability to make a high voltage appear across the inductance, or capacitance, in a tuned circuit depends therefore evidently on the ratio of the inductive reactance to the resistance of the coil. This ratio, sometimes called the "quality factor" of a coil, is of the utmost importance in the analysis and design of tuned circuits and is designated by the symbol *Q*. The coil in the above example has a *Q* of 100, for instance, which tells us that if this coil is used in connection with a capacitance in a circuit tuned to 16,000 cps, the voltage appearing across it, or across the capacitor, will be a hundred times greater than the voltage exciting the circuit.

17–8. Factors Influencing the Value of *Q*; Relation between Power Factor and *Q* of a Coil.—Since $Q = \omega L/r$, it would appear that the *Q* of a given coil will increase proportionally with an increase in frequency. In other words, the coil mentioned in Sec. 17–7 should have a *Q* of 200, if the frequency of the tuned circuit is raised to twice the original value. This is not the case, however, for the following reason. The resistance *r* of the coil, which in the above example was assumed as 1 ohm, is not necessarily the resistance that we measure with direct current. Owing to magnetic effects within the wire itself, an alternating current flowing through a wire is not distributed evenly over the whole cross section, as is the case with a direct current, but tends to flow in the outer layers of the wire. This phenomenon is known as "skin effect" and becomes more pronounced with an increase in frequency. At frequencies running into millions of cycles per second or higher, the alternating current is flowing almost exclusively in the outer skin of the wire; since the inner core of the conductor is therefore not helping to carry the current, it might just as well be considered as absent, which is evidently equivalent to reducing the cross section of the wire and increasing its resistance. In actual practice, it is quite often found that, owing to these effects, the ac resistance of a coil increases with frequency in about the same ratio as the inductive reactance does. It is

evident that under this condition the Q of the coil will remain constant over the range where this condition prevails. The Q of well-designed coils in radio receivers reaches a value of approximately 300.

It would seem that in low-frequency circuits (such as 60 cycles) inductances with a high value of Q could be obtained by the introduction of a laminated iron core in the coil. It should be pointed out, however, that the resistance r in Figs. 17–4*a* and *b* stands for more than simply the ohmic resistance of the coil; it symbolizes anything that causes losses to occur within the coil. The introduction of an iron core into a coil introduces eddy currents and hysteresis losses and is therefore equivalent to an increase of the resistance r. What values of Q can be expected in such a case will become apparent when we approach this problem from a slightly different point of view. We have seen that the Q of a coil was essentially determined by the component causing losses to occur in the coil; in ordinary ac theory, the power factor of a coil is also a figure conveying information on this subject. It is therefore justifiable to expect that there must be a relation between the power factor and the Q of a circuit component. We have for Q

$$Q = \frac{\omega L}{r} \tag{17–15}$$

The power factor of an ac circuit is equal to the cosine of the angle between the current and the voltage vector. It is therefore equal to the ratio between the resistance and the impedance of the circuit. We, consequently, have

$$\cos\varphi = \frac{r}{z} = \frac{r}{\sqrt{r^2 + (\omega L)^2}} \tag{17–16}$$

Squaring Eq. (17–16) and taking the reciprocal give us

$$\frac{1}{\cos^2\varphi} = \frac{(\omega L)^2 + r^2}{r^2} = Q^2 + 1 \tag{17–17}$$

and, consequently,

$$Q = \sqrt{\frac{1}{\cos^2\varphi} - 1} \tag{17–18}$$

In most cases of tuned circuits, the power factor of the inductive component of the tuned circuit is less than 0.1. Its reciprocal squared is therefore usually in excess of the value 100 and no serious error is committed when we neglect the 1 in Eq. (17–18). In almost all practical cases we can therefore write the following approximation:

$$Q \approx \frac{1}{\cos\varphi} \tag{17–19}$$

Equation (17–19) states the very important relation that the Q of a coil is approximately equal to the reciprocal of its power factor. It is a well-known fact that with a laminated iron core in a coil the power factor is usually in the neighborhood of 5 per cent, or 0.05. Equation (17–18) then tells us that the Q of such a coil would be approximately 20. The step-up of voltage obtainable in a low-frequency circuit by employing a tuned circuit is therefore considerably smaller than is possible with high-frequency circuits.

17–9. Excitation of Tuned Circuit by Mutual Inductance.—A tuned circuit in which the oscillating current is maintained by the introduction of a source of voltage in series with the inductance and the capacitance, as shown in Fig. 17–4, is called a "series-resonant" circuit. It must not be thought, however, that in order for this condition to exist we shall have to be able to point to a localized source of voltage. A very convenient and widely used method of introducing the required exciting voltage—*i.e.*, the voltage necessary to drive the oscillating current through the resistive or "loss" part of the circuit—is by mutual induction from a second coil brought in proper relation to the inductance in the tuned circuit. The circuit shown in Fig. 17–4 then becomes the one shown in Fig. 17–5. Let us assume that the mutual inductance M in Fig. 17–5 is 0.5 mh. The alternating voltage induced in one coil by an alternating current flowing in another coil, the two coils having a mutual inductance M with respect to each other, is given by

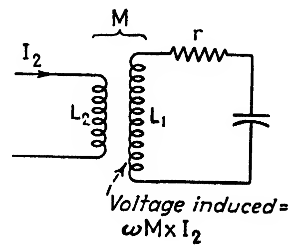

FIG. 17–5.—A voltage may be introduced into an ac circuit by mutual inductance from another circuit.

$$E_1 = \omega M I_2 \tag{17–20}$$

In the example treated in Figs. 17–4*a* and *b*, ω was equal to 100,000; ωM is therefore equal to 50 ohms. A current of 1/50 amp, or 20 ma, flowing in the coil L_2 at a frequency of 15,900 cps will therefore induce in the resonant circuit a voltage of exactly 1 volt. It should be pointed out, however, that the proximity of the coil L_2 also changes the apparent inductance and resistance of the resonant circuit. The subject of resonant circuits excited by mutual inductance is of the utmost importance to the radio engineer and will be found adequately treated in the better texts on radio engineering.

17–10. Excitation by a Source Parallel to the Tuned Circuit (Parallel Resonance).[2]—If the oscillating current in a resonant circuit is maintained by a source of voltage included in the circuit, as shown in Figs. 17–4*a* and *b*, this source must be able to furnish the full amount of oscillating *current*, but the *voltage* that it must supply is small compared to the voltages existing across the capacitive and inductive parts of the resonant circuit. The

losses occurring in the circuit are given by I^2r; this is also the power that the source of voltage must furnish. There is, however, another method available to maintain the oscillating current in a resonant circuit, and this is by applying the exciting voltage in parallel to the oscillating circuit, and not in series. This case is known as "parallel resonance" and is considerably more difficult to treat mathematically and to understand than is the case of series resonance. In Fig. 17–6 let the capacitor C and the inductance L with a resistance r be connected in parallel across a source of alternating voltage. Owing to the parallel connection the same voltage is now applied to the capacitive as well as to the inductive branch; the current in the capacitance will lead the applied voltage by an angle of 90 deg, while the current in the inductive branch will lag behind the voltage of the generator by an angle not quite 90 deg. If the inductance is perfect, *i.e.*, if the resistance in this branch is equal to zero, the current in this branch would lag exactly 90 deg behind the applied voltage. Let us assume that for a particular frequency the current through the inductance is 4 amp, while the current through the capacity is 1 amp. With a perfect inductance, the two currents would be exactly 180 deg out of phase, and the source of alternating current would have to furnish a current of only 3 amp. Now, suppose that we increase the frequency to twice its original value. This will double the inductive reactance of the coil and the current through this branch will, consequently, be reduced to 2 amp, while the capacitive reactance in the other branch will be cut in half. This results in doubling the current in that branch. Both branches would therefore now take a current of 2 amp and, since they are 180 deg out of phase (*i.e.*, when the instantaneous current through the inductive branch is, for instance, from point A to O, an exactly equal current flows through the capacitive branch from O to A), they add up to zero and the alternating-voltage source will not have to furnish any current whatsoever. It will have to furnish a voltage, however, equal to that desired across the inductance or capacitance. Take note of the fundamental difference between series and parallel excitation. A source used for series excitation of a perfect resonant circuit did not have to furnish any voltage, but it had to be able to pass the full oscillating current; in contrast to this, a source used for parallel excitation of a perfect resonant circuit will not be required to pass any current but must be able to furnish the full voltage existing across either component of the oscillating circuit.

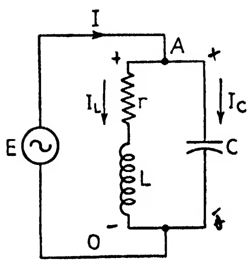

FIG. 17-6.—In the case of parallel resonant circuit, the source has to furnish less current than flows in each branch of the circuit.

17–11. Current Required from Source for Parallel Excitation.—How much current will the source in Fig. 17–6 have to furnish if the oscillating

circuit is not a perfect one, *i.e.*, if the inductive (or for that matter, the capacitive) branch contains a resistance as shown in Fig. 17–6? This question can be answered most conveniently and easily with the aid of the vector diagram of the voltages and currents of the circuit shown in Fig. 17–6. In Fig. 17–7 let the line OA represent the vector of the voltage supplied by the source in Fig. 17–6. This voltage will produce a current I_L in the inductive branch of the circuit, which will lag the voltage by an angle somewhat less than 90 deg. Let the current flowing in this branch

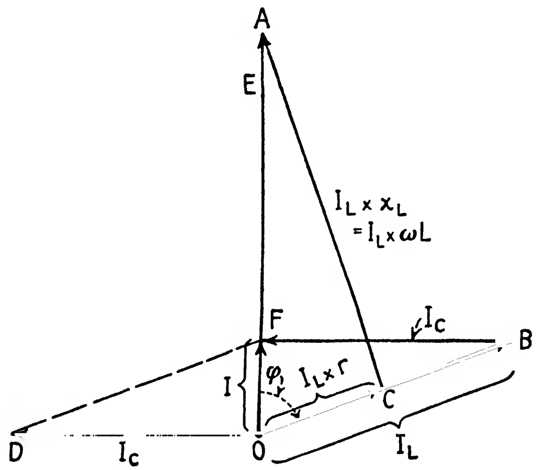

FIG. 17 7.—With the aid of this vector diagram, the current that the source must furnish to a parallel resonant circuit can be determined.

be shown by the vector OB in Fig. 17–7. This current produces alternating voltages across the resistance r and the inductive reactance x_L. These two voltages must add up vectorially to the total voltage OA. The voltage across r will be in phase with the current, while the voltage across the inductance will be 90 deg ahead of the current vector. In Fig. 17–7 OC represents the voltage across the resistance r, while the vector CA represents the voltage $I_L x_L$ existing across the inductance; these two vectors are seen to add up to the total applied voltage OA, as required. Now, let us turn our attention to the capacitive branch. The current in this branch will lead the voltage vector OA by 90 deg if the capacitor is perfect. The size of this current vector is evidently determined only by the size of the capacitance. The total current that the source must furnish is given by the vector sum of I_L and I_C. The vector OD must, therefore, be added to the vector OB. This is indicated in the vector diagram shown in Fig. 17–7. If the magnitude of the capacitance C is such that the current I_C, represented by the vector OD, is equal to the distance of the end point B of the vector I_L from the vertical vector OA, then it is evident that the total current that the source will have to furnish is given by the vector OF and that this vec-

tor will be in phase with the voltage vector OA. This means that, as far as the source of alternating voltage is concerned, the resonant circuit acts like a resistance. In the following the apparent resistance of a resonant circuit excited in parallel resonance will be designated by R.

17–12. Relation between the Apparent Resistance R of a Parallel Resonant Circuit and the Resistance r of the Inductance.[4]—We shall now study the relationship between the apparent resistance R, which a parallel resonant circuit seems to offer to the exciting voltage, and the resistance r of the inductance itself. Evidently there must be some kind of inverse relation between the two. If the resistance r in Fig. 17–6 is zero, *i.e.*, if the inductance is perfect, we saw that the alternating voltage would not have to furnish any current whatsoever. This is equivalent to saying that under this condition the resonant circuit would exhibit an infinite resistance to the voltage source exciting it in a parallel resonant circuit.

If a combination of circuit elements takes an alternating current that is in phase with the alternating voltage applied to it, its apparent resistance is evidently given by the ratio of the alternating voltage to the alternating current. In the case of the parallel resonant circuit shown in Fig. 17–6, the current taken by it is given by the vector OF, while the voltage applied to it is represented by the vector OA. Therefore, its equivalent resistance—or a resistance that will take exactly as much current as the parallel resonant circuit—is given by the ratio of the two vectors OA and OF. We, therefore, have

$$R = \frac{E}{I} = \frac{OA}{OF} \tag{17–21}$$

We shall now attempt to express the total current I in terms of the current I_L flowing in the inductance. The two triangles OAC and OBF are both right-angle triangles and have the angle φ in common. They are, therefore, similar and the following relation will hold:

$$\frac{I}{I_L} = \frac{r}{\sqrt{(\omega L)^2 + r^2}} \quad \text{and therefore} \quad I = I_L \frac{r}{\sqrt{(\omega L)^2 + r^2}} \tag{17–22}$$

The current I_L flowing in the inductive branch is given by the voltage E applied to this branch, divided by the impedance of it. We therefore have

$$I_L = \frac{E}{z_L} = \frac{E}{\sqrt{(\omega L)^2 + r^2}} \tag{17–23}$$

Substituting the value for I_L of Eq. (17–23) into Eq. (17–22) results in

$$I = \frac{Er}{r^2 + (\omega L)^2} \tag{17–24}$$

Substituting the value for I as given by Eq. (17–24) into Eq. (17–21) finally gives us the desired ratio that is equal to the apparent resistance of the parallel resonant circuit. We have

$$R = \frac{E}{I} = \frac{r^2 + (\omega L)^2}{r} \tag{17–25a}$$

Equation (17–25a) is one of the most important relations encountered in the analysis of parallel resonant circuits. It can be written in several different ways, as will be shown presently. At this point let it be noted that for $r = 0$, the value of the apparent or equivalent resistance R indeed becomes infinity as we had suspected.

The numerator of the fraction on the right-hand side of Eq. (17–25a) is evidently the square of the impedance of the inductive branch. Equation (17–25a) can therefore also be written in the form:

$$R = \frac{{z_L}^2}{r} = z_L \frac{z_L}{r} = z_L \cdot \frac{1}{\cos \varphi} \tag{17–25b}$$

In most resonant circuits r is less than 10 per cent of the inductive reactance ωL. It is obvious that, under such a condition, neglecting the square of r in the numerator of Eq. (17–25a) will lead to an error of only about 1 per cent in the calculation of the apparent resistance R. We then have

$$R \approx \frac{(\omega L)^2}{r} = \omega L \frac{\omega L}{r} = \omega L Q \tag{17–25c}$$

Equation (17–25c) again shows the inverse relation between R and r. The smaller the coil resistance r of the inductance, the higher will be the apparent resistance, if such a coil is shunted by a capacitor and connected to a source of alternating voltage with the resonant frequency. Equation (17–25c) also shows that this apparent resistance R is equal to the impedance or inductive reactance of the coil (there is less than ½ per cent difference between these two expressions if the resistance r is less than 10 per cent of the inductive reactance), multiplied with the Q of the coil.

There is one additional way in which Eq. (17–25a) can be written and which is sometimes preferable to the forms presented so far. The frequency with which a perfect resonant circuit comprising the inductance L and the capacitance C would oscillate is given by

$$\omega^2 = \frac{1}{LC} \tag{17–13}$$

If the value for ω^2 from Eq. (17–13) is substituted into Eq. (17–25c) we obtain

$$R \approx \frac{\omega^2 L^2}{r} = \frac{1}{LC}\frac{L^2}{r} = \frac{L}{Cr} \tag{17–25d}$$

The expression for R given by Eq. (17–25d) saves the trouble of calculating the resonant frequency of the combination. Note again that it shows the inverse relation existing between the apparent resistance R and the coil resistance r.

The reader should fix firmly in his mind that a parallel resonant circuit at the resonant frequency acts exactly like a resistance and that the value of this apparent resistance is given by any one of the Eqs. (17–25a) to (17–25d). Assume, for example, that the inductance L in Fig. 17–4 is 1 mh, the resistance of the coil 2 ohms, and the value of the capacitor 0.1 μf. The apparent resistance that this combination will have is, according to Eq. (17–25d), $R = L/Cr$, which, with the numerical values just given, results in 5,000 ohms. The resonant frequency can be found by substituting for L and C the correct values in Eq. (17–12). We obtain $\omega = 100{,}000$ or $f = 15{,}900$ cps. The inductive reactance of the coil or the capacitive reactance of the capacitor at this frequency turns out to be 100 ohms. The Q of the coil is seen to be 50; Eq. (17–25c) would give us then also an apparent resistance R of 5,000 ohms.

A more careful study of the parallel resonant circuit discloses that the above analysis is not quite correct. There is no question that a frequency exists at which the circuit will act entirely resistive, *i.e.*, a frequency for which the vector diagram shown in Fig. 17–7 is absolutely correct. This frequency is not, however, equal to the one given by Eq. (17–12), the frequency for which the inductive reactance of the coil is equal to the capacitive reactance of the capacitor. For oscillating circuits with a Q of 10 or higher, the difference between the frequency as obtained from Eq. (17–12) and the frequency at which the vector diagram shown in Fig. 17–7 is correct, is so small that the results based on Eq. (17–12) are sufficiently accurate for all practical purposes.

The resonant circuit in the above example will appear as a resistance of 5,000 ohms to a source of alternating voltage with a frequency of 15,900 cps. Two facts must be kept in mind firmly, however: (1) the circuit acts as a resistance of the given value *only* at the resonant frequency; (2) this resistance is a rather special one. When a self-respecting, legitimate resistance is disconnected from a source of voltage, the current through it and the voltage across it both cease to exist instantaneously. When a parallel resonant circuit is disconnected from the source of alternating voltage (to which it appeared exactly like a resistance, since it took a current in phase with the voltage), the current taken from the source will of course drop to zero instantaneously. The voltage across it, however, will keep on going with the resonant frequency, although the amplitude of successive cycles will become smaller and smaller, finally dying out after theoretically infinite time.

17–13. Voltage Relations of Two Coils Coupled by Mutual Inductance. The subject of mutual inductance was discussed in Sec. 2–3, and it has

been briefly mentioned in Sec. 17–9. Since a clear understanding of the voltage relations of two coils coupled by mutual induction is an absolute necessity for the analysis of oscillator performance, the inclusion of additional material in this chapter should help the reader in the study of oscillators in Chap. XVIII.

To the power engineer two coils coupled magnetically represent a transformer. When voltage is applied to one of them. the voltage appearing across the second coil is usually considered by him as given by the turns ratio, or at least very nearly so. This is the first thing we have to unlearn; it is true only when the magnetic flux passes in full strength through both coil . In the case of iron-core transformers the flux is usually confined to the iron core; therefore the two voltages are nearly in the ratio of the number of turns. This is decidedly not the case for two coils arranged close to each other in air and may also not be the case for an iron core, if the designer has deliberately provided large leakage paths. We shall then have to go back to the fundamental concepts of mutual and self-inductance, if we wish to calculate the voltages appearing across the two coils.

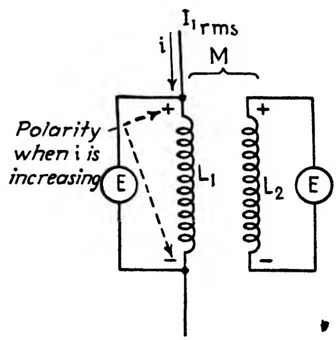

FIG. 17–8.—The voltages produced by self-inductance and by mutual inductance are of the same phase and polarity since both are produced by the rate of change of current in one coil.

In Fig. 17–8 two coils are shown side by side. Let us assume that they are actually wound on top of each other, with the same direction of the winding, and that the method of showing them side by side is used only as a matter of convenience. In other words, when there is a magnetic field in the direction from bottom to top in the one coil, it will have the same direction in the other coil. Let an alternating current flow through coil 1. The voltage appearing across this coil—if it has no resistance—will at any instant be equal to the self-inductance L_1 in henrys multiplied by the rate at which the current changes at this instant (remember Ohm's law for the inductance). When the current is increasing, for instance, in the direction as shown in Fig. 17–8, the voltage across coil 1 will have a polarity as indicated. The voltage across coil 2 will be equal to the rate at which the current changes in coil 1, multiplied by the *mutual* inductance in henrys. It is therefore at any instant of the same polarity as the voltage appearing across coil 1, and always in the ratio M/L_1. When a sinusoidal alternating current with an rms value I_1 and a frequency f flows through coil 1, we saw in Sec. 4–4 that the voltage appearing across this coil will have an rms value of $\omega L_1 I_1$ and will be leading the current by 90 deg. Owing to the condition outlined above, the voltage appearing across coil 2 —with no load connected to it—will therefore be $\omega M I_1$ (since it is always

in the ratio M/L_1) and will have the same phase as the voltage appearing across coil 1.

Coil 1 was assumed to have no resistance. Now let it suddenly acquire some resistance r. The magnetic field produced by a coil with or without resistance is exactly the same, as long as the current remains the same; therefore the voltage appearing across coil 2 will not change in either magnitude or phase: it will still be given by $\omega M I_1$ and will lead the current flowing in coil 1 by 90 deg. But the voltage appearing across coil 1 will now not only become larger but also change its phase, since it is the sum of the voltage $\omega L_1 I_1$ and, at right angles to this voltage, the voltage rI_1. These relations are presented in the vector diagram shown in Fig. 17–9. It is seen that the voltage across coil 2 now leads the voltage across the excited, or primary, coil. This angle of lead will be the smaller, the less the resistance r of the primary coil. (Let the reader be reminded that, although we have used the current in the primary coil as the starting point of this analysis, we have again solved the reverse problem too. With a voltage OC applied to the primary coil, the current flowing in this coil will be OD, and the voltage appearing across the secondary coil will be OA.) To summarize the discussion of this section: When an alternating current flows through a coil or when an alternating voltage is applied to a coil and a second coil is magnetically coupled to the first coil, the voltage across the second coil will lead the voltage across the first coil by an angle determined by the Q of the first coil; the higher the Q, the more nearly will the two voltages be in phase, and the more nearly will their values be in the ratio M/L_1.

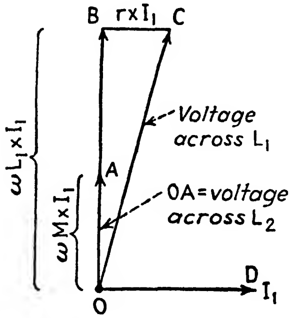

FIG. 17-9.—If the resistance of the coil carrying the inducing current cannot be neglected, the terminal voltages of the coils L_1 and L_2 of Fig. 17–8 are not exactly in phase any more.

PROBLEMS

17–1. A weight of 20 lb is suspended from a helical spring as shown in Fig. 17–10. It is found that a pull of 4 lb (in the downward or upward direction) causes a displacement of 3 in. from the position of rest (downward or upward, respectively). If the weight is pulled 6 in. from its position at rest and then released, with what frequency will it oscillate?

What speed will it have when it passes through the original (or neutral) position?

What will the kinetic energy be at the moment when it passes through the neutral position?

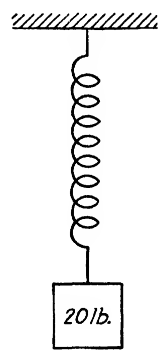

FIG. 17–10.—The mechanical oscillating system of Prob. 17–1.

Would the frequency increase or decrease if the displacement had been 8 in. instead of 6 in.?

17–2. A capacitor of 200 μf is charged to 100 volts. By closing switch S_1 (see Fig. 17–11) it is connected across a pure inductance of 5 henrys.

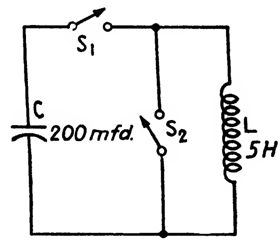

FIG. 17-11.—Circuit of Prob. 17–2.

a. If at the instant of complete discharge, *i.e.*, at the instant when the voltage across the capacitor and inductance has dropped to zero, switch S_2 is closed, what current would be flowing in the inductance?
b. What time will have elapsed between the closure of switch S_1 and the instant when the capacitor is discharged for the first time?
c. If the inductance had a resistance of 20 ohms, instead of being a pure inductance, what kind of voltage would have to be put in series with it and the capacitor to maintain the same current that would flow in the case of the pure inductance?

17–3. An air-core inductance of 0.4 henry has a resistance of 450 ohms. It is to be tuned to a frequency of 2,500 cps.

a. What size of capacitor must be used?
b. What resistance will the parallel combination have at the resonant frequency?
c. To keep an oscillating current of 0.15 amp flowing in the circuit, what generator voltage would be necessary for series excitation? for parallel excitation?
d. How much current will the generator have to furnish in the case of parallel excitation? What is the Q of the circuit?

17–4. The following observations were taken on a special transformer with negligible copper and iron losses. All measurements were taken with a frequency of 60 cps. 180 volts applied to winding 1 caused an open-circuit voltage of 67.5 volts to appear across winding 2. The current taken by winding 1 under this condition was 119.2 ma. With 180 volts applied to winding 2, this winding takes 53.1 ma.

Find the open-circuit voltage appearing across winding 1 in the second case (*i.e.*, with 180 volts applied to winding 2) as well as the values L_1, L_2, M, and the coefficient of coupling k.

17–5. Available is a calibrated oscillator with a range of 20 to 35,000 cps, and a mica capacitor with a known capacity of 0.01 μf. The inductance of a given coil is to be

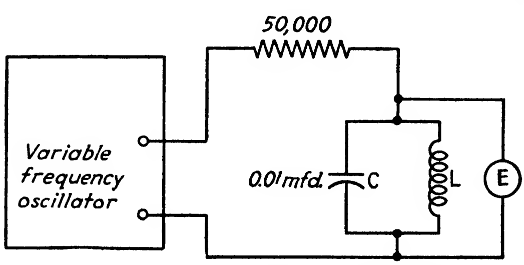

FIG. 17-12.—Circuit diagram of Prob. 17–5.

determined. Its dc resistance is 120 ohms. The coil is connected in a circuit as shown in Fig. 17–12. The voltage across the parallel combination of coil and capacitor is observed by means of an electronic voltmeter having a resistance of 250,000 ohms. It is observed that this voltage reaches its maximum when the frequency of the oscillator is

3,980 cps. It is furthermore noted that at this frequency the meter shows that 60 per cent of the output voltage of the oscillator appears across the resonant circuit.

a. What is the value of the inductance?
b. What is the Q of the circuit?
c. What is the ac resistance (*not* reactance or impedance) of the coil at this frequency?

17-6. A coil of 25 mh inductance and 500 ohms resistance is shunted with a capacitor of 0.001 μf. This combination is placed into the plate circuit of a 6SF5 tube operated at rated values of plate and grid voltage.

a. What is the resonant frequency of the circuit?
b. What is the voltage gain of the stage at the resonant frequency?
c. What is the voltage gain at a frequency 20 per cent lower than the resonant frequency?

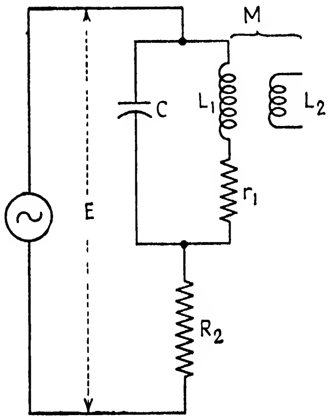

FIG. 17 13.—Circuit diagram for Prob. 17-7.

17-7. The following values apply to the circuit shown in Fig. 17-13: E, 100 volts; C, 0.02 μf; L_1, 80 mh; r_1, 50 ohms; L_2, 20 mh; k, 0.25 (coefficient of coupling between coils L_1 and L_2); R_2, 200,000 ohms.

a. What is the Q of the resonant circuit?
b. What is the resonant frequency of the circuit?
c. What voltage will appear across the resonant circuit when the applied voltage has the above frequency?
d. What voltage will appear across the second coil L_2 and what will its phase be with respect to the voltage E?

SUGGESTED ADDITIONAL READING

Everitt, W. L.: "Communication Engineering," Chap. 3, pp. 59-93, McGraw-Hill Book Company, Inc., New York, 1937.
Glasgow, R. S.: "Principles of Radio Engineering," Chaps. 2, 4, and 5, McGraw-Hill Book Company, Inc., New York, 1936.
Henney, K.: "Principles of Radio," Chap. 9, pp. 168-200, John Wiley & Sons, Inc., New York, 1945.
Terman, F. E.: "Radio Engineering," Chap. 3, pp. 49-89, McGraw-Hill Book Company, Inc., New York, 1932.

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1. Franks, C. J.: Values of Q for Various Component Parts, *Electronics*, April, 1935, p. 126.
2. Reed, M. B.: Frequency Response of Parallel Resonant Circuits, *Electronics*, August, 1941, pp. 43-44.
3. Eltgroth, G. V.: Frequency Stability of Tuned Circuits, *Electronics*, February, 1944, p. 118.
4. Teachman, A. E.: Equivalent Resistance Chart, *Electronics*, August, 1938, pp. 31-32

CHAPTER XVIII

INDUCTANCE-CAPACITANCE TUNED OSCILLATORS

18-1. Methods of Analyzing Oscillator Circuits.—After the discussion of the properties of series and parallel resonant circuits in Chap. XVII, we shall investigate what happens if such circuits are incorporated into vacuum-tube circuits. For the mathematical analysis of the combination of a resonant circuit and a vacuum tube there exists only one correct and unassailable approach: to set up the differential equations for the various currents in the tube and in the branches of the oscillating circuit. In order to be of help to that large group of readers unfamiliar with the methods of differential and integral calculus, let alone differential equations, we shall present this subject here by making use of certain simplified assumptions, so nearly fulfilled in actual cases that the results obtained by their use are an entirely satisfactory basis for the calculation of the performance of oscillating circuits.

18-2. Parallel Resonant Circuit as the Plate Load of a Tube.—In Fig. 18-1 a parallel combination of inductance and capacitance is placed in the plate circuit of a tube. Let an alternating voltage of variable frequency, but constant amplitude, be applied to the grid. An alternating voltage will then, of course, appear across the *L-C* combination. If the voltage applied to the grid is so small that the tube is operating in the linear part of the characteristics, the magnitude of the voltage appearing across the load in the plate circuit can be calculated with the aid of the equivalent-plate-circuit theorem. In other words, the tube is replaced by a black box containing a battery and a resistance equal to the plate resistance of the tube. In series with the battery is placed a generator furnishing an alternating voltage μ times the voltage applied to the grid, and 180 deg out of phase with the latter. This converts the circuit into the one shown in Fig. 18-2. Since we are interested only in the alternating voltage appearing across the load, all direct voltages, *i.e.*, the two batteries shown in Fig. 18-2, can be considered as short-circuited.

It was shown in Secs. 17-11 and 17-12 that a parallel combination of L and C will at one particular frequency—the resonant frequency—act like a resistance R; with L, C, and the resistance r of the coil given, R can be calculated by any of Eqs. (17-25*a*) to (17-25*d*). For any other frequency, the impedance of the *L-C* combination is less. The alternating voltage appearing across the plate load in Fig. 18-1 will therefore reach a maximum when the signal applied to the grid is of the resonant frequency.

The magnitude of this voltage can be determined very easily by assigning the value R to the plate load in Fig. 18–1 or 18–2. The reader should refer to Eq. (10–1), given in Sec. 10–6, where the equation for the voltage appearing across a resistance R_L placed in series with a tube is developed.

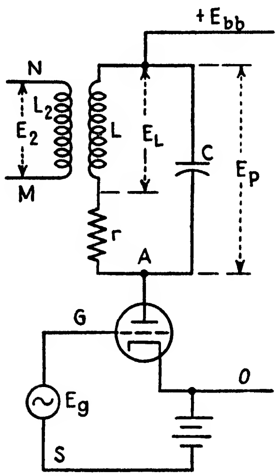

FIG. 18-1.—A parallel resonant circuit placed in the plate circuit of a tube acts like a resistance when the voltage applied to the grid is of the resonant frequency.

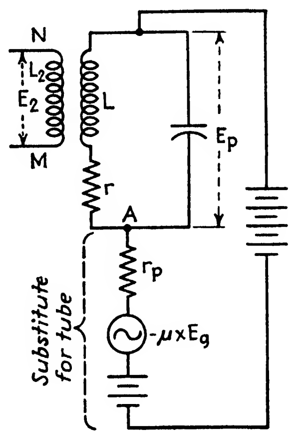

FIG. 18 2.—The performance of the circuit shown in Fig. 18–1 may be analyzed with the aid of the battery-resistance substitution for the tube.

Replacing R_L of this equation with the apparent resistance R of the parallel resonant circuit, we obtain

$$E_p = \frac{\mu E_g R}{R + r_p} \tag{18–1}$$

(Of course, as far as the dc component is concerned, the circuit does not act like a resistance R but has only the value of the coil resistance r. In most oscillating circuits the coil resistance is of the order of a few ohms, so that the quiescent plate current is practically equal to the current that would flow with the resonant circuit short-circuited.)

Having determined the alternating voltage E_p that will appear across the resonant circuit when a voltage E_g of the resonant frequency is applied to the grid [see Eq. (18–1)], we can, if we desire, calculate the amount of current that will flow in the right-hand and the left-hand branch of the parallel resonant circuit. As far as the phase relationship of the ac potential of point A and the ac potential of the grid G is concerned, these two are 180 deg out of phase just as they were with any resistive load. In other words, when point G is swinging up, point A is swinging down. As an example, let the resonant circuit discussed in the last few paragraphs

of Sec. 17–12 be connected to a tube with an amplification factor of 15 and a plate resistance of 10,000. This circuit, it will be remembered, resonated at about 16,000 cps and acted like a resistance of 5,000 ohms at this frequency. Let it be assumed that a voltage of 5 volts and of this frequency is applied to the grid of the tube. Replacing the tube with a black box containing a resistance of 10,000 ohms permits us to calculate the alternating current that will flow in the plate circuit. The alternating voltage acting in the fictitious circuit is equal to $5 \times 15 = 75$ volts, and the total resistance is equal to 15,000 ohms. The alternating current is, therefore, equal to 5 ma. The voltage across the resonant circuit (which acts like a resistance of 5,000 ohms) is consequently 25 volts. The impedance of the two branches of the parallel resonant circuit was found to be 100 ohms; with 25 volts applied to both of them the current in each one will be 250 ma, with a phase displacement of not quite 180 deg between the two currents, as shown in the vector diagram, Fig. 17–7. Note that the current circulating in the resonant circuit is fifty times as high as the current furnished by the vacuum tube. This is again equal to the Q of the coil.

18–3. Voltage Induced in a Coil Coupled to the Resonant Circuit.—Now let a coil L_2 be brought near the coil L in the resonant circuit. A voltage E_2 will appear across this coil, owing to the mutual inductance existing between the two coils. As shown in Sec. 17–13, for a reasonably high Q coil in the resonant circuit, the voltage across the second coil will be almost in phase with the voltage across the coil L, and its magnitude will be in the ratio M/L to the voltage appearing across L, *i.e.*, the voltage E_p. We therefore have

$$\frac{E_2}{E_p} = \frac{M}{L} \tag{18-2}$$

(It should be emphasized again that this relation is strictly true only when the resistance r of L is zero. The reader should examine the vector diagram in Fig. 17–9 where the above relation is seen to apply strictly to only the two values OA and OB. For a small resistance r, the length OC, representing the total voltage across the coil L_1 is so nearly equal to the length OB, representing the voltage across the inductive part of the coil only, that Eq. (18–2) gives a result accurate enough for all practical purposes.)

Solving Eq. (18–2) for E_2, and at the same time substituting for E_p the value given by Eq. (18–1), we obtain

$$E_2 = E_p \frac{M}{L} = \frac{\mu E_g R}{(R + r_p)} \frac{M}{L} \tag{18-3}$$

18–4. Self-excitation of the Resonant Circuit.[1–4, 7]—In the numerical example discussed in Secs. 18–2 and 17–12, the inductance L was 1 mh, and

the voltage appearing across the resonant circuit with the application of 5 volts (of the resonant frequency) to the grid of the tube was 25 volts. The coil L_2 can certainly be wound with such a number of turns and brought close enough to the coil L, so that the mutual inductance M between the two coils is exactly $\frac{1}{5}$ mh. The voltage appearing across it will then be, according to Eq. (18–2), 5 volts. The reader may substitute into Eq. (18–3) the numerical values applying to this case and convince himself that this equation too will yield 5 volts for E_2.

The voltage appearing across L_2 is now exactly of the same magnitude as the voltage applied to the grid by the ac generator. What about the phase relation? When point G in Fig. 18–1 was at the crest of its up swing, point A was at the lowest point (since the resonant circuit acts like a resistance). The voltage E_2 is very nearly in phase (actually leading by a small angle) with the voltage across L. This means that at the instant just considered point M is at its negative maximum with respect to point N (the two coils assumed as wound on top of each other as explained in Sec. 17–13), or N is at its positive maximum with respect to M—just as G is with respect to S! This gives us an idea, doesn't it? So we sneak up with two leads from L_2 and, when the tube is just looking the other way, take the ac generator away from the grid and connect the two leads from L_2 in its place! It's a dirty trick to play on an unsuspecting amplifier tube, but it works! The connection is shown in Fig. 18–3. The circuit is now a self-excited amplifier, or an oscillator; the particular type shown in Fig. 18–3 is called "tuned plate-circuit oscillator." (A strictly accurate mathematical treatment would show that the frequency adjusts itself to a slightly higher value. This value would be such that in the case of separate excitation, as shown in Fig. 18–1, the voltage E_2 would be exactly in phase with the voltage E_g applied to the grid.)

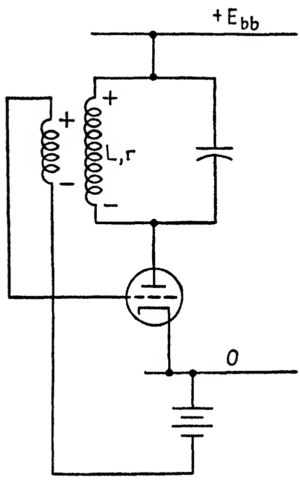

FIG. 18 3.—A self-excited circuit or oscillator results when we steal the voltage required for the excitation of the grid from the parallel resonant circuit by mutual induction.

Not only is the tuned plate-circuit oscillator shown in Fig. 18–3 the prototype of many circuit arrangements known under different names, but the method of analysis presented in the preceding paragraphs is applicable to most of the oscillator circuits, even if they are not using an L-C network. It is seen that, in the application of this principle, an oscillator circuit is simply treated as an amplifier circuit. It is then investigated whether at a particular frequency a voltage can be obtained from the output circuit of the amplifier equal to the one necessary to operate the circuit at the

particular frequency. If the investigation shows that this is possible, *i.e.*, that a voltage of the proper magnitude and phase relationship can be obtained from the output circuit of the amplifier, then the circuit will keep itself in oscillation upon properly connecting this voltage to the input circuit of the amplifier.

For the circuit to be self-excited, it is therefore necessary that the voltage E_2 appearing across L_2 as given by Eq. (18-3) is at least as large as the voltage E_g applied to the grid in Fig. 18-1. The ratio E_2/E_g must therefore be unity or larger. This ratio can be obtained from Eq. (18-3), which gives us

$$\frac{E_2}{E_g} = \frac{\mu R}{(r_p + R)} \frac{M}{L} \geqq 1 \tag{18-4}$$

Usually the quantity M is the unknown. Solving Eq. (18-4) for M and at the same time substituting for R the expression given in Eq. (17-25*d*) gives us

$$M \geqq \frac{L[r_p + (L/Cr)]}{\mu L/Cr} = \frac{rr_pC + L}{\mu} \tag{18-5}$$

If we insert in Eq. (18-5) the numerical values of the example treated above, we obtain

$$M \geqq \frac{2 \times 10{,}000 \times 0.1 \times 10^{-6} + 1 \times 10^{-3}}{15} \text{ henry}$$

$$\geqq \frac{2 \times 10^{-3} + 1 \times 10^{-3}}{15} = 0.2 \times 10^{-3} \text{ henry} = 0.2 \text{ mh}$$

This result agrees with the value for the mutual inductance arrived at in the course of the analysis of the circuit shown in Fig. 18-1.

18-5. Value of M in Practical Oscillator Circuits.—Equation (18-5) gives us the minimum value of mutual inductance necessary to furnish a voltage *exactly* equal to the one needed for excitation of the circuit. It will be noted that it does not give us any information about the amplitude of the oscillation that we can expect in the circuit. In the analysis of the circuit shown in Fig. 18-1 we assumed that a voltage of 5 volts was applied to the grid of the tube and found that, with a mutual inductance of 0.2 mh, a voltage of 5 volts will also appear across the coil L_2. It is self-evident that, as long as the tube is working in the linear part of its characteristics, an application of 6, 8, or 10 volts to the grid of the tube would result in the appearance of 6, 8, or 10 volts across the coil L_2. The circuit would keep itself in oscillation with any value of voltage. Now it is a physical impossibility to make the value of M *exactly* 0.2 mh, just as it is impossible to make a rod of steel *exactly* 12 in. long; it will always be a little larger or a little smaller than the desired value (that is why tolerances

should be given in all practical problems, electrical or mechanical). Even if this were possible, it would mean only that the circuit, once placed into oscillation by an outside source, would be capable of maintaining itself in this state, but it would not be able to start oscillation, by itself. It is, therefore, always necessary to make the value of the mutual inductance somewhat larger than the value given by Eq. (18–5). What will happen if M is made larger than the minimum required to keep the circuit in oscillation? As long as the tube is operating in the linear part of its characteristics, we shall feed back to the grid of the tube a voltage larger than that required for the operation of the resonant circuit in its plate circuit. This means that each succeeding amplitude of the oscillation taking place in the resonant circuit will be somewhat larger than the preceding one. Under this condition, the oscillating circuit is even capable of building up oscillations without help from an outside source. This is due to the fact that there are always small disturbances of current and voltage in a tube circuit, which excite the resonant circuit into oscillation with a small amplitude. With the mutual inductance M exceeding the minimum required value as given by Eq. (18–5), the amplitude of the small oscillations will keep on increasing with each cycle as outlined above. The next question, naturally, is how long this increase of amplitude will keep on. The answer to this question is rather unexpected: the amplitude of the oscillations keeps on increasing until Eq. (18–5) is satisfied. Examination of the right-hand side of Eq. (18–5) shows the plate resistance r_p in the numerator of the fraction. As the amplitude of the oscillations increases, the operating point will begin to swing into the nonlinear part of the characteristics of the tube. It will be remembered that the plate resistance becomes larger when the tube is operating with low values of plate current, as a look at the plate characteristics of any triode will disclose. In Eq. (18–5) r_p should be considered, not as the plate resistance existing at the quiescent point of operation but as an average value over one complete cycle of oscillations. With an increase of the amplitude of the oscillations, the right-hand side of Eq. (18–5) therefore also increases. This means that with M larger than required for the start of oscillations an equilibrium will finally be reached where Eq. (18–5) will be satisfied. This reasoning discloses another very important fact: a tube could not possibly operate as an oscillator if its characteristics were completely linear. If they were linear, oscillations, after having been started, would keep on increasing in amplitude until destruction of either the tube or the component parts of the oscillating circuit took place. It is the nonlinearity of the tube characteristics that limits the amplitude of the generated oscillation to a safe value. At the same time, it must be recognized that this fact also prevents us—at least theoretically—from obtaining a pure sinusoidal output of current or voltage from the oscillating circuit. As will be seen presently, actual oscillator circuits go so far in operating the tube in the nonlinear part of

the characteristics that the tube is actually cut off during the larger part of the cycle; the plate resistance during this part of the cycle is then evidently infinite.

18–6. Class C Operation of the Oscillator.—Derivation of Eq. (18–5) was based on the analysis of the circuit shown in Fig. 18–1, assuming the tube to be operating in the linear part of its characteristics so that the application of the equivalent-plate-circuit theorem was permissible. Equation (18–5) had then given us the minimum value of the mutual inductance required for the maintenance of oscillations. In the simplest terms, we could say that if we wish to maintain a voltage E_p across the resonant circuit shown in Fig. 18–1, it will be necessary to apply to the grid of the tube a voltage E_p/v_g, and that this voltage must be 180 deg out of phase with the voltage that we wish to maintain across the resonant circuit. (To be more specific, when the anode bobs down, the grid must be bobbing up.) It is common practice in the design of oscillator circuits to furnish a voltage to the grid much in excess—as a matter of fact, several times as high—of the one that would be just sufficient to maintain the oscillating current if the tube were operating in the linear part of the characteristics. The operation of an oscillator under such conditions is quite different from that analyzed in the preceding sections; it is not harder to understand, however, as the following considerations will show.

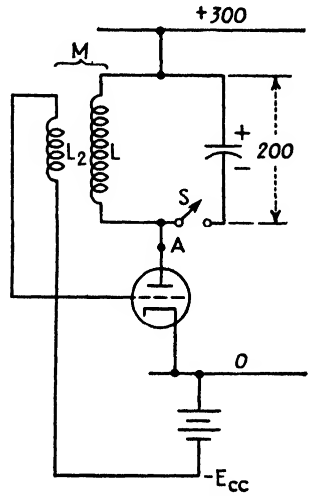

FIG. 18–4.—We could operate the tube as a Class C amplifier if we could charge the capacitor of the resonant circuit at the beginning of the operation.

Let the capacitor shown in Fig. 18–4 be charged to a voltage of 200 volts by an outside source, with a polarity as shown. Consider the tube at first as inoperative by keeping the cathode cold, for instance. With the supply voltage equal to 300 volts, as shown in the diagram, the lower end of the capacitor will be at a potential of +100 volts with respect to the cathode of the tube. Now, let us close switch S. This will start an oscillation in the circuit, as explained in Sec. 17–3. Point A will swing up to a level of 500 volts, returning again to the level of 100 volts after one complete cycle. However, this is exactly true only if there are no losses occurring in the oscillating circuit. Under actual conditions the capacitor, after one complete cycle, will not be charged to 200 volts, as it was originally, but to a lower value, say, to 190 volts. This is quite analogous to what happens in the mechanical system shown in Fig. 17–2a where we connected the stretched spring to a mass and then released the mass. If there were no friction present, the mass would return to the starting point after one

complete cycle; with friction present, it fails to do so by a certain amount. If we could get hold of the mass at this instant, or perhaps a little earlier, and help it along to the original starting point, then the next cycle of oscillation would be exactly like the first one. In a similar way, if we could persuade the tube at the instant when point A comes down to the 110-volt level, or perhaps a little earlier, to permit current flow, the effect would obviously be to force point A farther down than it would go without this help.

In Sec. 13–9, various classes of amplifiers were discussed. It was explained there that in Class C operation the grid bias of the tube is chosen beyond the cutoff grid voltage so that plate current will flow during only a relatively small part of the positive half wave of the grid voltage. In the preceding paragraph we saw that it would be possible to keep an L-C circuit oscillating by letting the tube pass current for a small part of the cycle when the anode was at or near its lowest level. This is then evidently a tailor-made condition for Class C operation of the tube. The bias voltage E_{cc} is made much larger than the cutoff grid voltage, and the mutual inductance M between the two coils is made so large that with an alternating voltage of 200 volts amplitude across the coil L, *i.e.*, across the resonant circuit, the voltage induced in L_2 will have an amplitude sufficient to overcome the bias voltage and to permit plate-current flow. The coil L_2 must, of course, be connected to the grid with such a polarity that the grid will swing positive at the instant when point A is farthest down. The reader will realize that this method of keeping an electric circuit in oscillation is entirely equivalent to keeping a church bell ringing; in the latter case, too, energy is not delivered to the oscillation system continuously (you cannot push "up" on a rope), but the required energy is delivered in spurts at the correct instant within the cycle. This is exactly the situation prevailing when a tube in Class C amplification is used for the excitation of a resonant circuit.

18–7. Automatic Grid Bias for Oscillators.—The circuit shown in Fig. 18–4 would operate exactly as described. It has, however, one disadvantage that would make it entirely impractical. We saw that in order to get it into operation it was necessary to charge the capacitor from an outside source to the desired voltage. The circuit cannot get into operation by itself because the grid bias E_{cc} is so large as to cause the plate current to be completely cut off before oscillations have started. One way to overcome the necessity of charging the capacitor would be to reduce the grid bias at the beginning so that the tube would operate in Class A. This can be accomplished by providing a voltage divider across the grid-bias battery that will permit reduction of the bias so that oscillations can start. As these oscillations increase in amplitude, the grid bias may then be increased until the tube is operating truly in Class C. Would it not be nice if this shift in grid bias could be obtained automatically? One of the curi-

ous facts about vacuum-tube circuits is that quite often a desired result, which seemingly requires complicated circuit arrangements, can be had with practically no effort at all. In the case of an oscillator circuit the inclusion of a simple parallel combination of a resistor and a capacitor, such as we have become familiar with when discussing grid-leak detection, is all that is needed to accomplish the desired result. This is indicated in Fig. 18–5. In this figure the grid-leak combination is indicated in the grid lead, but it can, of course, be as well placed in the cathode lead at the point marked X. Before the circuit starts to oscillate, the grid is at cathode potential or at least very near it. A large plate current will consequently flow, and the tube is at a point of the characteristic that makes it operate in Class A. Oscillations will now start in the resonant circuit, sometimes also referred to as the "tank" circuit, for the reasons outlined in the preceding paragraphs. The alternating voltage appearing across L_2 is now applied to the grid in series with the grid-leak resistor and capacitor. Owing to the rectifying action of the grid, as discussed under grid-leak detection, a direct voltage will build up across the capacitor approximately equal to the amplitude of the alternating voltage induced in the coil L_2 and with a polarity as shown. If a dc meter is included in the plate circuit of the tube, a large reduction of plate current will be observed at the instant when oscillations begin. As a matter of fact, the plate current in high-power oscillating tubes would reach destructive values very quickly if oscillations stopped for any reason whatsoever, which leads to the loss of the bias voltage developed across the grid-leak capacitor shown in Fig. 18–5. In low-power circuits, on the other hand, this change of plate current occurring when a tube goes into or out of oscillation has been put to practical use. Thus, in an elevator control offered several years ago, the coils L and L_2 were mounted facing each other on top of the elevator cage and extending out into the hatchway; at every floor a piece of metal was arranged vertically so as to get between the two coils when the cage passed this floor. The entrance of the metal between the two coils evidently reduced the mutual inductance existing between the two coils and thus stopped the oscillations. A relay in the plate circuit of the oscillating tube was then used to operate the leveling control of the elevator. The versatility of a vacuum-tube oscillator will be appreciated when it is realized that a tube operating in a circuit as shown in Fig. 18–5 is operating simultaneously as an oscillator, a Class C amplifier, and a grid-leak detector, and the results arising from all these three facts may be put to practical use.

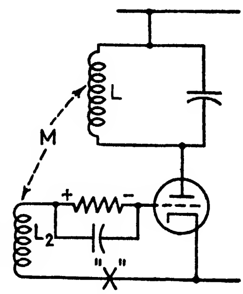

FIG. 18–5.—The inclusion of a grid resistor and capacitor provides zero bias at the beginning so that oscillation will start easily but will automatically furnish a high negative bias for Class C operation.

When the resistor-capacitor combination furnishing self-bias for the oscillator has a large time constant, such as results from the use of a high resistor value, a phenomenon known as "blocking" takes place. As the amplitude of the oscillations builds up, the voltage across the capacitor increases also since this voltage is the result of the grid rectifying the alternating voltage appearing across the grid coil. It may then happen that the negative bias thus developed becomes so large that the tube is cut off. The oscillations in the tuned circuit will then die down, just as the oscillations of a mechanical system will die out if no energy is furnished to keep them up. But the negative bias furnished by the charged capacitor will also disappear as the latter discharges through the resistor, and the tube will finally become conducting again. As this happens, oscillations will begin and their amplitude will increase until the negative bias is sufficient to cut the tube off again. Under such a condition the oscillations consist of trains of waves, building up and dying down in strength with a frequency depending on the time constant of the capacitor-resistor combination in the grid circuit. Although this is a convenient method of obtaining a modulated high-frequency wave, in most cases such operation is not desired. If it occurs, the remedy is to reduce the value of the grid resistor to about 25,000 to 50,000 ohms.

18-8. Modifications of the Tuned Plate-circuit Oscillator.—In the tuned plate-circuit oscillator the coil L_2 serves only to apply to the grid a voltage 180 deg out of phase with the plate voltage and of the proper magnitude to keep the tank circuit oscillating. Any circuit arrangement that will do this will be equally satisfactory. The tuned plate-circuit oscillator is the prototype of a number of circuits, all of which can be reduced to the fundamental circuits shown in Figs. 18-1 to 18-5. The first modification is shown in Fig. 18-6. In this circuit the coil L_2 is simply a continuation of the main coil in the tank circuit. The coil can be considered as an autotransformer tapped at point B, which is seen to be connected to the fixed potential of the plate-supply source. It is evident that the potential of point D will be 180 deg out of phase with the potential of point A and that this voltage is capacitively coupled through the capacitor C_b to the grid of the tube. The capacitor C_b not only serves the purpose of coupling the alternating voltage of point D to the grid of the tube but, in conjunction with the resistor in the grid circuit, takes over the function of the grid-leak combination shown in Fig. 18-5. The rectifying action of the grid prevents it from going positive when D swings positive with respect to point B. The capacitor C_b, therefore, not only

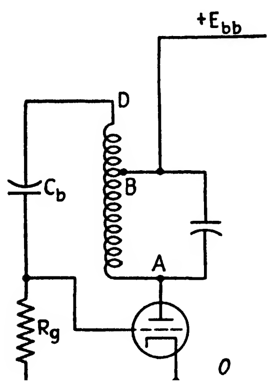

FIG. 18-6.—A modification of the circuit shown in Fig. 18-5.

becomes charged to a voltage equal to the supply voltage but acquires an additional voltage equal to the peak voltage existing across section *BD* of the coil.

The circuit shown in Fig. 18–6 is better known in a somewhat modified form. The modification consists of placing the capacitor across the whole coil instead of only a section of it. In this form it is known as the "Hartley circuit," shown in Fig. 18–7. The important fact to recognize with respect to this circuit is again that the potential of *D* is 180 deg out of phase with the potential of *A*. The alternating voltage of *D* is again coupled to the grid by means of the coupling capacitor C_b and the grid-leak resistor R_g.

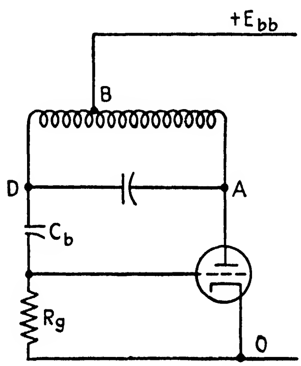

FIG. 18–7.—This circuit is known as the "Hartley circuit." It does not differ essentially from that shown in Fig. 18–6.

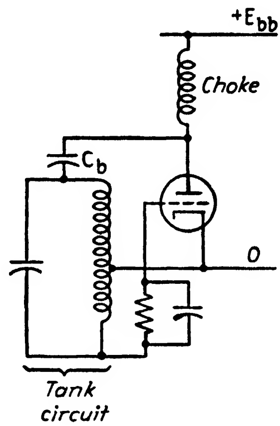

FIG. 18–8.—A shunt-fed Hartley circuit. This arrangement permits the placing of the resonant circuit at ground potential.

The tuning capacitor in all the circuits so far discussed had a dc potential with respect to the cathode equal to the plate-supply voltage. This is usually of not much consequence in low-power oscillator circuits, but it becomes an additional insulation problem, as well as an operating hazard, in the case of power oscillators operating from a high voltage. It is therefore generally desirable to place the tank circuit at cathode potential, the latter usually being connected to ground. A circuit accomplishing this result is shown in Fig. 18–8, which is known under the name "shunt-fed Hartley circuit." The choke shown in the plate circuit must have an inductive reactance large compared to the apparent resistance *R* of the tank circuit so that the alternating current will flow through the blocking capacitor C_b and a section of the tuning coil instead of flowing through the choke and the plate supply.

If in Fig. 18–8 we replace the single capacitor in the tank circuit by a series combination of two capacitors and use the junction point of the two

as a tap, as shown in Fig. 18–9, the circuit becomes the "Colpitts circuit." The fundamental condition that we have to feed to the grid a voltage 180 deg out of phase with the voltage on the plate is again fulfilled by this circuit arrangement.

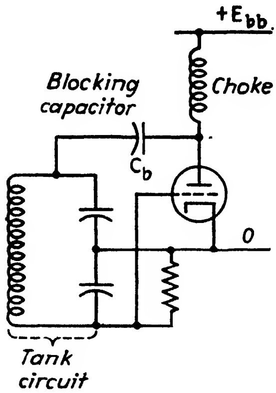

FIG. 18–9.—A shunt-fed Colpitts circuit.

18–9. Capacity Relays.[5, 6, 8, 10]—In Fig. 18–10 we return to the original and simple tuned plate-circuit oscillator. Let us assume that the mutual inductance M existing between the two coils in this diagram is much larger than stipulated by Eq. (18–5). We now place a voltage divider—which should be of as high a value as possible in order not to cause a loading effect on the tank circuit—across the coil furnishing voltage to the grid. It is evident that there will then exist a point on the voltage divider where the voltage will be just sufficient to start oscillations. If, for instance, the mutual inductance M is twice as large as required by Eq. (18–5), then oscillations can be expected to start when the arm of the voltage divider is just in the center of the total resistance.

The circuit shown in Fig. 18–10 is presented not because it is in itself of particular practical value but because it shows the underlying principle of some circuits that may be of interest. It is evident that the voltage divider in Fig. 18–10 can be replaced by a capacitive voltage divider; in other words, the two parts of the voltage divider may be replaced by the reactance of two capacitors. A change in value of one of these two capacitors has obviously the same effect as a movement of the arm of the potential divider shown in Fig. 18–10 would have. The two capacitances forming this capacitive voltage divider may be quite small; one of them may be, for instance, the capacitance to ground of a wire stretched in space. Any object coming between this wire—usually called the "antenna"—and ground will increase the capacitance of the antenna to ground and thus act in the same way as if the arm of the potential divider had been moved nearer to the cathode end in Fig. 18–10. Such an arrangement, the circuit of which is shown in Fig. 18–11, is usually referred to as a capacity relay because it operates when the capacitance of the antenna to ground is changed by an object in the manner outlined above. The circuit shown in Fig. 18–11 is not the only one used for this purpose, but it is one of the simplest ones and, at the same time, demonstrates a

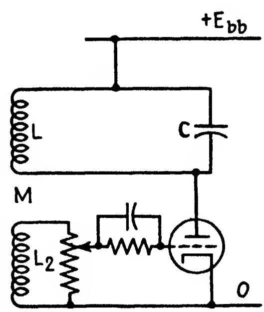

FIG. 18–10.—This circuit will evidently go into oscillation at one particular position of the arm of the voltage divider.

practical application of an oscillating circuit. In actual practice, a Hartley circuit with the tank circuit in the cathode lead has been used (see Fig. 18–12). Although this circuit arrangement looks somewhat strange, it is actually performing in exactly the same manner as the circuit shown in Fig. 18–6. In both figures, at an instant when the voltage across the tank circuit, *i.e.*, between points *A* and *B*, increases, the voltage across the tube will decrease. Owing to the autotransformer action of the coil *AD*, the potential of *D* is then swinging up with respect to the cathode. What fraction of this up swing of point *D* in Fig. 18–12 will reach the grid, however, depends on the voltage divider formed by the adjustable capacitor and the capacitance existing between the antenna and ground. This may be so adjusted that with no object between the antenna and ground the tube will be oscillating. An increase of this capacity will lead to a stoppage of the oscillation. The presence or absence of oscillations may be used to operate a relay or a meter by connecting the alternating voltage to any one of the detector circuits described in Chap. XVI.

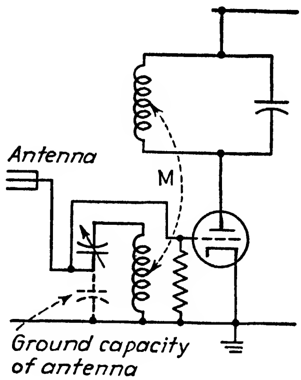

FIG. 18–11.—The action of the voltage divider shown in the circuit of Fig. 18–10 can also be obtained by means of a capacitive voltage divider, one of the capacitances being formed by the capacitance of antenna to ground.

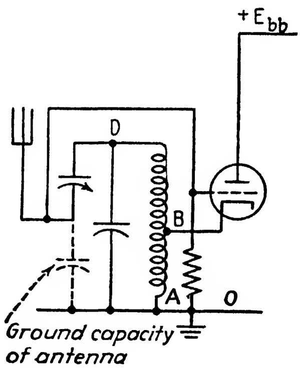

FIG. 18–12.—A capacity relay using the Hartley circuit.

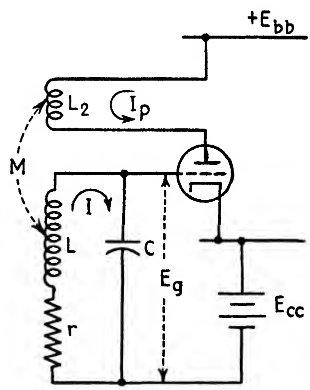

FIG. 18–13.—The fundamental connection of a tuned grid-circuit oscillator.

18–10. Tuned Grid-circuit Oscillator.—In all the oscillating circuits discussed so far, the tank circuit was placed in the plate circuit. It made no difference whether the tank circuit was connected to the plate lead, as shown in Fig. 18–6, or in the cathode lead, as shown in Fig. 18–12. In each case, it is seen that the plate current flows through the inductance in

the tank circuit. Another arrangement sometimes employed in the design of oscillator circuits is shown in Fig. 18–13. In this arrangement the tank circuit is seen to be connected to the grid of the tube; the exciting coil L_2 is connected in the plate circuit. Oscillators operating on this principle are called "tuned grid-circuit" oscillators. The mathematical treatment of the tuned grid-circuit oscillator is considerably more complicated than that of the tuned plate-circuit oscillator. For the tuned plate-circuit oscillator, Eq. (18–5) gave us the value of the mutual inductance M that had to exist between the two coils in order to be sure that oscillations would start. Although this equation was derived under a simplifying assumption, a strictly correct treatment leads to exactly the same result. The only difference between the strictly correct and the approximate treatment is found in the value of the frequency at which this circuit will oscillate. Examination of Eq. (18–5) discloses the fact that even for a perfect tank circuit, *i.e.*, for $r = 0$, the mutual inductance is not equal to zero. The reader will find it profitable to ponder on this statement.

18–11. Simplifying Assumption for Mathematical Analysis of Tuned Grid-circuit Oscillator.—A reasonably simple mathematical treatment of the tuned grid-circuit oscillator can be carried out under a simplifying assumption that can usually be satisfied in practice. The assumption is the following: let the current in the plate circuit be determined *only by the grid voltage.* This would evidently be very nearly true if we would use a pentode for this circuit. But even in the case of a triode, this condition will be nearly fulfilled if the inductance L_2 in the plate circuit and the reaction of the tank circuit on this coil through the mutual inductance M are small compared to the plate resistance of the triode. In the usual oscillator circuits, this condition is automatically met, or at least can be met if so desired, and the results obtained on the basis of this simplifying assumption are in most practical cases entirely satisfactory as far as predicting circuit performance is concerned.

Examination of Fig. 18–13 discloses the fact that the tank circuit in this case is not excited in a parallel resonant connection as in all previous circuits but that we are dealing with a series resonant circuit. As shown in Chap. XVII, the voltage necessary to excite a series resonant circuit at the resonant frequency is simply equal to the desired oscillating current multiplied by the resistance r of the inductance. Therefore, if in Fig. 18–13 we wish to maintain the current I in the tank circuit, it is necessary to introduce in this circuit by mutual inductance a voltage equal to Ir. The induced voltage must be in phase with the desired current. Figure 18–13 discloses further that the voltage applied to the grid of the tube is equal to the voltage existing across the capacitor in the tank circuit. This voltage lags the current in the tank circuit by 90 deg. The amount of voltage is equal to the current in the tank circuit times the capacitive reactance of

the capacitor. The voltage applied to the grid is, therefore, given by

$$E_g = Ix_c = \frac{I}{C\omega} \tag{18-6}$$

We now make use of our simplifying assumption. If the inductance L_2 in the plate circuit is small or if we should be dealing with a pentode, then the alternating current that will flow in the plate circuit will be found simply by multiplying the alternating voltage applied to the grid with the transconductance g_m of the tube. We therefore have

$$I_p = g_m E_g = \frac{I g_m}{C\omega} \tag{18-7}$$

This current will be in phase with the grid voltage under the assumption that we have made, but it will be 90 deg out of phase with the current flowing in the tank circuit since the voltage applied to the grid was 90 deg out of phase with the latter. Owing to the mutual inductance M existing between the two coils L_2 and L, there will now be induced in L a voltage given by the mutual inductance multiplied with the rate of change of plate current. The voltage induced in the coil of the tank circuit is therefore given by

$$E_{\text{in}} = I_p \omega M = \frac{I g_m \omega M}{C\omega} = I\frac{g_m M}{C} \tag{18-8}$$

The voltage induced in L is 90 deg out of phase with the current flowing in the plate circuit and is therefore either in phase or 180 deg out of phase, depending on the connection made to the coil L_2, with the current that we wish to maintain in the tank circuit. Consequently, if M is large enough so that the induced voltage as given by Eq. (18-8) is as large as that required to maintain the oscillating current, in other words, if it is equal to or larger than Ir, the tank circuit will maintain itself in oscillation. By equating these two values, we obtain

$$I\frac{g_m M}{C} \geqq Ir \tag{18-9}$$

This equation simply states that the induced voltage, as given by the left-hand side of the equation, must be equal to or larger than the voltage required to make the current I flow in the resonant circuit.

Solving Eq. (18-9) for M results in

$$M \geqq \frac{rC}{g_m} \tag{18-10}$$

or also, since $g_m = \mu/r_p$,

$$M \geqq \frac{r r_p C}{\mu} \tag{18-11}$$

If it is desired, use can be made of the relation $\omega^2 = 1/LC$. Equation (18–11) then becomes

$$M \geqq \frac{rr_p}{\mu L\omega^2} \tag{18–12}$$

Comparison of Eqs. (18–5) and (18–12) shows that for a tuned grid-circuit oscillator with a perfect inductance in the tank circuit, *i.e.*, with $r = 0$, the required mutual inductance M is zero. This indicates further that with a tuned grid-circuit oscillator a change of r of the tank circuit, or any factor that may have an effect on the losses occurring in the tank circuit, is much more liable to stop oscillations of the circuit if the value of the mutual inductance M is somewhere near the critical value.

In the diagram shown in Fig. 18–13 the source of grid bias was indicated as a battery. Evidently the same reasoning that led us to the use of a capacitor and resistor in the case of a tuned plate-circuit oscillator will also be valid in this case, and an automatic biasing arrangement may be obtained by replacing the battery with a resistor-capacitor combination.

18–12. Parasitic Oscillations and Their Suppression.—The discussion of oscillator circuits proceeded from the assumption that these oscillations were wanted. The designer of electronic circuits finds quite often to his sorrow that tube circuits generally are so willing to break out into oscillations that it is often a hard job to keep them from oscillating where they are not supposed to. Such oscillations, also referred to as "parasitic" oscillations, usually occur with very high frequencies. Oscillations of very high frequency are rather hard to detect because the amplifiers in such instruments as a cathode-ray oscillograph usually have an upper-frequency limit of the order of 100,000 cps. Hours and even days are sometimes spent trying to find out why a given tube does not behave as it should according to the published characteristics, and the reason for the erratic behavior is quite often the fact that parasitic oscillations take place somewhere in the circuit. This trouble is especially liable to occur with high-transconductance tubes, such as the beam-power tubes. A special warning should be sounded if it is contemplated to connect two beam-power tubes in parallel. In such a case parasitic oscillations are practically a certainty. It is almost an impossible task to analyze just what causes this action. It can be stated in general, however, that the oscillating circuit is usually formed by the electrode capacity, and the inductance is furnished simply by the wiring itself; since these are small, the oscillating frequency will naturally be extremely high. Experience has shown that the spurious oscillations can usually be suppressed by including a resistor of 100 ohms or more in the grid circuit; this resistor should be connected directly in series with the grid right at the socket of the tube. A small capacitor in the order of 10 to 50 $\mu\mu$f connected at the socket of the tube

from the grid terminal to the cathode terminal is also usually found effective in suppressing parasitic oscillations.

18–13. Oscillations with a Resonant Circuit in the Grid Circuit and an Inductive Load in the Plate Circuit.—One type of unwanted oscillation may occur when there is a tuned circuit in the grid circuit of a vacuum tube, similar to the case of a tuned grid-circuit oscillator, and an inductance in the plate circuit but with no mutual inductance between the two coils. In this case, the necessary coupling from the anode of the tube to the tank circuit is provided by the capacitance existing between the plate and the grid of the tube. The tendency of such an arrangement to break into oscillation was a serious problem in the early days of radio, and elaborate neutralizing schemes were employed in order to prevent these oscillations from occurring in the radio-frequency amplifier stages of a radio receiving set. In Fig. 18–14 is shown a tuned circuit connected to the grid of the tube, with an inductance L included in the plate circuit. The capacitance existing between the anode and the grid of the tube is shown in dotted lines outside the tube, connecting the points A and B. Assume for an instant that the tube is inoperative. If we wanted to excite the tank circuit as a parallel resonant circuit by connecting it to a source of alternating voltage, as shown by the dotted lines, this source would have to furnish a current in phase with the voltage across the tank circuit; the tank circuit would appear to this source as a resistance equal to the equivalent resistance R of the parallel resonant circuit, as given by any of the Eqs. (17–25a) to (17–25d). If, with the tube operating, a current of the required magnitude flows from A through the electrode capacity (also shown in dotted lines) to B, then the outside source can be omitted and the circuit will keep itself in an oscillating state. The alternating voltage applied to the grid is identical with the voltage existing across the oscillating circuit. If we can make again the assumption that we made when analyzing the tuned grid-circuit oscillator, namely, that the plate current was determined by the grid voltage only, or in other words, that the effect of the inductance L in the plate circuit was negligible, then the plate current will be in phase with the grid voltage. The potential of A will therefore be 90 deg out of phase with the plate current and, therefore, also with the voltage existing across the tank circuit. But the current that will flow through a capacitor is in turn 90 deg out of phase with the voltage producing it; therefore, the current flowing from A to B will be 90 deg out of phase with the potential of A. The current flowing from A to B is, therefore, either in phase or 180 deg out of phase with the voltage existing across the oscillating cir-

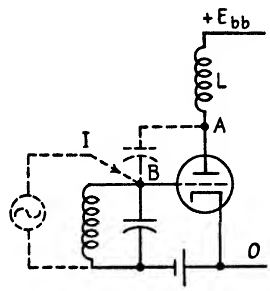

FIG. 18–14.—Due to the capacity existing between anode and grid of a tube, oscillations may result with a parallel resonant circuit in the grid circuit and an inductance in series with the plate.

cuit. The reader may prove to himself that the phase relationship is the first and that the circuit will therefore oscillate if the electrode capacity is of sufficient value. The introduction of the screen grid between the control grid and the anode in a tetrode or a pentode reduced the capacitance existing between anode and control grid to a fraction of its former value. This was the answer to the problem of suppressing the unwanted oscillation. It was for this reason and not for the high amplification of the tetrodes, which was merely incidental, that they were introduced into radio receivers around 1927.

18–14. Negative Resistance, or Dynatron Oscillators.—In the oscillator circuits so far discussed, two inductances or a single inductance with a

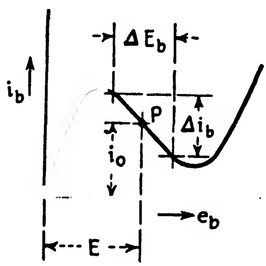

FIG. 18–15.—The plate characteristics of a screen-grid tube show a region of negative plate resistance.

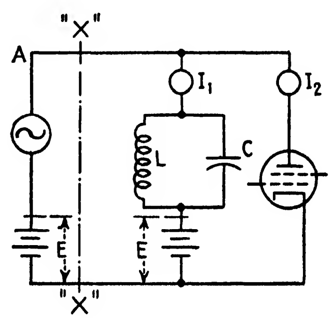

FIG. 18-16.—A parallel resonant circuit has a positive resistance. If a negative resistance is placed parallel to it, we can omit the exciting generator if the negative resistance is equal in magnitude to the positive resistance.

tap was needed in order to obtain the proper phase relationship. By the use of negative resistance, or negative-transconductance tubes, it is possible to do away with this requirement. Figure 12–2 showed that the plate current of a tetrode or screen-grid tube decreases with an increase of plate voltage over a certain part of the plate-voltage–plate-current characteristic of such a tube. It was pointed out in Chap. XII that in such a region the plate resistance of this tube is negative. Let Fig. 18–15 represent the plate-current–plate-voltage characteristic of a tetrode in the region where the plate current is falling with an increasing plate voltage. Let the plate voltage in the center of this region, *i.e.*, for point P, be E. Now consider the circuit shown in Fig. 18–16. Starting from left to right of this figure, we see a dc source with a voltage E in series with an ac generator connected to a tuned circuit and another dc source of the voltage E. Let us disregard for a moment the tube connected parallel to the combination of tuned circuit and dc battery. The two dc sources are seen to be opposing each other, and there is consequently no direct current in the

resonant circuit. Now, consider the ac generator. If the frequency of this generator is equal to the resonant frequency of the tuned circuit, then we know that the latter acts like an equivalent resistance R given by any of Eqs. (17–25a) to (17–25d). This means that during the half cycle when A is swinging positive, the current through the resonant circuit, registered by the meter I_1, flows from top to bottom and reaches a maximum when the voltage of point A reaches its maximum. Consider the effect that the parallel connection of the tube will have on this circuit. With the ac generator absent, or at the instant when the voltage of the latter is zero, the tube will be taking a current i_0, as indicated in Fig. 18–15. This current can be considered as being furnished by either one or both of the two dc sources E. During the half cycle when the terminal A of the ac generator becomes positive, the voltage across the tube is increasing and the current through it, indicated by meter I_2, will be decreasing, as shown by the characteristic shown in Fig. 18–15. Evidently if the decrease—or negative increase—of current through the tube is equal to the increase of current taking place in the tuned circuit, the ac generator does not have to furnish any current at all, and we might as well discard the part of the circuit shown to the left of the line XX. The dc component taken by the tube will then be furnished by the source of direct current E in series with the oscillating circuit. In order not to confuse the diagram with unessentials, the control grid and the screen grid are not shown connected in Fig. 18–16; it is, of course, understood that they must be connected to fixed voltages equal to those prevailing when the characteristic shown in Fig. 18–15 was taken.

The condition under which the circuit of Fig. 18–16 will maintain itself in oscillation is obviously very simple to state. If we wish to discard the ac generator to the left of the line XX, the alternating current through the tube, which is, as we have seen, 180 deg out of phase with the generator voltage, must be equal to, or even larger than, the in-phase current taken by the tuned circuit. In other words, the negative resistance of the tube must be equal to or smaller than the positive equivalent resistance of the tuned circuit. Suppose that in Fig. 18–15 the total region ΔE_b over which the plate current is falling had been found to be 40 volts and that the plate-current change occurring over this region, marked Δi_b, was 2 ma. The negative resistance of the tube over this region is therefore $-20{,}000$ ohms. The tube will then be able to keep in oscillation a resonant circuit with an equivalent resistance R in excess of 20,000 ohms [as given by any of the Eqs. (17–25a) to (17–25d)]. Dynatron oscillators, as these circuits are called, enjoyed a great popularity some years ago because the frequency of the generated oscillations was exceptionally stable. Changes in manufacturing technique of these tubes, however, resulted in an increase of the negative resistance exhibited by them, and consequently replacement of a tube with a later type quite often led to a failure to generate oscillations.

Even under the best conditions, the phenomenon of secondary emission depends on a great many factors; therefore the negative resistance exhibited by these tubes changed appreciably from one tube to the next.

18–15. Negative-transconductance Oscillators.[9]—The fundamental requirement to keep a tuned plate-circuit oscillator in oscillation is to feed to the grid of a tube a voltage 180 deg out of phase with the ac component of the plate voltage. In other words, when the potential of the plate swings down, the potential of the grid has to swing up. This can be accomplished by either a separate coil or by having a tapped coil in the plate circuit—such as the Hartley circuit—which then furnishes a voltage with the required phase relationship. If there existed a tube in which the plate current *increases* when the grid voltage becomes more negative, it is evident that the requirements as to the phase relationship between the anode voltage and the voltage applied to the grid would be reversed from those just outlined. In such a case we could couple directly from the anode by means of a coupling capacitor to the grid of the tube. Such a tube could be properly said to possess a negative transconductance. At the instant when the anode is swinging negative, the grid, through the action of the coupling capacitor, would also be swinging negative; with a negative-transconductance tube this would mean that the current through the oscillating circuit would be increasing which, as had been shown in the discussion of Fig. 18–4 in Sec. 18–6, will tend to maintain the circuit in oscillation by replenishing the capacitor charge at the proper instant.

Owing to a curious phenomenon within a pentode, such a tube can be converted into a negative-transconductance device. Figure 12–5 showed the rubber-sheet model of a pentode; in Sec. 12–7 the question was raised whether there would not be a great number of marbles falling in the funnel-shaped recess formed by the screen grid, but it was pointed out that as long as the saddles formed by suppressor grid were low enough, the marbles would be able to run over and through them. But suppose now that we should raise the row of pins representing the suppressor grid or, electrically speaking, make the suppressor grid more negative. It is obvious that when the row of pins representing the suppressor is raised so high that the bottom of the saddles is higher than the level of the cathode, the marbles are no longer able to negotiate this hump and consequently fall back into the funnel-shaped recess formed by the screen grid. This is actually what takes place in a pentode. The current flowing to the screen grid will increase when the suppressor grid is made more negative. Therefore, if we use the screen grid as the anode and the suppressor grid as the control grid, holding the regular control grid and the regular anode at a fixed potential, then the tube will exhibit a negative transconductance between the suppressor grid and the screen grid. Figure 18–17 shows the connection diagram of such a circuit. The resonant circuit is connected to the screen grid, but the voltage existing across it is also capacitively coupled to the

suppressor grid. For a type 57 or 6C6 tube the manufacturer gives the following operating values:

	Volts
Plate voltage	22.5
Screen-grid voltage	100
Suppressor-grid bias	−10
Control-grid voltage	0

The manufacturer states that with these operating voltages the negative resistance, which is the reciprocal of the negative transconductance, is between 3,400 and 4,000 ohms. With a dynatron the negative resistance obtainable is usually between 40,000 and 50,000 ohms. As an example, suppose that the circuit shown in Fig. 18–17 is to produce a frequency of 10 kc and that a coil with an inductance of 85 mh and a resistance of 1,000 ohms is available. The capacitance necessary to tune this inductance to 10,000 cps is found by solving Eq. (17–12) or (17–13) for C. This results in

$$C = \frac{1}{L\omega^2} = \frac{1{,}000}{85(2\pi \times 10{,}000)^2} \text{ farad} = 0.00298\ \mu\text{f}$$

The apparent resistance of the tuned circuit is given by Eq. (17–25d):

$$R = \frac{L}{Cr} = \frac{85 \times 10^{-3}}{2{,}980 \times 10^{-12} \times 1{,}000} = 28{,}500 \text{ ohms}$$

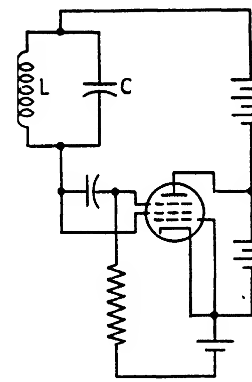

FIG. 18-17.—The circuit of a negative-transconductance oscillator.

The negative resistance of the tube is considerably smaller than the positive resistance of the tuned circuit; consequently, no trouble will be experienced in maintaining this circuit in oscillation. As a matter of fact, if it were required, it would be permissible to connect a load parallel to the resonant circuit and still maintain the oscillations. The old dynatron, on the other hand, would not be able to maintain this particular resonant circuit in oscillation.

As far as the coupling capacitor and the grid-leak resistor shown in Fig. 18–17 are concerned, it should hardly be necessary to point out that the remarks made in connection with capacitance coupling between amplifier stages also apply to this combination. In other words, at the operating frequency the capacitive reactance of the capacitor should be small compared to the resistance of the grid-leak resistor. Furthermore, the grid-leak resistor actually represents a load connected across the oscillating circuit. In the above example where we had calculated the equivalent resistance of the resonant circuit as 28,400 ohms, we shall have to consider the grid-leak resistor in parallel to this equivalent resistance. But since

the grid-leak resistor is as a rule in excess of 100,000 ohms, its loading effect can usually be neglected.

Negative-transconductance oscillators have the same frequency stability as found in the old dynatron oscillators without having their disadvantages. They are simple to construct and will operate up to frequencies of several megacycles; their upper-frequency limit is given by the electron transit time.

PROBLEMS

18–1. A resonant circuit consisting of an inductance of 200 mh with a resistance of 250 ohms and a capacitor of 0.05 μf is placed in the plate circuit of a 6J5 tube. The plate voltage is 250 volts, and the grid is biased to -8 volts. Under this operating condition the plate resistance is 7,700 ohms, the amplification factor 20, and the transconductance 2,600 micromhos.

a. What must be the least mutual inductance between the coil of the resonant circuit and a coil placed in the grid circuit to permit oscillations to start?

b. What will the ratio be between the voltages appearing across the plate coil and across the grid coil?

18–2. For measuring purposes, the coil in the tank circuit of Prob. 18–1 is disconnected from the capacitor and connected in series with the grid coil. The total inductance of the two coils in series measures 280 mh; with one reversed it measures only 200 mh. Under this condition would the circuit of Prob. 18–1 have a chance to oscillate after restoring the original connection, proper phasing of the coils assumed, of course?

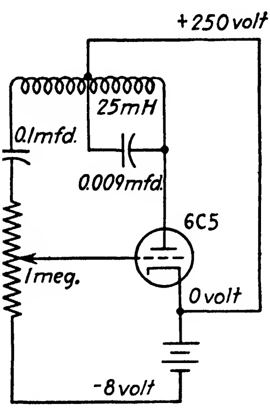

FIG. 18–18.—Circuit diagram for Prob. 18–3.

18–3. The total inductance of a single-layer, center-tapped coil has been measured as 58 mh; the inductance of each section (from center tap to either end) measured 25 mh. The resistance of the whole coil is 133.3 ohms. The coil is connected in a circuit, as shown in Fig. 18–18. For the operating conditions given in this figure, the 6C5 has a plate resistance of 10,000 ohms, an amplification factor of 20, and a transconductance of 2,000 micromhos. What fraction of the 1-megohm potential divider must be at least included in the grid circuit to permit oscillations to start?

18–4. A tuned circuit consisting of an inductance of 12.5 mh and a capacitor of 0.02 μf is placed in the grid circuit of a type 6J7 tube, triode-connected. The connections are as shown in Fig. 18–13. The resistance of the coil is 90 ohms. The coil in the plate circuit of the tube has an inductance of 2 mh. The tube is operating with 180 volts on the plate and -5.3 volts on the grid, under which condition the manufacturer gives the following values: $r_p = 20{,}000$; $\mu = 20$; $g_m = 1{,}800$ micromhos.

What mutual inductance must at least exist between the coil in the plate circuit and the tank circuit to make oscillations possible?

18–5. Two coils L_1 and L_2 are inductively coupled to each other. When connected in series, their inductance measures either 142 or 118 mh, depending on their connection. L_1 alone measures 100 mh and has a resistance of 40 ohms. The two coils are connected in a circuit as shown in Fig. 18–19. With operating voltages as shown in

this figure, the manufacturer gives the following values for a 6P5G: r_p = 9,500 ohms; μ = 13.8; g_m = 1,450 micromhos. What is the lowest frequency at which this circuit can possibly oscillate, and what size capacitor will be needed across L_1?

The 6P5G is now replaced with a 6N7. This tube is a double triode. With the two grids tied together, likewise both plates, the manufacturer gives the following values for a plate voltage of 250 volts and a grid voltage of −5 volts: r_p = 11,300 ohms, μ = 35, g_m = 3,100 micromhos. What will the lowest frequency be now? What size of capacitor must be used in the tank circuit?

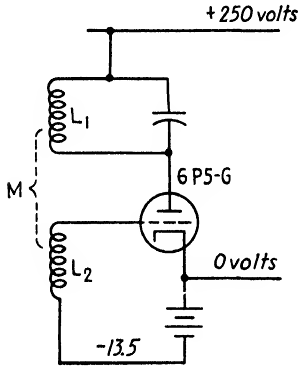

FIG. 18-19.—Circuit diagram for Prob. 18-5.

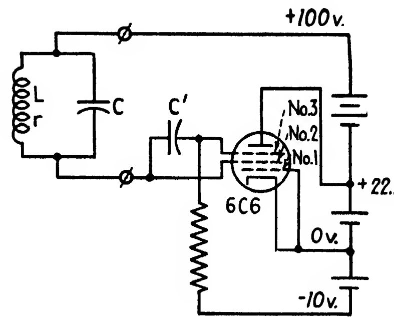

FIG. 18-20.—Circuit diagram for Prob. 18-7.

18-6. The plate characteristic of a tetrode for a certain grid and screen-grid voltage has been determined, giving the values shown in the accompanying table.

e_b, volts	10	20	30	40	50	60	70	80	90	100
i_b, ma	2.0	2.6	2.3	2.0	1.75	1.4	1.0	0.9	1.1	1.5

A parallel resonant circuit consisting of a coil with an inductance of 80 mh, a resistance of 60 ohms, and a capacitor of 0.02 μf is connected to the tube in a dynatron circuit. Will oscillations be established? If so, how much resistance could be placed in series with the coil before oscillations will stop?

18-7. *R.C.A. Application Note* 45, February, 1935, states that a type 6C6 tube operating with the voltages given in Fig. 18-20 exhibits a negative resistance on the screen grid if the latter is coupled through a sufficiently large capacitor (or coupling battery, of what value?) to the suppressor grid. The value of the resistance is given as −4,000 ohms. (This statement means that if screen grid and suppressor grid are both made 1 volt more positive, the current to the screen grid will *decrease* ¼ ma.) The tuned circuit shown in Fig. 18-20 consists of L = 0.1 henry, r = 80 ohms, C = 0.1 μf.

a. At what frequency will the circuit oscillate?
b. How much resistance could be placed parallel to the tuned circuit without stopping oscillations?

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CHAPTER XIX

BEAT-FREQUENCY OSCILLATORS; RESISTANCE-CAPACITANCE TUNED OSCILLATORS; PHASE-SHIFT OSCILLATORS

19–1. Frequency Limitations of Inductance-capacitance Tuned Oscillators.—Inductance-capacitance tuned oscillators, such as described in Chap. XVIII, are very satisfactory generators of ac power of any frequency. If the tank circuit has a high Q, the current flowing in it and the voltage appearing across it will be very nearly of sinusoidal wave shape. It is only in one respect that these circuits leave something to be desired: when it is required to produce a generator with a frequency variable over a fairly large range. Evidently, when it is desired to change the frequency of the generated oscillations, it is necessary to change either the inductance or the capacitance (or both) of the tank circuit. Since these values appear under the square-root sign in the fundamental formula for the frequency, this means that it requires, for instance, a capacitance or an inductance change in the ratio 9:1 to produce a frequency change in the ratio of 3:1. It is not too difficult to manufacture variable air capacitors of this range, such as used in any radio receiving set. Even there the minimum capacitance, which is determined by the stray capacitance existing between the movable plates and the fixed plates after the former have been turned completely out of mesh with the fixed plates, is usually approximately 10 per cent of the maximum capacitance. This is a 10 to 1 change in capacitance, and the maximum frequency change obtainable if such a capacitor is included in a tank circuit would be, roughly speaking, 3 to 1. Apart from the considerations just outlined, air capacitors are usually of low-capacitance values, seldom exceeding a value of 0.002 μf. This restricts their use to high-frequency circuits, such as about 100,000 cps, because for lower frequencies the inductance would become impractically large. Variable inductances with a range of 10 to 1 or so are even harder to construct than variable capacitors. Therefore, if it is desired to design an oscillator with a frequency range of, say, 100 to 10,000 cps, such as may be required in the investigation of audio-frequency problems, it would be rather difficult to use an inductance-capacitance tuned tank circuit.

19–2. Phenomenon of Beats.—One of the earliest solutions of this problem was the beat-frequency oscillator. The phenomenon of beat frequencies is of great importance, not only in the construction of oscillators of this kind, but also of practical use in many other electronic devices. A

complete and thorough discussion of this subject such as the radio engineer would demand is beyond the scope of this book, but the reader should be familiar with the fundamentals underlying it.

In Fig. 19–1*a* let two ac generators be connected in series. One furnishes a peak voltage of 100 volts at a frequency of 100 cps; the second furnishes a voltage of 25 volts peak with a frequency of 101 cps. What will be the total voltage appearing across the series combination of these two generators? From the vector representation of alternating voltages discussed in Sec. 3–6, the instantaneous values of voltage existing across the two generators can be considered as the projections of two rotating vectors, one of the length 100 and rotating with a speed of 100 rps, the

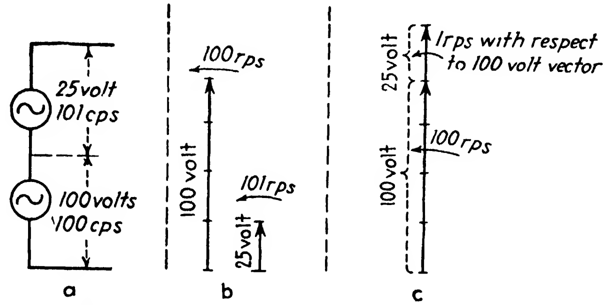

FIG. 19-1.—The voltage resulting from the series connection of two voltages of slightly different frequencies can best be visualized by using their vector diagram.

other a vector of the length 25 and rotating with a speed of 101 rps. Assume that we can catch the two vectors at the instant when they are both in a vertical position as shown in Fig. 19–1*b*. At this instant, evidently, the two voltages are exactly in phase, and the total voltage across the two generators at this instant will be 125 volts. After $\frac{1}{200}$ sec, the 100-volt vector will have made exactly $\frac{1}{2}$ revolution; the 25-volt vector will have made approximately $\frac{1}{2}$ revolution, being slightly ahead of this position. The total voltage will therefore still be very nearly 125 volts but of opposite polarity. Now, consider the situation after $\frac{1}{2}$ sec has elapsed. At this time the 100-volt vector will have made 50 complete revolutions and will therefore be in exactly the same position as at the beginning. The 25-volt vector, however, will have made 50.5 revolutions and will therefore be 180 deg out of phase with the original position. This means that the instantaneous values of the two voltages now oppose each other, with the result that the amplitude of the total voltage is now only 75 volts. If we place an ac voltmeter across the two generators, its indication would evidently fluctuate between 89 volts (which is the rms value of a sinusoidal alternating voltage with a peak of 125 volts) and 53 volts; these fluctuations would take place at a frequency of 1 cps. An even clearer insight can be obtained by applying the rules of vector addition of alternating

voltages to this problem. If the two voltages of Fig. 19–1*a* were of the same frequency, say, 100 cps, their vectors would simply be added together, and the resulting figure would then have to be considered as rotating with a speed of 100 rps. In Fig. 19–1*b* the 100- and the 25-volt vectors are shown separately. As outlined above, the instantaneous value of the total voltage is found by adding the length of their projections at any given instant. The same result would obviously be obtained if we add the 25-volt vector to the 100-volt vector, as shown in Fig. 19–1*c*. Instead of keeping them in a fixed relation to each other, however, as would be the case when the two frequencies are alike, we consider the 25-volt vector rotating with respect to the 100-volt vector with a speed of 1 rps. This means that while the 100-volt vector makes 100 revolutions (taking 1 sec), the 25-volt vector makes 1 revolution with respect to the 100-volt vector. The total length of the resulting vector—the projection of which gives us the instantaneous value of the total voltage—is seen to vary between the extreme limits of 125 and 75. It cannot be emphasized enough, and the reader should be entirely clear about the fact, that the resulting voltage *does not* contain a component with a frequency of 1 cps. The manner in which it was originated alone indicates that not more or less than two voltages, one of a frequency of 100 cps and the other of 101 cps, are present in the resulting voltage. Suppose we were to increase and decrease a 60-cycle voltage rhythmically two times per second. A meter connected across such a voltage would give an indication fluctuating with a frequency of 2 cps. But this *does not* mean that the 60-cycle voltage now contains a component with a frequency of 2 cps, any more than that a steady reading of the ac voltmeter is an indication that the alternating voltage has a dc component.

19–3. Production of Beat Voltage by Rectification.—Now let the frequencies of the two alternating voltage sources of Fig. 19–1*a* be stepped up to 1,000 times the value given in this example; in other words, let the frequency of the 100-volt source be 100,000 cps and the frequency of the 25-volt source be 101,000 cps. The voltage resulting from the addition of these two will now be a voltage of essentially 100,000 cps frequency but varying in amplitude between the limits 125 and 75 volts at a rate of 1,000 times per second. This is indicated in Fig. 19–2, although, for the sake of clarity, only a few cycles of the high frequency are shown. If such a voltage is connected to a telephone, we shall hear nothing because this

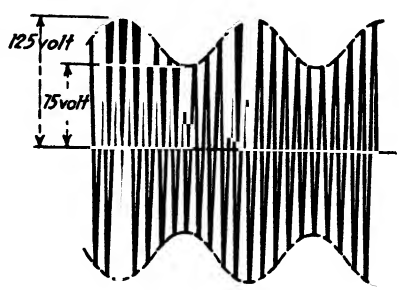

FIG. 19-2.—The sum of the two voltages shown in Fig. 19–1 is a voltage whose amplitude varies with a frequency equal to the difference of the frequencies of the two original voltages.

voltage attempts to move the diaphragm of the telephone back and forth at a rate of 100,000 times per second. In the first place, the diaphragm cannot move with this frequency, but even if it could, we could not hear it. Now let the voltage pictured in Fig. 19-2 be rectified. A rectified alternating current has, of course, a dc component, and if the alternating current changes in magnitude, so will the direct current. If we therefore apply the sum of the two high-frequency voltages to a detector circuit such as was described in Chap. XVI, there will appear in the output of the detector a current or voltage with a frequency equal to the difference or "beat" frequency of the two high frequencies. To be sure, the original frequencies will also be contained in the output of the detector because the latter will consist simply of a series of half waves of the high frequency, the amplitudes of which will vary with the beat frequency. In other words, the output of the detector will look simply like Fig. 19-2 with the negative half waves removed. These high-frequency variations could be considered simply as the ripple and can therefore be filtered out in a manner entirely analogous to the method described in connection with filter circuits for rectifiers.

19-4. Difficulties in the Design of Beat-frequency Oscillators.[1, 2]—The application of this principle makes it possible to construct a low-frequency generator with a very wide range of frequencies by employing two high-frequency generators, one of which can be varied in frequency over a comparatively narrow range. Thus, if one of the oscillators furnishes a fixed frequency of 100,000 cps and the other is equipped with a variable capacitor that will permit a change of frequency from 100,000 to 110,000 cps, then their beat frequency will vary from 0 to 10,000 cps when the variable-frequency oscillator is varied between the limits stated above. The circuit arrangements by means of which the principle just outlined is put in practice are manifold; for this reason, no particular one will be shown. In the simplest case two coils, each of which is coupled by mutual inductance to the two tank circuits of the two oscillators, are placed in series, and the sum of the two voltages is applied to a vacuum-tube detector. The scheme seems to be simple and straightforward enough, but in practice things are usually not quite so rosy. Suppose we wish to obtain a beat frequency of 100 cps by beating a frequency of 100,000 cps against one of 100,100 cps. If this scheme is to be successful, the two frequencies must remain at the exact values just stated; if either one of them should drift only 10 cps, which is 1 part in 10,000 of their operating frequency, the beat frequency will change to 90 or 110 cps, thus upsetting the calibration of the dial by means of which the frequency is adjusted. Such small drifts are hardly avoidable, and the designer of beat-frequency oscillator circuits usually does not even attempt to avoid them. He rather tries to build the two oscillator circuits, the fixed and the variable one, physically as nearly alike as possible, even to the point of making sure that

both circuits and their components will be subjected to the same temperature rise; with such an arrangement, it can be assumed that the frequency will drift the same amount in both oscillator circuits, which means that the beat frequency will remain unchanged. The problem of frequency stability of oscillator circuits has received a great deal of attention, but a more detailed discussion would be beyond the scope of this book. Another trouble commonly experienced with beat-frequency oscillators is their tendency to pull into step with each other if their operating frequencies are very near to each other, *i.e.*, if the beat frequency is low. In such a case it may be found necessary to apply the two voltages at first to two buffer amplifiers before combining them and applying them to a detector. The two buffer amplifiers prevent interaction between the two oscillator circuits.

19-5. Applications of the Principle of Beat Frequencies.—The principle of comparing two frequencies by observing their beat frequency is one of the most sensitive methods in electrical measuring technique. The capacitance of two similar flat plates facing each other is proportional to their area and inversely proportional to the distance between them. The capacitance of two flat plates in air is given by the following relation:

$$C = 0.0885 \frac{A}{d} \mu\mu\text{f} \tag{19-1}$$

A is the area of each plate in square centimeters; d is the distance between them in centimeters.

Suppose that two plates spaced 0.05 in., or 50 mils, apart form the capacitor in an oscillating circuit and that the inductance is of such a value as to cause the frequency to be 1,000,000 cps. If these two plates should change their distance 1/10,000 in., or 0.1 mil, this would represent a change of 1 part in 500 of their original distance. Owing to the inverse relationship existing between the distance and the capacitance, the latter will also change 1 part in 500. The frequency of the oscillations, being inversely proportional to the square root of the capacitance, will therefore change 1 part in 1,000. The original frequency was 1,000,000 cps and will therefore now change by 1,000 cps. If this frequency is made to beat against a fixed frequency of 1,000,000 cps, it is evident that the small motion of the two plates with respect to each other will produce a beat-frequency change from 0 to 1,000 cps. Instruments for the measurement of displacement as low as one-millionth of 1 in. have been designed with this principle as a basis. Generally speaking, any physical quantity that can be converted into a capacitance can then, by incorporating this capacitance into an oscillating circuit, be converted into a frequency that in turn can be used for measuring or controlling purposes.

19-6. Tuning Effect of Resistance-capacitance Combination.[3]—The analysis of certain resistance-capacitance networks shows that they exhibit to

some degree the same properties as inductance-capacitance tuned circuits. These circuits permit the design and construction of oscillators with a wide range of frequencies in the audio band, without having to resort to the principle of beat frequencies. Figure 19–3*a* shows a series combination and a parallel combination of two equal resistors and capacitors. Suppose that we apply to the combination a voltage E_i of variable frequency and try to determine the voltage E_o that will appear across the parallel combination of the resistor and the capacitor. Evidently when the input voltage E_i is of very low frequency—in the extreme let it be a direct voltage —the output voltage E_o will be zero because no direct current can flow through the circuit continuously. On the other hand, if the frequency is

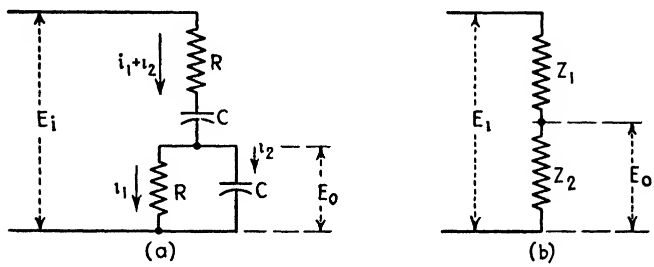

FIG. 19 3—At a frequency for which the capacitive reactance of the two capacitors is equal to the resistances, the output voltage E_o will reach a maximum and will be in phase with the input voltage

extremely high, the capacitive reactance of the capacitors C will be negligibly small compared to the resistance of the upper resistor R, and the voltage appearing across the lower capacitor, *i.e.*, the output voltage E_o, will again be zero. This reasoning indicates that there must be between zero and infinite frequency a certain frequency at which the output voltage E_o will be a maximum.

19–7. Calculation of Ratio of Input to Output Voltage by Complex Method.—Instead of calculating what the output voltage E_o will be for a given input voltage E_i, we shall reverse the problem. Let us find the required input voltage E_i to produce a given and desired output voltage E_o. Those familiar with the complex method of calculating ac circuits will have no difficulty in solving this problem in a quick and convenient way. The circuit shown in Fig. 19–3*a* is, after all, nothing but a special case of the circuit shown in Fig. 19–3*b*. For this latter circuit we have

$$E_i = E_o \frac{z_1 + z_2}{z_2} \tag{19–2}$$

In our case, the impedance z_1 is equal to the series combination of the resistor and the capacitor, and the impedance z_2 is equal to the parallel com-

bination of the two. With $x_c = 1/2\pi fC$, Eq. (19–2) will assume the following shape for the circuit shown in Fig. 19–3:

$$E_i = E_o \frac{R - jx_c - \dfrac{Rjx_c}{R - jx_c}}{-\dfrac{Rjx_c}{R - jx_c}} \tag{19–3}$$

Equation (19–3) can be simplified by multiplying the numerator and denominator with $R - jx$. This results in

$$E_i = E_o \frac{R^2 - 2jRx_c - x_c^2 - jRx_c}{-jRx_c} = E_o\left(3 + j\frac{R^2 - x_c^2}{Rx_c}\right) \tag{19–4}$$

The result obtained in Eq. (19–4) discloses a very interesting fact. For a frequency at which the capacitive reactance x_c is equal to the resistance R, the second member in the parenthesis becomes zero and the input voltage will be equal to three times the required output voltage; at the same time the input voltage will be exactly in phase with the output voltage. For any other frequency the input voltage will be larger than three times the output voltage and will also be phase-displaced with respect to the output voltage. It is obvious that the circuit exhibits somewhat the properties of a tuned circuit. With a constant input voltage of variable frequency, the output voltage will reach a maximum and will be in phase with the input voltage at a frequency for which the capacitive reactance of the capacitor is equal to the value of the resistor. The maximum value of the output voltage will be one-third the input voltage under this condition.

19–8. Alternate Method of Calculating Voltage Ratio.—A treatment not requiring the knowledge of the complex method is given in the following, which will be found instructive even to those who have been able to follow the solution presented in Sec. 19–7. Referring again to Fig. 19–3*a*, let us suppose that we wish to establish a voltage E_o across the lower resistor R. Through the resistor will then be flowing a current i_1, and the current through the capacitor C will be i_2. The magnitude of i_2 will depend on the frequency, but under all conditions it will lead the current i_1 by 90 deg. The sum of these two currents i_1 and i_2 flows through the series combination, establishing a voltage across this combination. Since the principle of superposition is valid, this voltage may be obtained by finding the voltages produced by the two components i_1 and i_2 separately. In other words, let us find the voltage that the current i_1 *alone* would produce in the series combination of R and C, then repeat this for the current i_2, and then add the two voltages to obtain the total. The current i_1 flowing through the lower resistance R in Fig. 19–3*a* produced the voltage E_o. Flowing through the upper resistor R of identical value, it

will consequently produce again a voltage E_o that will be in phase with the output voltage. This current i_1 flowing through the capacitor C will produce a voltage across the latter that will lag the current—and therefore also the output voltage E_o—by 90 deg. Now, let us turn to the voltages produced by the current i_2 in the series combination R and C. Since, in the parallel combination, the voltage E_o produces a current i_2 through the capacitor C, then logically this same current flowing through the upper capacitor C of the identical value would cause a voltage E_o across this capacitor too. This voltage will be in phase with the output voltage E_o. The voltage produced by the current i_2 flowing through the upper resistor R will be in phase with this current i_2, and since the latter leads the voltage E_o, the voltage caused by it across the upper resistor R will also lead the voltage E_o. To sum up, we have then the four components of voltage shown in the accompanying table (see Fig. 19-3a).

	Voltages produced	
	In upper R	In upper C
By current i_1	E_o in phase with output voltage E_o	i_1x_C lagging output voltage E_o by 90 deg
By current i_2.	i_2R leading output voltage e_o by 90 deg	E_o in phase with output voltage E_o

This tabulation shows that the voltage existing across the series combination R and C consists of four components. Two of them are equal to and in phase with the output voltage E_o, which means that the input voltage will always have an in-phase component with the output voltage equal to three times the latter. The input voltage will also have a leading and a lagging component with respect to the output voltage, as given in the table. When these two are numerically alike, they cancel so that the input voltage will be simply three times the output voltage. This will be the case when i_1x_C is equal to i_2R, which condition will be fulfilled when x_C is equal to R. This also makes i_1 equal to i_2.

19-9. Use of the *R-C* Network for Oscillators.[4-9]—Let us assume that we can obtain an amplifier furnishing an output voltage equal to slightly more than three times the input voltage and *in phase* with the input voltage. Since in a resistance-coupled stage a 180-deg phase reversal takes place, it is obvious that an amplifier consisting of two such stages will fur-

nish an output voltage that will be in phase with the input voltage. The adjustment of the voltage gain to the desired value of slightly above three could be obtained by the proper choice of load resistors or also by controlling the amount of voltage coupled from the first to the second stage, by making the grid resistor of the second stage a potential divider, for instance. (A different method will be discussed presently however.) Now, if we should couple the output of such an amplifier to the input terminals of the network shown in Fig. 19–3*a* and connect the output terminals of this network to the input terminals of the amplifier, it is clear that the conditions are fulfilled for the system to maintain itself oscillating at a frequency for which R is equal to x_C. For any other frequency, the voltage gain of the amplifier of slightly above three would not be sufficient, nor would the phase relationship be correct, to furnish a voltage to the input terminals of the amplifier of sufficient magnitude to keep the system oscillating.

The principle outlined in the preceding paragraph has been very successfully applied in a commercially available audio oscillator. Several models are available, ranging from as low as 2 cps to as high as 200,000 cps. The frequency-determining network is exactly as shown in Fig. 19–3*a*. The capacitors are variable air capacitors mounted on the same shaft so that their values change together. Going from the minimum to the maximum value of these capacitors usually changes the frequency in the ratio 10:1; by means of a tap switch different pairs of resistors can then be inserted in the frequency-determining network. This provides several ranges, each covering a 10 to 1 change in frequency.

19–10. Automatic Adjustment of Voltage Gain to Maintain Oscillations.[4–8]—In Secs. 19–6 to 19–8 it was shown that the network of Fig. 19–3*a* exhibits a tuning effect. This effect is, however, not anywhere near so sharp as an inductance-capacitance resonant circuit. For this reason a voltage gain of the above-mentioned amplifier even slightly in excess of what is needed to maintain the circuit in oscillation will lead to considerable distortion. The commercial oscillator mentioned in Sec. 19–9 employs a very ingenious scheme to maintain automatically the voltage gain of the amplifier at exactly the value required for the maintenance of oscillations in the system. The fundamental principle accomplishing this is shown in Fig. 19–4. Both tubes are shown as triodes for simplicity's sake; in the actual equipment pentodes are employed, but this has no bearing on the fundamental principle. The output voltage of the first tube T_1 appearing across the plate load R_1 is transferred by means of the capacitive coupling C_2-R_2 to the grid of the second tube. The output voltage of this tube appears across its plate load R_3. First consider the resistor-capacitor network to the left of the line XX as well as the capacitor C_4 and the resistor R_4 as absent. If we now apply an alternating voltage e_i to

the grid of the first tube, there will appear on the anode A_2 of the second tube an output voltage e_o determined by the over-all voltage gain of the two stages. Thus, if the first stage has a voltage gain of 50, and the second stage has a voltage gain of 10, the total voltage gain from the grid of the first tube to the anode of the second tube will be 500. Let us consider what happens when we place the capacitor C_4 and the resistor R_4 back into the circuit. Suppose that the capacitor C_4 is of high value so that its capacitive reactance is low compared to the sum of the two resistors R_4 and R_5 for any frequency for which we wish to investigate the

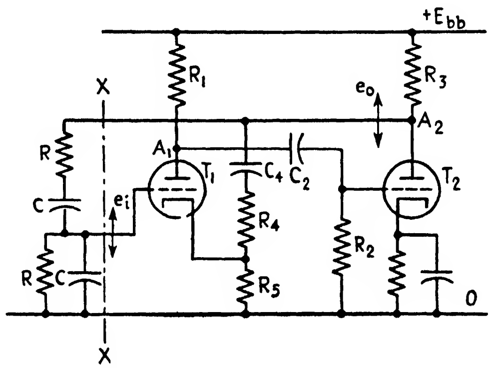

FIG. 19-4.—The circuit utilizing the tuning effect of the simple network shown in Fig. 19-3; the two resistors R_4 and R_5 provide negative feedback, the amount being controlled by the size of R_5.

performance of the amplifier. The potential variations of point A_2 (*i.e.*, the output voltage) will be partly transmitted to the cathode of the first tube, the amount being determined by the ratio of the two resistors R_4 and R_5. But any "bobbing" of the cathode with respect to the grid means just as much an input to the first tube as the more conventional bobbing of the grid with respect to the cathode. We have therefore a feedback system in this circuit. The next question is whether this feedback is regenerative or inverse. Let us analyze what happens when the grid of the first tube swings positive, or bobs up. This makes the anode of the first tube swing down. The grid of the second tube will also swing negative, owing to the coupling provided by capacitor C_2. The anode A_2 will therefore swing up, and so will the cathode of the first tube, owing to the coupling provided by C_4 and the action of the voltage divider R_4-R_5. The swing of the cathode is seen to be in the same direction as the originally assumed swing of the grid and is therefore evidently opposing what the grid is trying to do. (To be sure about this matter consider some numerical values. When the grid bobs up 1 volt, and the cathode 0.9 volt, then the grid has really become only 0.1 volt more positive with respect to the cathode than it was before the commotion started.) We have, therefore,

a case of negative or inverse feedback, and the over-all voltage gain of the amplifier is given by the following:

$$\frac{e_o}{e_i} = \frac{A}{1 - \beta A} \tag{15-13}$$

Here A is the voltage gain without feedback and β is the fraction of the output voltage fed back to the input. This was discussed in detail in Secs. 15-8 and 15-9.

In our case the feedback ratio β is $R_5/(R_4 + R_5)$ and is negative (since we have inverse feedback). The over-all voltage gain will therefore be the higher, the smaller R_5 is, reaching the full gain A of the amplifier when R_5—and with it β—becomes zero. For instance, when $R_5 = \frac{1}{2}R_4$, the feedback ratio β is equal to $\frac{1}{3}$, and the over-all voltage gain will be somewhat less than 3 (for a large value of A, as was shown in Sec. 15-10). If we should decrease R_5 to $\frac{1}{3}R_4$, β will be $\frac{1}{4}$, and the voltage gain will be approximately 4.

Our tuned capacitance-resistance network, connected as shown in Fig. 19-4 to such an amplifier, will therefore cause oscillations or fail to do so, depending entirely on the value of R_5. For low values of R_5, the amplifier will have a high over-all gain and is therefore certain to start and maintain oscillations; for higher values of R_5, it may stop oscillating.

The ingenious scheme referred to consists of automatically keeping R_5 at the exact value required to provide an over-all gain of 3. For this purpose R_5 is made a nonlinear resistance; in this particular case, a small Mazda lamp (3 to 6 watts, 110 volts). When the filament of this lamp is cold, it has a low resistance, amounting to a few hundred ohms. Before oscillations start, only the small quiescent plate current of the first tube flows through R_5 (which serves at the same time to provide bias); the filament of the lamp is fairly cold, the over-all gain high, and oscillations are certain to start. As soon as this takes place, however, an alternating current will begin to flow through the capacitor C_4 and the two resistors R_4 and R_5. The filament of the lamp will therefore become hotter, and the over-all gain of the amplifier will be reduced. If it should be reduced below a value of 3, the oscillations would stop and lead to a cooling off of the filament, which, in turn, would raise the over-all gain again. The output voltage e_o will therefore reach an amplitude of not more or less than required to pass an alternating current through the resistors R_4 and R_5, such that the value of R_5 will result in an over-all gain of 3. In discussing inductance-capacitance tuned oscillators, we saw that it was the nonlinearity of the tube characteristics that limited the amplitude. In the scheme just described, the tubes T_1 and T_2 are kept working in the linear part of their characteristics, but the required over-all gain is adjusted to the exactly required value by the rms value of the output voltage. Under

such a condition, the wave shape of the output voltage is a practically pure sine wave; the harmonic content of the output wave of such an oscillator is approximately 1 per cent.

19-11. Phase-shift Oscillators.[10-12]—Within the past few years another type of oscillator, making use of resistance-capacitance tuning, has been proposed. It is known under the name "phase-shift oscillator" and requires only one tube. The circuit is not so suitable for obtaining a wide range of frequencies since it requires the simultaneous variation of three components instead of just two. Where a single frequency is wanted, on the other hand, the extreme simplicity of the circuit makes it very attractive. The principle underlying the phase-shift oscillator is very simple. We saw that the voltage variations appearing on the plate of a tube with a resistive load are 180 deg out of phase with the voltage variations applied to the grid. Suppose that the voltage gain of a particular tube with a given resistance in the plate circuit is 10. This means that the application of an alternating voltage with a 1-volt amplitude to the grid will result in an alternating voltage 180 deg out of phase and an amplitude of 10 volts appearing on the plate of the tube. Now let us assume that we could design a capacitor-resistor network which, at one particular frequency, will give an output voltage 180 deg out of phase with the input voltage and with a value of the output voltage not less than 0.1 of the input voltage. If such a network is connected with its input terminals to the anode of the above-mentioned tube and its output voltage connected to the grid of the latter, it is evident that the conditions for the self-maintenance of oscillations at that particular frequency are fulfilled. This is exactly what has been accomplished in the phase-shift oscillator. In Fig. 19-5 the resistor R_1 and the capacitor C_1 are connected across a source of alternating voltage. The current in this circuit and also the voltage appearing across the resistor R_1 will lead the applied voltage by an amount determined by the ratio of the capacitive reactance x_C to the resistance R. Evidently, there must exist a frequency for which the voltage across the resistor R_1 will lead the applied voltage by exactly 60 deg. Now let us apply the voltage existing across R_1 to a second series combination of resistor and capacitor. Let us assume for a moment that the impedance of this series combination is made much higher than the value of the resistance R_1 so that its loading effect can be neglected. It is evident that a voltage may be obtained across R_2 that will again lead its input voltage, *i.e.*, the voltage across R_1, by 60 deg. This means that the voltage across

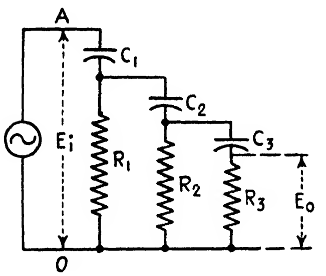

FIG. 19-5.—The network representing the heart of a phase-shift oscillator. For one particular frequency the output voltage E_o will be 180 deg out of phase with the input voltage E_i.

R_2 will lead the input voltage between points O and A by 120 deg. The addition of another series combination C_3-R_3 will finally give us a voltage across R_3 that will be 180 deg out of phase with the input voltage.

When the resistors and the capacitors in the network shown in Fig. 19–5 are alike, the assumption that the addition of each successive series combination will have no effect on the voltage that would appear across the preceding resistor without the additional stage can no longer be maintained. The basic reasoning still applies, however, even if each of the three phase shifts produced by the individual stages is no longer 60 deg. There will exist one frequency, for which the three shifts will add up to 180 deg, thus fulfilling the prerequisite for the use of this circuit in connection with a single tube. It can be shown that with three equal resistors and three equal capacitors the frequency at which the output voltage of the circuit shown in Fig. 19–5 is exactly 180 deg displaced from the input voltage is that frequency for which the capacitive reactance x_C bears the following relation to the resistance:

$$x_C = \sqrt{6}R \tag{19–5}$$

By substituting in Eq. (19–5) the expression $x_C = 1/2\pi fC$, we obtain the frequency at which the circuit will oscillate:

$$f = \frac{1}{2\pi\sqrt{6}RC} \tag{19–6}$$

The analysis of the circuit furthermore shows that with this frequency the output voltage will be one twenty-ninth of the input voltage. Tubes used for this circuit must therefore have an over-all gain—including the loading effect of the frequency-discriminating network—of not less than 29. Pentodes are usually employed with this circuit, but there are, of course, a number of triodes that would satisfy this condition. The diagram of a phase-shift oscillator is shown in Fig. 19–6. Variations of frequency in the order of 10 to 20 per cent may be obtained by making the last resistance, *i.e.*, the one connected to the grid of the tube, variable. This permits the adjustment of the frequency to an exact value, even if the capacitors and the resistors do not exactly correspond to the values used in conjunction with, or obtained by, Eq. (19–5) or (19–6).

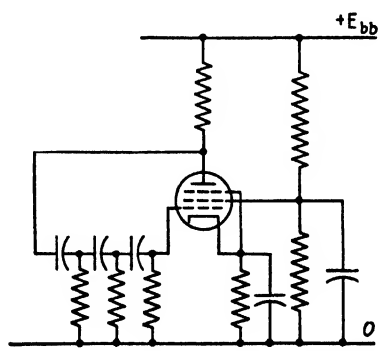

FIG. 19–6.—A single tube of sufficient voltage gain (usually a pentode) is sufficient to provide a phase-shift oscillator.

Inductance-capacitance tuned oscillators, resistance-capacitance tuned oscillators, and phase-shift oscillators all work essentially on the same principle. A frequency-discriminating network is placed between the output and the input of an amplifier, and at one particular frequency this network furnishes a voltage of the right phase relationship to maintain the circuit in oscillation. The output of these oscillators will be essentially sinusoidal if care is taken that the voltage fed back to the grid is only slightly in excess of what is needed for the maintenance of the oscillations. The operating point of the tube will then not swing very far into the nonlinear part of the characteristic. There are a number of circuits, however, that produce periodic variations of current or voltage on an entirely different basis. These will be taken up in the next chapter.

PROBLEMS

19-1. A pair of flat metallic plates, each measuring 2 by 2 in., face each other, with a clearance of 0.050 in. (50 mils) between them. One is mounted in a fixed position, and the other is fastened to a part that may move a few thousandths of 1 in. The capacitor formed by the two metal plates is part of a tuned circuit that is to have a frequency of 1,000 kc (1,000,000 cps).

a. What inductance will be needed?
b. If the tuned circuit is used as the tank circuit of an oscillator, beating its own frequency against an oscillator of a fixed frequency of 1,000 kc, what beat frequency will be generated if the two plates change their distance by ½ mil (0.0005 in.)?

19-2. Draw a graph showing the ratio of output voltage to input voltage as a function of the frequency of the applied voltage for the circuit shown in Fig. 19-3*a*. Let $R = 50{,}000$ ohms; $C = 0.1$ μf; suggested frequencies = 10, 15, 20, 30, 40, 60, and 90 cps. With the aid of the result, explain the principle of resistance-capacitance tuned audio oscillators.

19-3. A phase-shift oscillator is to be constructed as shown in Fig. 19-6, except that a 6SF5 triode is to be used in place of the pentode shown there. The plate characteristics of the tube are shown in Fig. 10-24.

a. If the resistors in the frequency-determining network are each 250,000 ohms and the desired frequency is 1,000 cps, what capacitors will have to be used?
b. If the available supply is 240 volts, if the resistor in the plate circuit is 50,000 ohms, and if it is desired to operate the tube with 200 volts actual plate voltage, what bias resistor should be used?
c. What would you do if an oscillographic observation shows a poor wave shape of the oscillations?

SUGGESTED ADDITIONAL READING

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CHAPTER XX

RELAXATION OSCILLATORS; MULTIVIBRATORS; TRIGGER CIRCUITS

20-1. Difference between the Operating Principles of Various Classes of Oscillators.[1, 2]—In Chaps. XVIII and XIX, oscillator circuits were discussed which furnish an output voltage of essentially sinusoidal wave shape. The fundamental principle underlying these circuits was to have the output terminals of an amplifier connected to a network, the output voltage of which in turn depended in size and phase on the frequency of the voltage applied to its terminals. The output voltage of this network was connected to the input terminals of the amplifier, and at one, and only one, frequency, the voltage applied to the amplifier was of the right value and phase relationship to maintain oscillations. Thus, in a tuned plate-circuit oscillator the amplifier is just one tube, which is connected to a parallel resonant circuit; part of this circuit, or a coil coupled to it by mutual induction, is connected to the input of the amplifier, *i.e.*, to the grid. At one particular frequency the voltage coming back to the grid is of sufficient magnitude and of the right phase relation to keep the circuit self-excited.

There are a number of circuits, however, the operation of which is based on an entirely different principle. Their common and distinguishing feature is that their output is a series of transients. The circuits usually do not contain a frequency-determining network, in the sense of the networks described in the preceding two chapters, but contain a combination capable of producing a transient. Included in the circuit are nonlinear elements that go into action when the transient has reached a certain value. The nonlinear elements then either start a new transient or nullify the effect of the first transient and cause the circuit to start all over again.

20-2. Glow-tube Relaxation Oscillator.[2]—One of the simplest oscillators operating on this principle is the relaxation oscillator employing a glow tube. Although gaseous tubes have not been discussed so far, there should be no difficulty in understanding the principle on which the glow-tube relaxation oscillator operates. The simplest glow tube available is an ordinary neon lamp such as is used where a small amount of light for signaling purposes is desired. The volt-ampere characteristic of a neon tube is as follows. Up to a certain voltage, called the "ignition" voltage or "breakdown" voltage, the tube does not conduct any current. After this voltage is reached, the tube will conduct current, but the amount of cur-

rent that it would take if the ignition or breakdown voltage were maintained would be so high as to cause destruction of the tube. In all practical applications of the neon tube, we must, therefore, provide a resistance in series with the lamp or tube that will limit the current after conduction has started. With ordinary neon lamps, rated at 110 volts, the breakdown voltage is usually approximately 60 volts. In the lamps designed for use in the ordinary light socket, a resistor is incorporated within the base of the bulb and limits the current flowing through the lamp itself after the voltage has reached the breakdown voltage, or exceeds it, as it naturally does when these lamps are used on ac circuits. After conduction has started, the voltage across the lamp itself will drop to a lower value, owing to the action of the series resistor. This lower value is called the "extinction" voltage of the lamp, and current flow will be maintained as long as the source of supply to which the combination of lamp and resistor is connected has a value exceeding the extinction voltage. When the supply voltage becomes less than the extinction voltage, however, current through the tube will cease. To reestablish the current flow, it is necessary to raise it to the breakdown voltage or slightly above it.

20-3. Frequency of Glow-tube Oscillators.—With the behavior of a glow lamp as described in the preceding section in mind, let us examine the circuit shown in Fig. 20-1. Let a voltage E in excess of the ignition voltage E_i of the neon lamp N be connected to the capacitor C and resistor R. Let us assume, for a moment, that the neon lamp N is absent. Upon closure of the switch S the capacitor C begins to charge, its voltage rising along the well-known exponential curve. This process was described in detail in Chap. II, and the reader will find the circuit shown in Fig. 20-1 exactly the same as that shown in Fig. 2-6. With the neon lamp placed across the capacitor, the process will be exactly the same as without it, as long as the voltage across the capacitor does not exceed the ignition or breakdown voltage of the neon lamp. But at the instant when the capacitor voltage has reached the ignition voltage of the neon lamp, the latter will ignite and discharge the capacitor almost instantaneously to the extinction voltage of the lamp. At this instant the lamp will extinguish, and the capacitor will begin to charge again until the voltage across it reaches the breakdown voltage, when the whole cycle just described repeats itself. The potential of point A therefore swings between the two values of ignition and extinction voltage; it rises exponentially from the

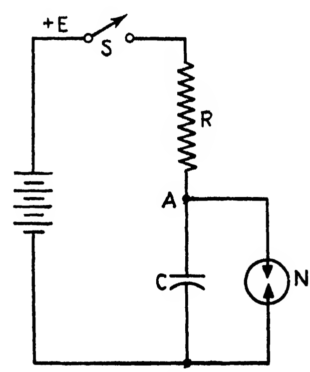

FIG. 20-1.—The fundamental circuit of a relaxation oscillator. The charging process will be interrupted when the voltage across the capacitor reaches the breakdown voltage of the neon lamp.

lower to the higher point and then suddenly drops back to the lower value. This condition is outlined in Fig. 20-2. It is apparent that the voltage variations of point A are essentially shaped like the teeth of a saw, and it is for this reason that voltages with this wave shape are often referred to as "saw-tooth" voltages. It is also apparent that the higher the supply voltage in comparison to the breakdown voltage, the more linear will be the rising part of the saw tooth. The frequency of such a circuit can evidently be predicted very easily when it is recognized that the time for one period is equal to the time it takes for the capacitor to charge from a voltage equal

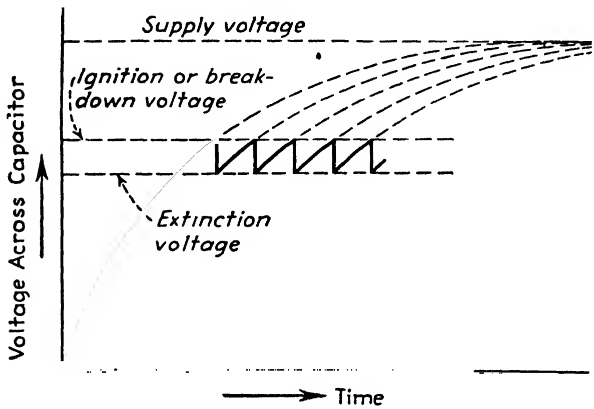

FIG. 20-2.—The performance of the circuit shown in Fig. 20-1 can be visualized with the aid of the figure presented here. The voltage across the capacitor is seen to fluctuate between the breakdown and extinction voltage of the neon lamp.

to the extinction voltage to a voltage equal to the breakdown voltage. Let us assume for a moment that the following values were given: supply voltage, 100 volts; ignition voltage, 60 volts; extinction voltage, 50 volts. Let the capacitor be 0.1 μf and the resistor 1 megohm. The time constant T of the combination is therefore 0.1 sec. When the capacitor is charged to the extinction voltage, or 50 volts, it will have to go another 50 volts before it can reach the maximum and final voltage; but breakdown of the glow tube will occur when the voltage has risen another 10 volts, or 20 per cent of the distance still separating it from the final value. In other words, we may consider the 50-volt point as the starting point and the 100-volt point the voltage that the capacitor would reach after an infinite time, provided that nothing interfered with the circuit. According to the capacitor chart shown in Fig. 2-8, it requires 0.22 time constant to swing through 20 per cent of the total voltage still separating the capacitor voltage from the final value. Since the time constant in this particular example was 0.1 sec, the period for one cycle will be 0.022 sec, which gives us a frequency of $1/0.022 = 45$ cps.

20–4. Mathematical Derivation of the Frequency of Glow-tube Oscillators.—Those who prefer to solve the problem strictly mathematically can make use of Eq. (2–13). With the extinction point regarded as the starting point for the process of charging, it is seen that the charging voltage E of Eq. (2–13) now becomes equal to the difference between the supply voltage and the extinction voltage. The problem is then simply to find the time in which the capacitor will charge to a voltage equal to the difference between ignition voltage E_i and extinction voltage E_e. With these substitutions Eq. (2–13) becomes

$$E_i - E_e = (E - E_e)(1 - \epsilon^{-t/T}) \qquad (20\text{–}1)$$

Equation (20–1) may now be solved for t. This results in

$$\frac{E_i - E_e}{E - E_e} = 1 - \epsilon^{-t/T}$$

$$\epsilon^{-t/T} = \frac{E - E_e - E_i - E_e}{E - E_e} = \frac{E - E_i}{E - E_e}$$

and

$$t = T \ln \frac{E - E_e}{E - E_i} \qquad (20\text{–}2)$$

The reader may easily convince himself that the application of Eq. (20–2) will give exactly the same result as the one predicted on the basis of the preceding explanation.

20–5. Approximate Determination of Frequency.—Another method, giving only approximate results but giving a better insight in the process, is as follows. At the instant when the capacitor has been discharged to the extinction voltage, the voltage remaining across the resistor R is 50 volts; at the moment of the breakdown of the tube, this voltage will be 40 volts. The average voltage existing during the charging process can be considered as 45 volts, although this is, of course, not strictly correct since the voltage rises not linearly but along an exponential curve. In this particular example, however, it is evident that the error made by this assumption is negligible. The average current flowing through the resistor while the voltage on the capacitor rises from 50 to 60 volts is therefore 45 μa. The rate of rise of voltage across the capacitor is given by the fundamental Eq. (2–9). In this particular case the voltage across the capacitor will therefore rise at an average rate of 450 volts per sec. The time required to make the voltage rise by 10 volts is therefore given by $10/450 = 0.0222$ sec, which is exactly the same value as the one obtained by the other two methods.

20–6. Variation of Actual Frequency from Calculated Value.—The calculation of the frequency of a glow-tube relaxation oscillator by the methods

just described will be found to give results agreeing with actual performance only for frequencies not exceeding a few hundred cycles per second. The reason for the discrepancy between calculated and observed values for the frequency is that the extinction and ignition voltages themselves depend on the frequency. When the lamp has been extinguished but the voltage is raised to the breakdown voltage extremely fast, the lamp will not have regained its original dielectric strength and therefore is liable to breakdown at an earlier voltage value. The upper limit of frequencies obtainable by this method is approximately 10,000 cps. This limit is due not only to the factor just mentioned but also to the fact that the value of the charging resistor R cannot be made too low. Reduction of the value of this resistor will lead to a condition where the tube will not extinguish any more but will conduct current continuously. It is evident that under such a condition no further oscillations can take place. As a matter of fact, the question why a tube should extinguish at all, even with a high value of resistance R, seems to be one that most books avoid either posing or answering. Here we shall be content with posing the question and leave the answer to the physicists.

20–7. Relaxation Oscillators Using Thyratrons, or Hot-cathode Gaseous Tubes.—The foregoing discussion has centered around a neon lamp, which is a cold-cathode gaseous tube. Exactly the same considerations do apply, however, also when the neon lamp is replaced by a hot-cathode gaseous tube, also known as a "thyratron." These tubes will be discussed in great detail in a later chapter. As far as this particular application is concerned, however, the only differences between them and a neon lamp are that their breakdown voltage can be adjusted to almost any desired value by adjusting the voltage applied to the grid of the thyratron and that their extinction voltage is much lower than that of a neon lamp, being in the order of 15 volts. The circuit shown in Fig. 20–1, with the neon tube replaced by a thyratron, is a very popular one and is found in most commercial cathode-ray oscilloscopes where the saw-tooth voltage produced by it is used as a time base. A more detailed description of the circuit will therefore be deferred until that instrument and its requirements are discussed in a later chapter. This circuit is known as a "sweep circuit."

20–8. Multivibrator and the Fundamental Considerations Applying to It.—Another circuit making use of the charge and discharge of a capacitor is the multivibrator, which is of very great practical importance. It was first suggested in 1919 by Abraham and Bloch.

Various methods have been used to investigate the performance of a multivibrator circuit. In this book more emphasis will be placed on obtaining a clear understanding of the fundamental principle of the circuit than in the development of complicated formulas or the discussion of finer details. In Fig. 20–3*a* two tubes are shown operating from the same plate-supply voltage, both with -4 volts on the grid. If the tubes are two

6C5's and the resistors in the plate circuit are 50,000 ohms, the drawing of a load line with 200 volts plate-supply voltage into the plate characteristics shown in Fig. 9–5 will show us that the voltage across the tubes will be 100 volts, and the same amount of voltage will appear across the resistors. Points A and B will therefore be at exactly 100 volts. Now, it might be argued that anything that will place -4 volts on the grid of the two tubes will lead to exactly the same condition as just outlined. Instead of obtaining the grid bias by placing a 4-volt battery between the cathode and the grid, we might conceivably obtain it by the method shown

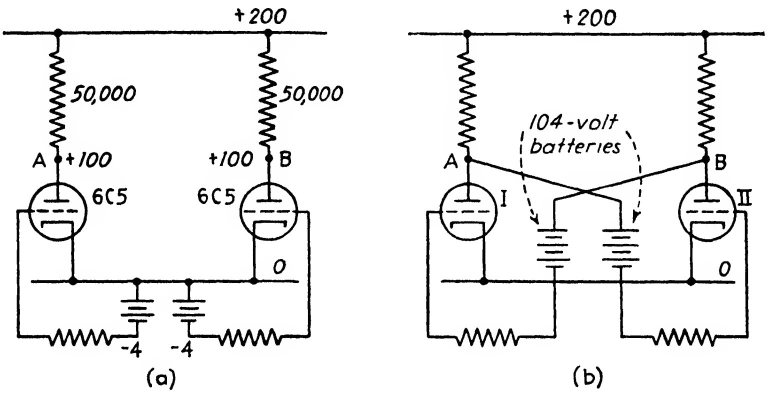

FIG. 20 3.—With -4 volts on the grids of the two 6C5's, the points A and B will both be at a potential of 100 volts. The circuit shown in (b), although apparently giving the same potentials as the circuit shown in (a), is not in a stable equilibrium.

in Fig. 20–3b. The grid bias of tube II is obtained by connecting a bias battery of 104 volts between A and the grid of tube II, reminiscent of the coupling scheme used in dc amplifiers. In the same manner, the bias of tube I is obtained from the anode of tube II, *i.e.*, point B. This circuit should result in exactly the same condition as the circuit shown in Fig. 20–3a. If it is set up, we shall not be able to obtain the same conditions as in circuit a. Let us now investigate why the two tubes will not operate exactly as in Fig. 20–3a. There is no denying the fact that when A is at exactly 100 volts the grid of tube II will be at -4 volts, its anode at exactly 100 volts, and the grid of tube I at exactly -4 volts, as it must be to make the anode A 100 volts positive. But now let us assume that because of a slight disturbance in the tube I, owing to a sudden burst of electrons from the cathode, for instance, the current in this tube increases slightly so that the potential of A comes down a small amount, say, 0.1 volt. This will make the grid of tube II also 0.1 volt more negative. The voltage gain of each of these two tubes is somewhere around 12 to 14, and the potential of point B will therefore go up 1.2 to 1.4 volts. But this point is coupled by the left-hand battery to the grid of tube I, which will therefore also go up this amount. With a voltage gain of this tube also

of from 12 to 14, the 1.2-volt change on this grid will make A come down about 15 volts, reinforcing the original disturbance of 0.1 volt approximately 150 times. Since this voltage is impressed on the second tube, it is clear that the state of affairs outlined in the beginning of this paragraph is physically impossible; in this condition the circuit would be in an unstable equilibrium, a slight disturbance making it go either one way or another. Now let us investigate in what state it will finally wind up. To continue our reasoning from above, if A comes down approximately 15 volts from its original 100-volt level, the grid of tube II would be at about -19 volts. A look at the plate characteristics of this tube indicates that this is enough to cut the tube off completely. With no current flowing through tube II, B will therefore be at the level of the plate-supply voltage, *i.e.*, 200 volts. This would make the grid of tube I 96 volts positive if it were not for the fact that a high resistance has been placed in series with the grid. The grid current that will begin to flow as soon as the grid gets near zero potential will cause a voltage across this resistor, taking up the difference in potential between the lower end of the battery and the cathode potential. If this resistance were, for instance, 1 megohm, a current of approximately 96 μa could be expected in this resistance. Tube I will consequently operate with a grid voltage of approximately zero volts; a load line drawn with 50,000 ohms intersects the zero grid-volt characteristic at a voltage of approximately 45 volts, which means that point A will be at a potential of 45 volts. This will make the grid of tube II 59 volts negative. Since this tube was already cut off with a grid voltage of -19 volts, it evidently would not make any difference how much more negative its grid becomes. The final state of affairs is as follows: one tube operates with approximately zero grid voltage, which makes its anode operate at a potential of approximately 45 volts. This depresses the grid potential of the other tube to a value of -59 volts, which is several times as much as is needed to produce cutoff in this tube. The cutoff in this tube in turn causes its plate to be at a potential equal to the plate-supply voltage, which in turn makes the grid of the first tube operate at the zero grid voltage with which this statement started. Which of the two tubes will be conducting and which will be nonconducting is seen to depend entirely on the first disturbance occurring in either one of them and the direction in which it occurs. If we force tube I by some outside disturbance into a conducting state, no matter for how short a period the disturbance imposed from the outside lasts, the tube will remain in this state, and tube II will be in the nonconducting state. Circuits which have two states of equilibrium and which can be thrown from one state into another by the introduction of a transient disturbance are also called "trigger circuits" and will be discussed in more detail later in this chapter.

20-9. Basic Multivibrator Circuit.[3, 4]—When discussing capacitance coupling, we started with direct coupling by means of a battery and then

replaced the battery by a capacitor. In a similar way, the trigger shown in Fig. 20–3b becomes a multivibrator circuit by the same process that converts a direct-coupled amplifier stage into a capacitance-coupled stage. The result of this modification is shown in Fig. 20–4, which is the basic multivibrator circuit. The discussion of the circuit of Fig. 20–3b ought to make an analysis of the multivibrator circuit relatively easy; nevertheless, there always seems to be uncertainty as to where to start with this circuit. One method of attack, which seems to be beyond reproach, is the following. Assume any state of affairs that you think might be possible in the circuit; then trace the phenomena taking place in the circuit with the assumed state of affairs as a starting point. If during the analysis of the further performance of the circuit a state should be reached equal to the one assumed as a starting point, then, obviously, the original assumption is tenable. Let us therefore start with the assumption that tube II is nonconducting, which means that its grid is somewhere below the cutoff value. Tube I, on the other hand, we shall assume as conducting. Remember that no claim is made that this assumption is correct, but if the subsequent analysis of the circuit starting with this assumption somehow leads us back to the same condition, it certainly will seem reasonable to consider the assumption as correct. If tube II is nonconducting, its plate, *i.e.*, point B, will be at a potential equal to the plate-supply voltage; if at the same time the grid of tube I is assumed at or near zero potential, capacitor C_2 must be charged to a voltage equal to the supply voltage; in our case, 200 volts. This is evidently a completely stable condition. There is no reason why the voltage across the capacitor should change as long as the two electrodes are at levels just mentioned. When the grid of tube I is at zero potential, its plate, *i.e.*, point A, will be at a level of approximately 45 volts, as shown by the load line. Since tube II was assumed as nonconducting, its grid must be below the cutoff value, say, at -40 volts. The voltage across capacitor C_1 will therefore have to be, at this instant, 85 volts. In contrast to the potentials of the plate of tube II and the grid of tube I, the condition existing on the grid of tube II is not a stable one. If a voltage of -40 volts exists on the grid of tube II, there must be a current flowing through the grid resistor R_1; if the latter is, for instance, 1 megohm, a current of 40 μa is flowing in this resistor. Since the grid of the tube is not taking any current when it is negative, this current must be flowing through the capacitor C_1, the voltage across which must therefore be changing. The final state—if nothing else inter-

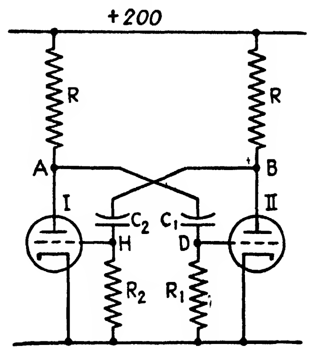

FIG. 20-4.—The basic multivibrator circuit. The capacitors C_1 and C_2 take the place of the coupling batteries shown in Fig. 20–3b.

feres before it can be reached—would be when the current through the resistor R_1 is zero, *i.e.*, when point D has reached zero potential and the voltage across the capacitor C_1 is equal to 45 volts. Point D, which, according to our assumption, was 40 volts negative, is therefore coming up along an exponential curve with the good intention of settling at the zero level. Its intentions may be very good, but it is due for a very unpleasant surprise. The load line indicates that tube II will begin to conduct current at the instant when its grid becomes less than approximately 15 volts negative. Therefore, at the very instant when D on its travel from the -40-volt level to the zero level reaches the value -15, tube II will begin to conduct current. But at the very instant when B begins to move downward, capacitor C_2, acting like a battery charged to 200 volts, will move H downward from its zero level. The grid of tube I is therefore made more negative, with the result that A goes up. This action takes D along, and the action taking place now is exactly the same as that described in conjunction with Fig. 20–3*b*. Point D is suddenly yanked upward; at the instant when this jump starts, the voltage across the capacitor C_1 is equal to approximately 60 volts since the potential of A is $+45$ volts, and the cutoff voltage of tube II is equal to -15 volts. Since A jumps suddenly to a level of 200 volts, D, as a matter of fact, would be instantaneously lifted to a potential of $200 - 60 = 140$ volts if it were not for the action of the grid current of tube II. This grid current very quickly establishes D at zero level and permits A to assume the level of $+200$ volts; in other words, by means of the grid current, the capacitor C_1 is very quickly charged to a voltage of 200 volts. Points A and D are therefore now established at potentials originally assumed for B and H, placing the grid of tube II at zero level. B is now at the 45-volt level. But just before B jumps from a level of 200 volts down to its new level of 45 volts, the capacitor C_2 was charged to 200 volts. Since the capacitor cannot change its voltage suddenly, the jump of B from the 200-volt level to the 45-volt level will mean a jump of H—which is the grid of tube I—from 0 to -155 volts. But it cannot stay at the -155-volt level since capacitor C_2 will discharge from its voltage of 200 volts to a voltage of 45 volts. Point H therefore starts out at a level of -155 volts and tends to go along an exponential curve to a level of zero. Evidently when it reaches the -15-volt level, tube I becomes conducting, and the whole process just described repeats itself on the other tube. Point A will now jump down from the 200-volt level to the 45-volt level, and point D will move correspondingly from the zero level to the -155-volt level. Tube II will be nonconducting, and tube I will be conducting. Point D will come up along an exponential curve from the -155-volt level to the zero level. At some instant during this interval it will clearly be at a level of -40 volts, and the whole circuit will be exactly in the condition we assumed for the starting point. Consequently, the condition assumed at the start of the analysis is one

that actually occurs sometime during the performance cycle of the circuit. A graph of the potentials of the four points of interest in this circuit is shown in Fig. 20-5.

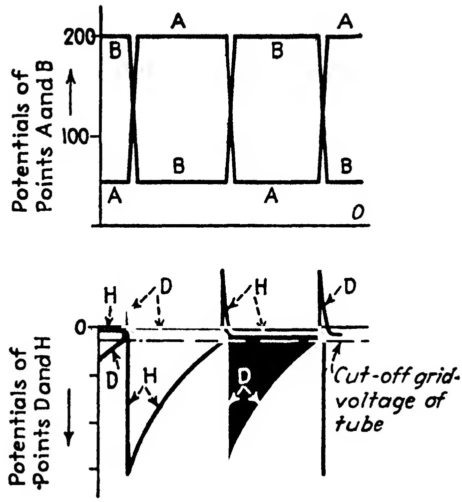

FIG. 20-5 —The potentials of the plates and grids for a multivibrator circuit.

20-10. Frequency of the Multivibrator.—If the foregoing analysis is clearly understood, there should be no difficulty in determining the frequency of the oscillations produced by this circuit. It is obvious that the most important factors are the two time constants of the resistor-capacitor combinations R_1-C_1 and R_2-C_2. It is also evident, however, that the size of the load resistor in the plate circuit of the tube will have an influence since it determines the jump of points A and B; furthermore, the cutoff voltage of the two tubes will have a bearing on the frequency. Until the recent appearance of an exhaustive treatise on the subject of multivibrators, various formulas had been given in the literature, all of them taking into account only the time constants of the two combinations just mentioned. The formulas given by various authors gave results differing as much as 100 per cent and were therefore all but useless. With the aid of the analysis just presented, on the other hand, there is no difficulty in arriving at surprisingly accurate values for the frequency. As far as the tube is concerned, the analysis shows that there are only two values that are of interest and must be known. The first is the amount of jump that the plate makes, and the second is the cutoff grid voltage for the tube operating with the chosen plate-supply voltage. The magnitude of the jump is the difference between the plate-supply voltage and the plate voltage when the tube is operating with approximately zero grid bias. In an actual circuit, the grid will probably not operate at exactly zero bias since, owing to the initial speed of the electrons, a voltage in the order of

approximately ½ to 1 volt may appear across the grid resistors R_1 and R_2. In the foregoing example, for instance, it is quite probable that the plate potential of the tube that is conducting will be nearer to 50 volts than to the assumed 45 volts. As far as the cutoff potential of the two tubes is concerned, it is also safer to assume that conduction begins with a voltage 1 or 2 volts more negative than that read from the plate characteristics. It takes only an extremely small plate current to initiate the switchover from one tube to the other, and the plate characteristics do not permit the accurate determination of the grid voltage that permits such a small current flow. Suppose that in the circuit shown in Fig. 20–4 the two grid resistors R_1 and R_2 are each 1 megohm, and the two capacitors C_1 and C_2 are 0.1 μf. When either tube becomes conducting, its plate jumps from a level of 200 volts to a level of 50 volts, which in turn makes the grid of the other tube jump from a level of 0 volts to a level of -150 volts. This tube will then remain in the nonconducting state until its grid has risen from the -150-volt level to the level of the cutoff voltage. Suppose that examination of the load line indicates that the cutoff voltage is approximately -16 volts. Conduction and therefore switchover from one tube to the other take place when the grid, during its travel from the -150-volt level to the zero level, has reached a level of -16 volts; in other words, when it has swept through 134 volts of the total sweep of 150 volts. This represents 89.2 per cent of the total sweep, and consultation of the capacitor charge and discharge chart discloses that a time equal to approximately 2.2 time constants is required. The time constant for the above chosen values for resistor and capacitor is 0.1 sec. Consequently each tube will be nonconducting (while the other tube will be conducting) for 0.22 sec. The total cycle will be 0.44 sec, and the frequency will be the reciprocal of this value, or approximately 2.3 cps. If the two capacitors are not alike, the two time constants will not be alike either, and the nonconducting time will be different for the two tubes. It is therefore possible to produce square waves with this circuit, the two half periods of which are not alike.

20–11. Wave Shape of the Voltages Produced by the Multivibrator.—From the preceding analysis of the circuit, one would be justified in assuming that points A and B will jump absolutely instantaneously from one level to the other and thus produce a rectangular wave with absolutely sharp corners. When the circuit is used for the production of low-frequency square waves, an oscillogram of the voltages of A and B seems to bear out this statement; but when an attempt is made to produce square waves of much higher frequency, a certain amount of rounding of the corners is noticeable, which for some applications is entirely undesirable. In Sec. 20–10, we saw that things began to happen when point D on its travel upward toward the cathode crossed the line of the cutoff voltage; at this instant the capacitor C_1 had a voltage of approximately 66 volts

across it. The resulting switchover made tube I nonconducting, and we assumed that this action would make its anode jump to the level of the plate-supply voltage. But can it actually do so? The capacitor C_1 has only 66 volts across it when point A starts going up toward the level of the plate-supply voltage. A complete jump of A to this level would therefore make the grid positive approximately 134 volts, under which condition it would take a current of many milliamperes. Whatever does take place, it is evident that the capacitor C_1 will be charged to the final 200-volt level by a current that will have to come through the plate resistor R of tube I. One could properly say that the charging of the capacitor C_1 to the final 200 volts is determined by the time constant of the circuit formed by this capacitor and the resistor R in the plate circuit of the tube plus possibly the effective resistance of the grid-to-cathode path of the tube II. Since the latter is a rather low value for positive grid voltages, the charging will be essentially determined by the time constant RC_1. It is for this reason that point A cannot jump suddenly to the level of the plate-supply voltage after the tube I has become nonconducting. The jump in the other direction, on the other hand, is a different matter. During this jump the grid of the opposite tube is being made more negative and consequently will not take any current. There will even be an overshoot in the negative direction because of the fact that the grid of the tube was pulled above the zero level for a short instant until the capacitor connected to it had an opportunity to charge to the full value of the supply voltage.

20–12. Use of the Equivalent Circuit for the Analysis of Multivibrator Circuits.—An attempt to use the equivalent-plate-circuit theorem for the solution of the multivibrator problem would seem to be utterly out of the question because the two tubes are certainly working outside of the linear range of operation. However, they are working at two distinct points of operation. One point is cutoff, during which time the plate resistance is infinite and the tube could therefore be considered as an open switch. The other is with zero-grid potential, during which time the tube is equivalent to a resistance equal to its plate resistance at the particular operating point and a battery in series with it.

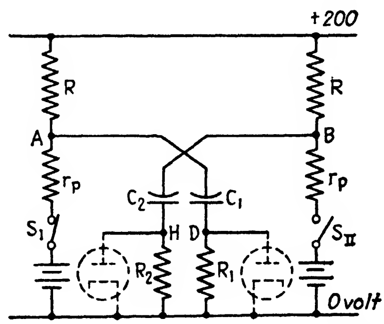

FIG. 20–6.—The multivibrator circuit may also be analyzed on the basis that the tubes are switches operated alternately.

With this in mind, Fig. 20–4 can be replaced by the equivalent circuit shown in Fig. 20–6. Points A, B, H, and D in this figure correspond to the same points in the diagram shown in Fig. 20–4. The two switches S_{I} and S_{II} are alternately closed, the

switchover occurring when D or H in its approach to the zero level becomes equal to the cutoff voltage of the tube. The rectifying action of the grid, which is of so much importance in the analysis of the multivibrator circuit, is indicated by having H and D connected to two anodes through which current will pass to two cathodes as soon as these two points cross the zero level. The rectifying elements are indicated by the dotted lines. With S_{I} closed, for instance, and switch S_{II} open, A will be at a level of 45 volts, and B will be at a level of $+200$ volts. Capacitor C_2 will be charged to 200 volts. Assuming again a starting point for our consideration at which point D is -40 volts, the reader will find that each st p as presented in the analysis of the circuit shown in Fig. 20–4 also applies to the circuit shown in Fig. 20–6. The two tubes can be considered simply as two switches operating alternately. Almost all the phenomena taking place in the multivibrator circuit can then be analyzed with the aid of this equivalent circuit.

20–13. Synchronization of the Multivibrator; Frequency Division.[5, 6]— The frequency of a multivibrator was seen to depend on the time constant of the capacitors C_1 and C_2 and the resistors R_1 and R_2; fundamentally, it is determined by the time it takes for the grid of the nonconducting tube to come up from its most negative value to the cutoff grid voltage of this tube. (When it crosses this value, conduction begins.) Since we have seen that there are a number of factors affecting this time, it can be expected that the frequency stability of a multivibrator is not of the highest degree. However, this very weakness may be turned into one of its strong points. It is possible to make a multivibrator operate in synchronism with a signal voltage, not only in such a manner that every cycle of the signal voltage will cause one cycle of the multivibrator but so that each cycle of the multivibrator will be equivalent to a number of cycles of the controlling frequency. The multivibrator may therefore be used for the purpose of frequency division. Figure 20–5 illustrated how the potential of the grid rises exponentially from its most negative value and how the switchover from one tube to the other occurs when the grid crosses the cutoff grid voltage. Let us assume that this time, sometimes referred to as "free running time," is approximately 0.1 sec. Variations in supply voltage, temperature changes in the resistors, and other factors may change this time slightly in either direction. Now, on one of the tubes, let us superimpose on the exponential sweep of the grid potential a 60-cycle signal, preferably, but not necessarily, of a peaked wave shape, as shown in Fig. 20–7. The injection of this additional signal can be accomplished in several ways. We may, for instance, include in the cathode lead a small inductance and apply to it a peaked 60-cycle wave. Or we may couple this voltage capacitively to the plate of the other tube. If we use the numerical values of our preceding example, for instance, the upper terminal B of the capacitor C_2 would, during the conductive period of tube II, not

be at a steady level of 45 or 50 volts, but the injected voltage will "wiggle" this point up and down. As shown in Fig. 20–7, on the average the grid is still following the exponential curve, but tripping or switchover will now take place after a time t_2, which is exactly 6 cycles of the injected voltage. Even if the free running time should change slightly, t_2 will still remain equal to 6 cycles of the synchronizing voltage. If the circuit constants of the other tube in the circuit are the same and if the same voltage is injected on this tube also, the complete cycle of the particular arrangement will take 12 cycles of the 60-cycle voltage. This means that

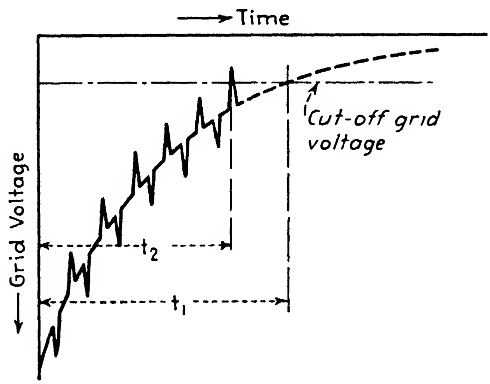

FIG. 20–7.—The superposition of a peaked voltage on the exponentially rising grid voltage causes the switchover from one tube to the other in a time which is an exact multiple of one period of the controlling voltage.

the frequency of the multivibrator will be exactly 5 cps. For more detailed information on this subject the reader is referred to the comprehensive literature on it.

20–14. Multivibrator as a Regenerative Amplifier; Single-tube Circuits. Although in the preceding paragraph the multivibrator was treated as a symmetrical arrangement of two tubes that are capacitance-coupled to each other, some investigators have preferred to look upon it as a two-stage resistance-capacitance-coupled amplifier with the output of the second stage coupled to the input of the first stage. Although it is not felt that this treatment gives a clearer understanding of the performance of a multivibrator, nevertheless, there is one feature that this point of view brings out very clearly. We have seen that a phase reversal takes place in a resistance-capacitance-coupled stage. If two such stages are employed, it is evident that the output voltage of the second stage—*i.e.*, the anode of the second tube—will swing positive, or up, if the input signal of the first stage swings the grid of this stage up. If the two stages are identical, the over-all voltage gain from the grid of the first tube to the anode of the second tube is given by the square of the voltage gain of each individual stage. Obviously then, if the output voltage is coupled back to the in-

put, the circuit will keep on changing in the direction of the original signal. As far as voltage gain is concerned, even a single stage would be sufficient to produce this result if it were not for the fact that with a single stage the phase relationship is not correct. If we should couple the plate of a single tube to its grid, for instance, and any slight disturbance within the tube or within the grid circuit should increase the flow of current, the anode of the tube comes down; if it is capacitively coupled to the grid of the tube, the latter would also be moved in a downward direction. This would obviously try to decrease the plate current; in other words, it would act in such a direction as to oppose the original disturbance. On the other hand, if this voltage variation of the anode is coupled to the grid of a second tube, the phase reversal taking place in the second tube will bring the signal back to the first tube in such a direction as to reinforce the original disturbance. This is exactly what is taking place in the multivibrator. From this discussion, it is evident that a single tube would be sufficient to produce a similar result if the relation between plate voltage and grid voltage were in a direction opposite to that found in an ordinary tube; in other words, if making the grid more negative, for instance, would make the plate also more negative. When discussing the dynatron and negative-transconductance tubes, we saw that in the case of a pentode it was possible to obtain a negative transconductance between the suppressor grid and the screen grid; it was shown that making the suppressor grid more *negative* caused an *increase* of current to the screen grid. Figure 20–8 shows a circuit incorporating this principle. It will be noted that the connections are identical with those shown in the circuit of Fig. 18–17, except that the resonant circuit of the latter figure is replaced by a resistance. The frequency of the circuit is determined essentially by the time constant of the capacitor C and the resistor R in the suppressor grid circuit. The load resistance R_L will also have an effect on the frequency although not so pronounced as the time constant of the resistor and capacitor in the grid circuit. Although the circuit shown in Fig. 20–8 is simpler than the multivibrator circuit, it is not so flexible as the latter; thus, the wave is not symmetrical owing to the fact that during the positive swing the suppressor grid takes current so that the charging period of the capacitor C is not equal to the discharge period. Considering the fact that a number of double triodes such as the 6SN7 or the 6SL7 are available, permitting the construction of a multivibrator with two triodes housed in the same envelope, it seems that the regular multivibrator circuit is more desirable from every point of view.

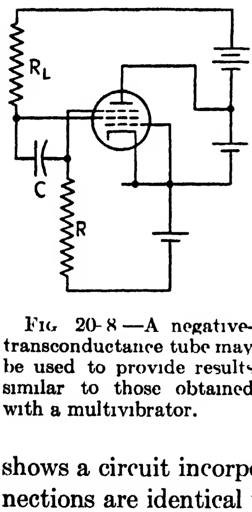

FIG. 20-8.—A negative-transconductance tube may be used to provide results similar to those obtained with a multivibrator.

20-15. Trigger Circuits.[7-11]—It will be remembered that the analysis of the multivibrator circuit used as its starting point the circuit shown in Fig. 20-3*b*. It was mentioned at that time that circuits of this type are also called "trigger" circuits. These circuits are very useful if it is desired to obtain operation of a circuit from an impulse of extremely short duration. The basic trigger circuit was first described in 1919 by Eccles and Jordan. The diagram of this circuit is shown in Fig. 20-9, and comparison of it with the circuit shown in Fig. 20-3*b* discloses that the two circuits perform in an identical manner. Both can be considered as two-stage dc amplifiers with the output of one stage coupled to the grid of the other. The only difference between the two circuits is the method of coupling. The circuit shown in Fig. 20-3*b* makes use of the system described in Sec. 14-4, and the circuit shown in Fig. 20-9 employs the circuit discussed in Sec. 14-5. In each case, when one tube becomes conducting, the voltage appearing across the load resistor makes the grid of the other tube more negative.

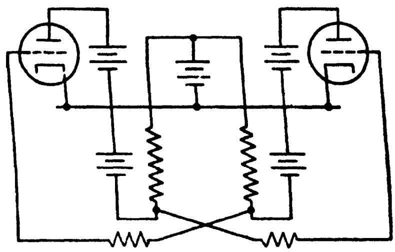

FIG. 20-9.—The basic Eccles-Jordan trigger circuit.

It is clear that it is desirable to eliminate the coupling batteries or the plate-supply batteries of Fig. 20-3*b* or 20-9, respectively. A practical form of the Eccles-Jordan trigger circuit is shown in Fig. 20-10. Since in any properly designed multivibrator or trigger circuit one of the tubes is always conducting, bias may be obtained by means of a cathode resistor common to both tubes. Again the reader should have no difficulty in recognizing that the circuit represents simply two dc amplifiers, the output of each coupled to the input of the other. The method of coupling is the one described in Sec. 14-6, *i.e.*, a voltage divider is used to transfer the voltage variations occurring at the plate of the one tube to the grid of the other.

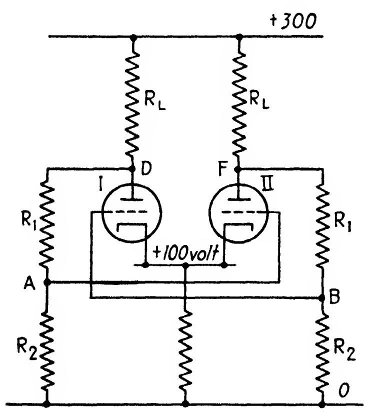

FIG. 20-10.—A modification of the basic trigger circuit to eliminate the need of separate batteries.

20-16. Example of Design of a Trigger Circuit.—The design of a circuit such as is shown in Fig. 20-10 is a relatively easy matter. All that is required is to make sure that with one tube conducting the grid of the other is below the cutoff value, and that the tube which is nonconducting makes the grid of the opposite tube zero or even positive. Assume that in Fig. 20-10

the two tubes are 6C5's and that the total voltage available for the circuit is 300 volts. In order to be sure that we have plenty of room "below" the cathode for negative bias, we decide to place the cathode at a level of 100 volts. This leaves 200 volts for the tube and its load resistor. Choosing the latter as 20,000 ohms and drawing a load line with this value and 200 volts in the plate characteristics of the 6C5 gives an intersection with the zero-grid voltage characteristic at a current value of 6.25 ma with a voltage of 75 volts across the tube and 125 volts across the resistor. Since only one tube is conducting at any time, the current through the cathode resistor will also be 6.25 ma, and in order to have the cathodes at the postulated level of 100 volts, the value of the cathode resistor must be $100/0.00625 = 16{,}000$ ohms. The cutoff grid voltage for a 6C5 operating at 200 volts plate voltage is -14 volts, according to the characteristics. In order to be sure that the circuit will not trigger spuriously, we shall be safer by placing the grid of the nonconducting tube at a level considerably more negative with respect to the cathode than this value; let us decide, for instance, at -25 volts. With the cathode established at a level of 100 volts, point A or B—whichever one is supposed to make its tube nonconducting—must therefore be at a level of 75 volts. Let us assume that tube I is conducting and tube II is nonconducting. Point D will then be at a level of 175 volts, and point F will be very nearly at a level of 300 volts if the resistances R_1 and R_2 of the voltage divider are of high value compared to the 20,000-ohm load resistor. The grid of tube II, which is nonconducting, is to be at 75 volts, which means that A must be at this potential. Since D is at 175 volts, the resistances R_1 and R_2 must be in a ratio of 100:75. They should be of high value compared to the resistance R_L; otherwise, this simplified method of design will not give accurate enough results since the current drain through these two resistors will, of course, upset the voltage distribution calculated above. In this particular case, R_1 may be made 500,000 ohms, and R_2 will then have to be 375,000 ohms. It would seem that the design of the circuit is now finished because all resistor values have been determined. (Remember that both sections are identical.) We assumed tube I conducting and, with zero-grid voltage on this tube, found that point D would be at a potential of 175 volts. This zero-grid voltage was only an assumption, however, and we must now check whether the resistor values R_1 and R_2, which will make tube II nonconducting (by making its grid 25 volts negative with respect to the cathode), will also make the grid of tube I zero with respect to the cathode, as we had assumed. With tube II nonconducting (as it certainly will be with a negative grid voltage of 25 volts), point F will be just a few volts below the 300-volt level (owing to the small current taken by the voltage divider). With the resistors R_1 and R_2 equal to 500,000 and 375,000 ohms, respectively, point B, if it were free, would be at a level

of approximately 130 volts, or 30 volts positive with respect to the cathode level. When B is connected to the grid of tube I, grid current will begin to flow and will prevent the grid from becoming more than a fraction of a volt positive with respect to the cathode. We see, therefore, that our original assumption of zero-grid voltage for the conducting tube is correct for all practical purposes. If this check had revealed that the grid failed to become zero or positive, another start with different load resistor, cathode resistor, etc., would have to be made.

20–17. Methods of Triggering; Modifications for Triggering by Impulses of Extremely Short Duration.—The triggering of the circuit shown in Fig. 20–10 from one tube to the other is most easily accomplished by coupling the triggering impulse of voltage by means of a small capacitor to either grid, *i.e.*, to either point A or point B. If tube I is conducting, for instance, transfer to tube II can be accomplished by applying either a negative pulse to B or a positive pulse to A. If the tripping impulse is of very short duration, such as 1 μsec, the circuit may be found to fail. The reason for this is that the capacitance existing between grid and cathode opposes a sudden change of potential. Although the tube to the grid of which the tripping impulse is delivered may be able to change its plate potential suddenly, this change, or rather the fraction determined by the voltage divider R_1-R_2, is supplied to the grid of the second tube over this voltage divider, consisting of resistors of high value. Since the whole action of the circuit depends on the fact that the original disturbance is reinforced by its amplification in the two stages, it is evident that failure may occur if the amplified impulse comes so late that the original impulse has already passed. The remedy for this situation is quite obvious. The grid-cathode capacitance in this circuit is evidently parallel to the resistor R_2 since the cathode potential is assumed to remain constant during the switchover. If we place a capacitor across R_1 also, such that its reactance is in the same ratio to the reactance of the grid-cathode capacitance as the value of resistors R_1 to the value of R_2, any voltage variations of F and D, no matter how sudden, will be transferred to A and B in the same ratio as the resistors R_1 to R_2. It usually will be found that capacitors in the order of 50 $\mu\mu$f will take care of this situation.

PROBLEMS

20–1. By applying a varying direct voltage over a protective resistance to a given neon lamp, it has been found that the lamp breaks down at 70 volts and establishes a glow. This glow is extinguished if the voltage is reduced to 50 volts. The lamp is connected in a circuit as shown in Fig. 20–11. The value of R is 500,000 ohms; the value of C is 0.25μf. The supply voltage is 150 volts, as shown on the diagram.

a. If an oscillation is established, what will be its frequency?

b. Draw a graph showing the voltage across the capacitor.

20–2. A multivibrator, or square-wave generator, is to be designed, employing two 6C5's, as shown in Fig. 20–12. Let $E_{bb} = 100$ volts; $R_1 = 10{,}000$ ohms; $R_2 = 250{,}000$ ohms; $C = 0.1\ \mu$f.

a. What will be the frequency of this circuit?
b. If the plate-supply voltage is raised to 200 volts, to what values will the frequency change?
c. If the plate resistors are changed in value to 25,000 ohms (with the supply voltage remaining at 200 volts), what will the frequency be?

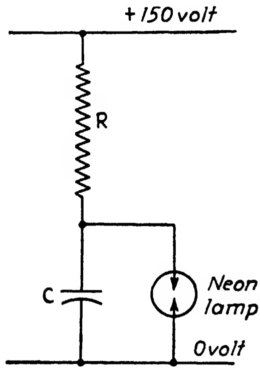

FIG. 20–11.—Circuit diagram for Prob. 20–1.

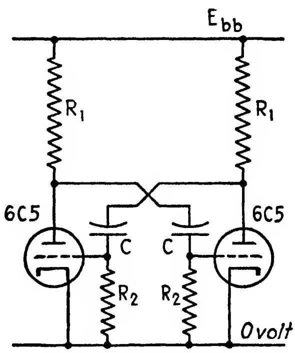

FIG. 20–12.—Multivibrator circuit of Prob. 20–2.

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CHAPTER XXI

GASEOUS TUBES AND THEIR CHARACTERISTICS

21–1. Comparison of Volt-ampere Characteristics of a Vacuum Tube and a Gaseous Tube.—In Secs. 6–16 to 6–18, the effect of gas in a diode was discussed. This subject will be expanded now, but the reader will find it worth while to go back to those sections and refresh his memory on the matter presented there.

We have seen that gaseous diodes are rectifiers that operate with a considerably lower voltage drop than diodes of the vacuum type. The fundamental difference between the two types of rectifier can perhaps be seen best by comparing their volt-ampere characteristics, shown in Fig. 21–1.

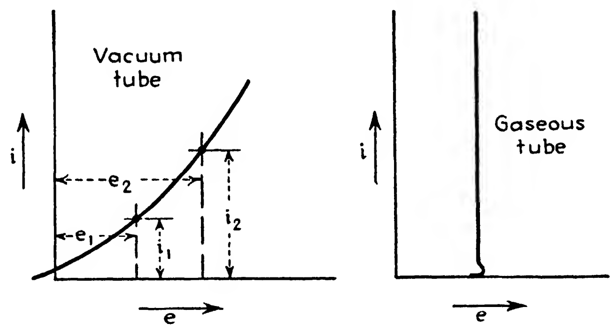

FIG. 21–1.—The application of a given voltage to a vacuum-type diode always results in a definite current. In contrast to this, the application of a rising voltage to a gaseous tube without means to limit the current may lead to destructive currents.

A vacuum-type diode—and, for that matter, a triode—has a volt-ampere characteristic, the slope of which is at all times positive. That statement means that the application of a certain voltage always produces a definite current and that in order to increase this current it is necessary to increase the voltage also. Thus, with a voltage e_1 applied to the vacuum-type diode the current will be i_1; in order to increase this current to a value i_2, it will be necessary to increase the voltage to e_2. A diode of the vacuum type can therefore be placed directly across a voltage (*i.e.*, without any resistance in series with it) if this voltage is low enough so that the current produced by it, as given by the volt-ampere characteristic, does not exceed the maximum permissible current of the tube. Now compare this volt-ampere characteristic with that of the gaseous tube shown in the same

figure. The volt-ampere characteristic is seen to be practically a vertical line. This means that for any voltage to the left of this line no current will flow while destructive current would flow at the very instant when the tube becomes conducting. As a matter of fact, the voltage required to start current flow is usually slightly (and in some cases quite considerably) in excess of the voltage required to maintain current flow, no matter how large. We therefore must apply to the tube a voltage in excess of the distance between the Y axis and the vertical line representing its volt-ampere characteristic in order to start conduction. But this voltage, if it were not reduced immediately after conduction started, would be too high and would lead immediately to the destruction of the tube. In other words, there must always be in series with such a tube a resistance or some other load that will limit the current to a value within the capacity of the tube. A switch can make and break a circuit, but it cannot control the amount of current that will flow after the switch has been closed; it is not the switch but the characteristic of the load that determines the amount of current that flows after the closure. In a similar way, a gaseous tube must never be placed alone across a fixed voltage because it cannot limit the current itself. As in the case of a switch, there must always be a device in series with the tube that will produce a voltage as soon as conduction has started and thus limit the current. In the case of a series combination of a switch and another device, there is usually not much question that this device represents the load. In other words, the emphasis rests on what is going on in this device, whether it is a light bulb, a motor, or any of the other various types of load. In the case of a series combination of a gaseous tube and another device, on the other hand, the desired and important phenomenon is often taking place in the tube itself. The series device serves no other purpose than to limit the current. A fluorescent light bulb, or any neon lamp for that matter, is an example of this. In both cases, the light produced is the important part, and it would be highly desirable if they could be placed directly across the line, but since they have a volt-ampere characteristic similar to that of any gaseous discharge device, a current-limiting resistor or reactor must be placed in series with them. In this case, the resistor or reactor is usually referred to as a "ballast," and the voltage appearing across it represents a loss.

21-2. Types of Cathodes in Gaseous Tubes.—The discussion in Chap. XVII dealt with only one kind of gaseous tube. It was shown there that in a gaseous diode with a heated cathode the ionization of the gas taking place nullified the effect of the negative space charge of the electrons so that high currents could be obtained with a voltage essentially equal to the ionizing voltage of the gas contained in the tube. Although it is true that a large number of the gaseous-tube types used for industrial control purposes use heated cathodes as the source of the electrons, there are a number of electronic devices using gaseous discharges that do not have

heated cathodes. For this reason, we shall have to examine the subject of gaseous conduction somewhat more in detail.

21-3. Fundamentals of Gaseous Conduction.—Conduction of electric current through a vacuum by means of electrons is a very simple and straightforward process compared to the phenomenon taking place when gas is included in the discharge device. Volumes have been written covering nothing but the scientific side of this subject. In line with the purpose of this book, the discussion of the subject will be held to a minimum. It would be possible to dispense entirely with a description of the internal phenomena taking place in a gaseous tube and to present them simply as devices with certain volt-ampere characteristics. But these characteristics have many peculiarities that are somewhat easier to understand if the user has at least a rough idea of the processes involved in the conduction of current through these tubes or devices.

In an ordinary neon lamp, current flows without either of the electrodes being heated so that it would become the emitter of electrons. Current cannot flow unless there are electric charges traveling, either electrons or negative or positive ions. The neon lamp is only one example of a number of tubes operating without the principle of thermionic emission; for this reason they are called "cold-cathode tubes." What is the mechanism of current flow in such devices?

21-4. Current Flow Due to Outside Sources of Ionization.—When we apply a voltage to electrodes disposed in a gas, a small current begins to flow immediately. The reason for this current is that the gas between the electrodes is continuously being ionized by outside influences; cosmic rays or photons may be striking the molecules of the gas. Ordinarily, these products of ionization recombine so that there exists a state of equilibrium between the number of newly formed ions and those that recombine. When these products of ionization find themselves in an electric field, such as produced by the application of voltage between two electrodes, the charges begin to travel toward the electrode of opposite polarity. Many of them will recombine before they have reached the electrodes, but it is evident that the chance of recombination will be less if the speed with which they travel is higher or, in other words, if the voltage is increased. We can therefore expect an increase in current with an increase in voltage, until a voltage is reached where substantially all the ions produced by the outside source of ionization are swept up and carried to the electrodes; from then on any further increase in voltage should not lead to a further increase in current. This is exactly what takes place, but the amount of current is ordinarily of such a low value that it is without any practical importance. It should be noted that with this type of conduction the current flow is entirely dependent on the outside source of ionization and ceases if the latter stops. Devices of this kind may be used for the measurement of the ionization taking place. X rays are known to produce

ionization, the amount of which is dependent on the intensity of the X ray. Therefore, if X rays are permitted to strike the gas between the electrodes, a considerable increase in current will be observed which may be used as a measure of the intensity of the X rays. When the X-ray radiation ceases, the current will then drop to its former value. The current produced in such "ionization gauges" is extremely small, in the order of a fraction of a millimicroampere. In order to make it useful for measuring purposes, it is usually made to flow through a resistor of very high value and the voltage appearing across this resistor is then measured by a vacuum-tube voltmeter, such as was described in one of the earlier chapters.

21-5. Nonself-maintaining Discharge.—When the voltage is increased further, an increase of current will be noticed, which is due to the fact that the voltage is now high enough to give some of the electrons sufficient speed to ionize additional gas molecules. The electron in so doing gives up most of its energy. Whether it or the newly liberated electron can repeat this trick once more before reaching the anode will evidently depend on the anode voltage. The number of times that a given electron can produce an additional one by collision will therefore be definitely a finite number and will depend on the anode voltage and the spot where the original electron was liberated. Although the current may become ten or twenty times as large as originally, it still depends entirely on the original supply of electrons due to the original ionizing agents. Such a type of discharge is called "nonself-maintaining." The practical application of this phenomenon is found in the gas-filled phototube. In this tube the original electrons are liberated from the photosensitive cathode by the process of photoelectric emission; in their flight toward the anode they acquire enough energy to ionize several gas molecules. This results in a current about ten times as high as that flowing if the tube were evacuated so that only those electrons originally emitted from the cathode would carry the current. This cumulative process is also known as an electron "avalanche." In the case of the example just stated, it should be noted again that the current will cease when light is no longer falling on the cathode of the phototube; in other words, when the original source of electrons or source of ionization has ceased to operate.

21-6. Self-maintaining Glow Discharge.[1]—When the voltage is increased further, the current increases steeper and steeper until finally the voltage reaches a value at which an infinitesimally small further increase leads to a sudden jump of the current to destructive values. If a ballast resistor is included in the circuit, the voltage will drop back to a lower value. The transition from the previous stages to the new stage is usually accompanied by the emission of light; the discharge begins to glow softly and for this reason is called "glow discharge." The glow discharge is now found to be independent of the original sources of ionization; it has become self-maintaining or self-sustaining. A theory explaining this

phenomenon was first proposed by Townsend, and another similar to the Townsend theory was proposed by J. J. Thomson. In our discussion of the electron avalanche in the preceding paragraph we paid attention to only one product of the ionization caused by the repeated collision of the original electrons; but besides the new electrons there will also be ions formed by these collisions. These ions, being positive charges, will travel toward the cathode and will also acquire kinetic energy. Ions are not anywhere near so capable of knocking a neutral gas molecule apart as electrons are. Ordinarily the ions simply land on the cathode and recombine there with an electron to form a new neutral gas molecule. Occasionally, however, the condition seems to be just right for a traveling ion to knock apart another gas molecule or also to liberate a new electron by secondary emission from the cathode. If on the average the products of one electron avalanche are capable of producing one new electron near the cathode, then evidently this new electron can take the place of the original one, and the discharge will no longer depend on the external source of electrons. This is the condition that exists in the glow discharge, which then obviously becomes self-sustaining. The potential distribution within the tube also changes completely. The voltage along the glow is usually not very high per centimeter; most of the voltage appears very close to the cathode (except of course in extremely long tubes such as neon signs, where even a relatively small drop per inch will begin to add up if the glow, or "positive column," as it is sometimes called, is made several feet long). If the tube is 1 ft or so long, the character of the discharge will vary along the tube. These various regions all have names such as "cathode dark space," "cathode glow," "Faraday dark space," "positive column." If the distance between the electrodes is shortened, some of these regions disappear. The reader interested in the reasons and characteristics of these various regions should refer to a book dealing with the subject of gaseous conductors. A further discussion would be beyond the scope of this book.

21-7. Normal and Abnormal Glow Discharges.—As soon as a glow discharge has been established, it will be observed that the amount of cathode area covered by the glow depends on the amount of current flowing through the discharge. When the glow discharge in a cold-cathode tube does not cover the full area of the cathode, the discharge is called "normal" glow discharge. The current density at the cathode will be constant under this condition, which means that the area covered by the glow is proportional to the current permitted to flow through the tube. The current density of the normal glow, also called the "normal" current density, is a function of the cathode material, the gas, and the pressure of the gas. As long as a cold-cathode tube operates with a normal glow discharge, it will be found that the voltage across it will increase very little with an increase in current. A practical application of this phenomenon is found in the so-called "voltage-regulator" tubes. These tubes have a small center post serving

as an anode, and the cathode is a cylinder approximately ¾ in. in diameter and 1 in. long. When operating within their ratings, the cathode is only partly covered by the glow. Under this condition the voltage across the tube remains essentially constant for a wide variation of current. Practical circuits of these tubes will be discussed later.

When the current through the gaseous tubes operating with a normal glow discharge is further increased—by reducing the ballast in series with the device—the cathode will finally become completely covered by the glow. A further increase in current is now accompanied by an increase in voltage, in contrast to the condition encountered during the operation with normal glow. Such a condition is called "abnormal" glow. The expressions "normal" and "abnormal" glow are rather confusing because they seem to indicate that one mode of operation is less desirable; this is not the case, however, since both types of operation are encountered in industrial electronic devices. The voltage rise encountered with an increase in current during operation with abnormal glow is not very steep, however, and is by no means proportionate to the current.

21-8. Arc Discharge.—If the current through the tube is increased further, another very sudden transition will occur from the condition of abnormal glow to a state known as an "arc." The current at which this transition occurs is not easily predictable. When it happens, the current density on the cathode increases suddenly, a phenomenon that is produced by the whole discharge suddenly concentrating on a small spot on the cathode, and the voltage across the discharge suddenly decreases to a much lower value than the one encountered under the abnormal glow condition. High temperatures of the gas and of the electrodes follow, suggesting that the cathode now produces electrons copiously by thermionic emission. Sputtering of the cathode material, or even the melting of it, may follow if the arc current is permitted to increase to high values. The volt-ampere characteristic of an arc exhibits definitely a negative resistance. With an increase of current through the arc, the voltage across it not only remains not constant, as it did essentially across a normal glow discharge, but actually decreases. The operation of an arc will therefore depend, even to a larger degree than that of the glow discharge, on the presence of a stabilizer in series with the arc.

If ever there was an incarnation of the devil and a saint in the same creature, it certainly is the arc. One-half of the electrical engineering profession is busily engaged in snuffing out arcs, and the other half is just as busy keeping them going. In the first camp we find the boys designing circuit breakers and other control apparatus, the contacts of which suffer from the arcing; in the other camp we find the welding engineers and the fellows trying to make mercury-arc rectifiers behave. But even those who are trying to snuff out their arcs should recall the statement made by Dr. Slepian that, if no arc existed, the electrical engineer would

have to invent something to take its place. No circuit containing inductance could be opened without destroying the insulation if no arc were permitted at the spot of interruption; the only way to interrupt would then be by gradual insertion of resistance in series with the load. To explain what happens within an arc seems to have even the experts stumped, which is a good reason for not making an attempt here. As far as the electronic engineer is concerned, the mercury-arc rectifier is the best known device making use of an arc discharge. A smaller one also of importance to him is the strobotron,[10] which is used as a light source in stroboscopes.

21-9. Summary of Gaseous Discharge.—Figure 21-2 shows the general characteristics of a gaseous discharge. It must be remembered that the

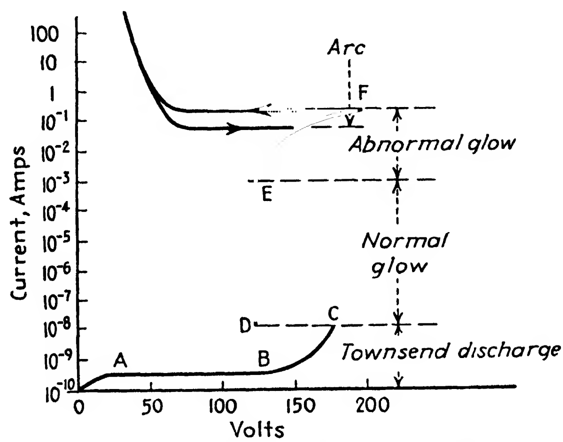

FIG. 21-2.—The volt-ampere characteristics of a gaseous discharge over wide ranges of current. Note that the current scale is logarithmic

voltages depend on the cathode material, the nature of the gas, its pressure, as well as on the distances between the electrodes. The figure is meant to indicate only the general character of such a discharge. It should be noted that the vertical scale, *i.e.*, the scale for the current, is not linear in this figure but logarithmic. To summarize the results discussed in the preceding paragraphs with the aid of Fig. 21-2, part *OA* represents that part of the nonself-maintaining discharge when the applied voltage is not yet sufficient to sweep all the ions produced by outside influences out of the interelectrode space. At *A* this state has been reached and a further increase in voltage does not increase the current until *B* is reached. At this point the effect of ionization by collision makes itself felt, although the discharge is still nonself-maintaining. At *C* the electron avalanches become numerous and the produced ions have sufficient energy to produce new electrons near the cathode so that the discharge becomes a normal glow and is now self-maintaining. The current must

now be limited by a device in series with the gaseous discharge tube. If the current is increased by reduction of this ballast resistance, the voltage will remain constant over a wide range until the whole cathode is covered by the glow. This is the region between D and E of the characteristic. If the current is permitted to increase further, the discharge enters the region of abnormal glow shown between E and F. The voltage then suddenly collapses with a further increase in current, and the discharge becomes an arc with a definitely falling volt-ampere characteristic.

21–10. Types of Gaseous Tubes.[3]—The variety of gas tubes is considerably greater than that of vacuum tubes. Not only are there all the varieties encountered among vacuum tubes, such as diodes, triodes, and pentodes, but also, although the cathode in the case of vacuum tubes is always of the thermionic type, it may be either of the cold-cathode type or of the heated-cathode type in the case of gaseous tubes. Furthermore, the gaseous tubes may be operating with a normal or abnormal glow discharge, or they may be operating with an arc. It seems as if every month brings a new combination of operating conditions and adds another type to the ever-growing list of gaseous tubes.

Gaseous diodes with heated cathodes were discussed in Chap. VI. It was shown there that the presence of the gas in the tube had for its purpose the reduction of the space-charge effect. Almost all the current is still due to the electrons emitted from the cathode. Owing to the cancellation of the space-charge effect, however, it was possible to construct the cathode with a considerably higher efficiency than the cathode for vacuum tubes. It is possible to use corrugated or pleated surfaces so that with the same wattage expended for the heating of the cathode, the emission increases many times. The gas used for the filling of such a tube is mercury vapor, which is obtained simply by introducing a drop of mercury into the tube before it is sealed. Mercury vapor is, however, not the only gas used for this purpose. In the Tungar or Rectigon rectifier, for instance, argon gas under a pressure of approximately 5 cm is employed. The type of gas used and its pressure affect the operation of the tubes in several ways. These are questions, however, of more interest to the tube designer than to the application engineer. Let it be said here that the argon filling of the Tungar rectifier makes it possible to operate the cathode at a higher temperature and thus obtain more current; on the other hand, it lowers the maximum voltage that the tube can withstand in the opposite direction, which in the case of the Tungar rectifier is in the order of a few hundred volts. In contrast to this, a mercury-vapor tube can stand considerably higher voltages.

21–11. Cold-cathode Rectifiers.—At first glance, it seems that cold-cathode diodes could serve only for the production of light as in a neon lamp, or as voltage-regulator tubes, but that they could not be used as rectifiers. This would indeed be the case if the two electrodes were ex-

actly identical. In Sec. 21–7 it was stated that more voltage is required for a discharge operating under abnormal glow than for one operating with a normal glow. If a gaseous tube with two cold electrodes of unequal size is included in an ac circuit, it is evident that a condition may be obtained where the tube is operating with normal glow when the larger electrode is the cathode, but during the half cycle when the smaller electrode becomes the cathode, the glow may be abnormal. Such an arrangement will obviously result in a rectified current. The first *B*-battery eliminators used in connection with radio receivers made use of a glow-tube rectifier; owing to their high voltage drop and resulting low efficiency, as well as their small current-carrying ability, glow tubes are no longer used for rectifying purposes, except where the fact that they do not need any filament heating outweighs the disadvantages just mentioned.

21–12. Voltage-regulator Tubes and the Design of Circuits Containing Them.[2]—As mentioned in Sec. 21–7, cold-cathode diodes operating with a normal glow discharge have found application as voltage regulator tubes. At present four types are commercially available and enjoy great popularity: VR-75, VR-90, VR-105, and VR-150. The figures in these designations indicate the voltage existing across the discharge. For the VR-105, for instance, the rating and characteristics quoted from the manufacturer's information are given in the accompanying table.

OC3/VR-105

Dc anode supply voltage *	133 volts min
Dc operating current (continuous)	40 ma max; 5 ma min
Ambient temperature range	−55 to 90°C
Characteristics:	
Dc starting voltage (approximate)	115 volts
Dc operating voltage (approximate)	105 volts
Dc operating current (continuous)	5 to 40 ma
Regulation (5 to 30 ma)	1 volt
Regulation (5 to 40 ma)	2 volts

* Not less than indicated supply voltage should be provided to ensure "starting" throughout tube life.

It will be noted that the circuit must be arranged in such a way that removal of the tube from its socket would cause the voltage to rise to a value of at least 133 volts so that the tube will start under all conditions. The current through the tube should be not less than 5 ma or more than 40 ma. As indicated by the table, the voltage variation across the tube will be 1 volt when the current is kept between 5 and 30 ma. It will be noted that the operating voltage is given as approximately 105 volts; it will be found to vary from tube to tube by from 2 to 3 volts. A conventional circuit arrangement is shown in Fig. 21–3. Let us assume that the load is to be operated from a voltage of 105 volts and that the current taken by it varies between 10 and 20 ma; let it be further assumed that the

voltage delivered by the rectifier system, owing to line-voltage variation, fluctuates between 225 and 275 volts. The problem is to find the best value for the series resistance R. The tube will have to take the maximum current at an instant when the load current is at its minimum value, *i.e.*, at 10 ma, and the supply voltage is at its highest value, 275 volts. If we decide to limit the current through the tube to 30 ma, the sum of the currents taken by the load and the tube will be 40 ma; with the supply voltage equal to 275 volts, the voltage across the resistor R under this condition will be $275 - 105 = 170$ volts. The required resistance R will consequently be $170/0.04 = 4{,}250$ ohms. We now have to check whether under the extreme condition in the opposite direction the current through the tube will not be less than 5 ma. When the supply voltage drops to 225 volts, the voltage left for the resistor R will be $225 - 105 = 120$ volts. The current that will be flowing through the resistor R will then be $120/4{,}250 = 0.0284$ amp, or 28.4 ma. Since the maximum value of the load current was given as 20 ma, this will leave a current of 8.4 ma through the tube, which is in excess of the minimum value given by the manufacturer. As a final check of the design, one should make sure that even under adverse conditions the starting voltage for the tube will be in excess of 133 volts. It is left to the reader to check whether, with the maximum load current and the minimum supply voltage, the voltage that will exist across the tube terminals, if the latter were removed from the circuit, exceeds this value.

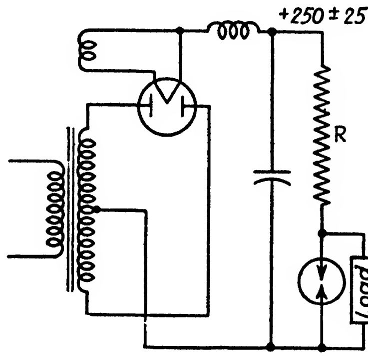

FIG. 21-3.—A voltage regulator to keep the voltage across a load constant.

If a regulated voltage in excess of the rating of the commercially available tubes is required, several of them may be connected in series. Thus a VR-150 and VR-105 will provide a total voltage of 255 volts. It is evident that the junction between the two tubes may be used as a tap if such a voltage should be required. If the regulated circuit contains pentodes, for instance, such a tap may be used to supply the required screen voltage.

21-13. Equivalent Circuit of a Voltage-regulator Tube.—Sometimes a somewhat different method of looking at a voltage-regulator tube may be of advantage. In Chap. V it was shown that, regardless of the volt-ampere characteristic of a device, it can always be considered over a limited region of its operating range as consisting of a battery with a resistance in series with it. If we are told that for a VR-105 voltage-regulator tube a change of current from 5 to 30 ma is accompanied by a rise of voltage of 1 volt,

then we could say that the tube acts like a battery of 105 volts and a resistance equal to 1/0.025 = 40 ohms in series with it. As a matter of fact, some investigators prefer to characterize the performance of voltage-regulator tubes by giving their "internal" resistance in the manner outlined. The substitution just discussed will be found of particular help in the analysis of circuits containing a certain type of small regulator tube not mentioned in the list previously given. The type 991 is a small glow-discharge tube capable of carrying between ½ and 2 ma of current; the voltage existing across the tube with these two values of current is 55 and 62.5 volts, respectively. The internal resistance of the tube, as defined above, is therefore 5,000 ohms, which certainly does not look conducive to very good voltage regulation. The tube could be considered as a battery of 52.5 volts with a resistance of 5,000 ohms in series with it. In Fig. 14–6 discussed in Sec. 14–6, it was shown that in the method of dc coupling described there it was not possible to obtain the full voltage fluctuation of the anode of the preceding tube on the grid of the following tube, owing to the voltage-dividing action of the two resistances R_1 and R_2. By replacing resistance R_1 with one or several of these small voltage-regulator tubes, the circuit assumes the form shown in Fig. 21–4. If the reader now imagines the two tubes replaced by a battery of 105 volts and a resistance of 10,000 ohms in series with this battery, it should be clear to him that under this condition most of the voltage variation appearing on the anode of the first tube is transferred to the grid of the second. The scheme is usually applicable only in the case where the plate current of the first tube is considerably larger than the current taken by the voltage-regulator tube. This would hardly be the case for the 6SF5 shown in the circuit of Fig. 14–6.

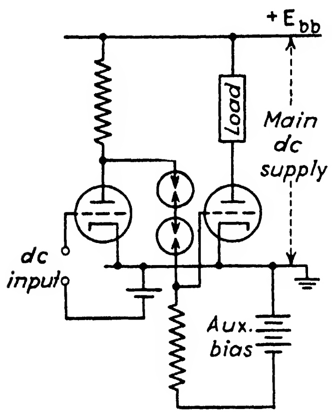

FIG. 21–4.—If direct coupling is desired between two amplifier stages, the output voltage of one stage may be transferred to the grid of the following stage, without any appreciable loss, by means of small voltage-regulator tubes.

21–14. Grid-controlled Gaseous Tubes; the Thyratron.—We have thus far discussed several types of gaseous diodes. Since the introduction of a third electrode, the grid, into a vacuum diode converted a mere rectifying device into a control device with extremely valuable properties, it was natural to attempt combining the higher current-carrying capacity of the gaseous tube with the control features of a grid. This attempt led to the development of a great variety of grid-controlled gaseous tubes; as a matter of fact, the variety is so bewildering that an all-embracing method of classification is almost impossible. Let us start with the discussion of

the counterpart of familiar vacuum tubes, such as a triode or a pentode, employing a heated cathode as the source of electrons. Gaseous tubes making use of a heated cathode and containing a grid are known under the name "thyratrons." The name, originally applied only to tubes using mercury vapor as a gas, now includes also tubes making use of other gases for filling.

21-15. Fundamental Experiment with a Thyratron.[4]—Suppose someone gave us a tube that looked like the familiar 6J5 vacuum-type triode but told us that the tube had a gas filling, in other words, that it was a thyratron. If no other information was given, we might have the bright idea of taking a set of transfer characteristics. Following the procedure with a vacuum tube, we might connect the tube to a source of supply of, say, 200 volts, making the grid at first quite negative and then reducing this negative bias gradually. If the gas filling increases the current-carrying ability of the tube, we might expect a larger current than with a vacuum tube but a characteristic similar, in general, to that of a vacuum tube. When in this experiment we decrease the negative grid bias, we shall all of a sudden have a very unpleasant surprise. At some particular value of the grid voltage the tube will suddenly blow up, and the meter in the plate circuit will be burned out. We have learned our first lesson the hard way: never run a gaseous tube without a load. The gaseous tube is not able to exercise control over the amount of current as the vacuum tube does. After this sad experience we get outselves another tube and another meter to repeat our test, but this time we include in series with the tube a resistance of sufficient magnitude to limit the current to a safe value, no matter what may happen within the tube. With a supply voltage of 200 volts, a resistance of 2,000 ohms in series with the tube will certainly limit the maximum current to 100 ma even if we should find that the voltage across the tube dropped to zero. Repeating our experiment and starting with, say, −30 volts on the grid, we find that the current through the load resistor is zero and remains at this value until we have reduced the grid voltage to a certain voltage, in this case, about −20 volts. At this instant the current suddenly jumps from zero to a value of approximately 92 ma; the voltage across the resistor will be 184 volts and across the tube about 16 volts. Reducing the grid voltage further, i.e., making it less negative, does not change the situation one bit. Furthermore, if we should repeat the experiment with resistors of different values, we would find that the voltage across the tube would always be approximately 16 volts, regardless of whether the current was larger or smaller than in the original experiment. The tube simply acts as a switch, although there remains a voltage of 16 volts across it, which would not be true in the case of a regular switch. Note again that, just like a switch, the tube has no control over the amount of current that flows in the circuit; it only determines the instant of closure of the circuit. When a gaseous tube closes a circuit in this manner, it is

said to "fire." Now assume that in our original experiment we decided to open the circuit again. We therefore make the grid of the tube more negative. To our consternation we find that making the grid 30, 40, and even 50 volts negative does not have any effect on the current whatsoever. We are not able to interrupt the current by grid control after it has once started to flow. This statement is not quite correct because it will be found that very small tube currents, such as a few milliamperes, may be interrupted by making the grid 100 volts or more negative; but since this represents an operation much below the rated values, it still can be said that for all practical purposes it is impossible to interrupt the plate current of a thyratron by making the grid negative. Before discussing this certainly highly undesirable state of affairs any further, let us go back to the original experiment. Since the grid is incapable of interrupting the current flow, there is not much we can do except open the anode circuit or reduce the supply voltage to zero, both of which will, of course, extinguish the tube.

21–16. Control Characteristics of Grid-controlled Gaseous Tubes.—In the original experiment we were using 200 volts as a supply voltage. If we repeat the experiment with 300, 250, 150, and 100 volts as a plate-supply voltage, we find that the grid voltage at which the tube fires, *i.e.*, at which it closes the circuit, also changes. With 100 volts as a plate-supply voltage, for instance, we find that the grid voltage has to be reduced to -11 volts before the tube will fire. The size of the load resistance has, of course, nothing to say about this; it only determines the amount of current that will flow after firing has taken place. The tube in turn has nothing to say about this current. For this reason we evidently cannot plot a relation between the current and the voltage in a gaseous tube; however, we can plot the relation between the supply voltage (which will, of course, be equal to the anode voltage before firing takes place) and the grid voltage at which firing takes place. Such a graph is called the "control characteristic" of the gaseous tube. Figure 21–5, for instance, shows the control characteristic of an FG 81, an argon-filled triode. This curve tells us that with a supply voltage of 100 volts, for example, the grid voltage will have to be slightly more negative than 3 volts in order to prevent firing. Figure 21–6 shows the control characteristics of an FG 27A, a mercury-vapor-filled tube. The characteristics show a feature common to all mercury-vapor tubes. Instead of a single control characteristic, as with an FG 81, it will be noted that there are now several curves, marked from 20 to 80 deg. These figures refer to the temperature of the condensed drop of mercury contained in the tube. It is a well-known physical fact that the temperature of a liquid determines the pressure of the vapor that surrounds it. The temperature of the liquid mercury, therefore, determines the pressure, which in turn has an effect on the firing or control characteristics. With a supply voltage of 800 volts, for instance, this tube requires

approximately 3 volts negative grid voltage to prevent it from firing when the mercury is at a temperature of 20°C. It requires approximately 9 volts negative to achieve the same results when the tube is operating at 80°C. This is not so much of a handicap as it seems since it is the usual practice to bias the tubes with a voltage much in excess of what is required to keep them just from firing and then to introduce a sharp impulse of positive polarity to offset this bias.

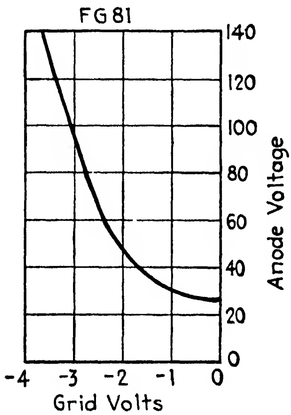

FIG. 21-5.—The control characteristic of an FG 81 grid-controlled gaseous tube. (*From "Electron Tubes in Industry," by K. Henney.*)

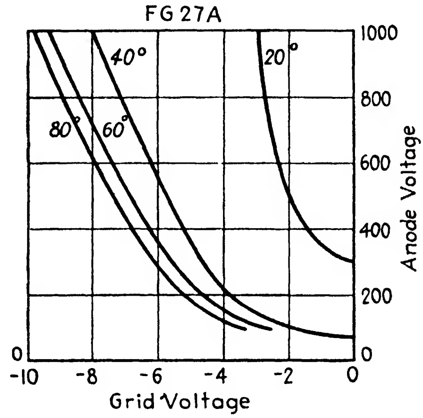

FIG. 21-6.—The control characteristics of a mercury-vapor-filled grid-controlled tube depends on the temperature of the condensed mercury.

21-17. Grid-controlled Gaseous Tube as a Self-latching Relay.—Our inability to interrupt the plate current by making the grid negative certainly is a great disappointment after having become used to the smooth and complete control that we can exercise with a vacuum tube. As a matter of fact, we are certainly justified in questioning the use of a relay or switch that can be closed but not opened. However, our switch possesses certain advantages over an ordinary relay, which makes it a very useful device even with the handicap outlined. In the first place, is it always a disadvantage that the controlled current keeps on flowing even after the controlling impulse is over? After all, manufacturers of electrical control apparatus have designed and used self-latching relays, and a gaseous grid-controlled tube obviously is equivalent to such a relay. In what respect will the gaseous tube be a self-latching relay superior to the magnetic kind? In order to close an ordinary magnetically operated relay, voltage will certainly have to be applied to the coil of the relay for a period not much less than $\frac{1}{60}$ sec, for instance. Furthermore, the faster the relay is to operate, the higher the current that will flow during the application of the control voltage. In a gaseous tube, on the other hand, the grid voltage will have to be reduced below the firing point only for a time suffi-

cient to permit ionization of the gas in the tube. This time is usually in the order of a few microseconds. In other words, if the tube is biased negatively beyond the firing point, it will take a positive impulse of only a few microseconds to close the circuit.

21–18. Influence of the Character of the Load on the Performance of Gaseous Tubes.—The current that flows after a gaseous tube has been permitted to fire will of course depend on the nature of the load, just as it does in the case of a switch. If the load should be highly inductive, for instance, the current cannot build up to any appreciable value during a few microseconds. Since ionization depends on the amount of current that is permitted to flow, it is sometimes found that a gaseous tube will not fire in the case of an inductive load with a short tripping impulse, such as would be sufficient to operate the tube if it had a resistive load. By-passing such an inductive load with a series combination of resistance and capacitance, preferably with a time constant equal to the time constant of the inductive circuit, will usually remedy such a situation.

An idea of the capabilities of gas tubes may be obtained when we learn that the shadow of a projectile passing in front of a photoelectric cell will produce an impulse long enough to fire a gaseous tube. This high operating speed alone would be sufficient to merit the attention of the electrical engineer; but combined with this highly desirable feature is the fact that control can be exercised without the expenditure of appreciable energy, similar to the condition encountered with vacuum tubes. No wonder then that the gas tube has assumed a place of as much importance as the vacuum tube.

21–19. Need of a Grid Resistor; Deionization Time.—When first explaining the performance of a gaseous tube with the aid of an example, we assumed a tube operating with a supply voltage of 200 volts and a resistance of 2,000 ohms. We noted that the tube fired when the grid became less negative than -20 volts, but that after firing had taken place the current could not be interrupted by making the grid negative again. It is always desirable, or even necessary, to place in series with the grid of a gaseous tube a protective resistor. As soon as the tube becomes conducting, the grid is embedded in a gaseous discharge taking place between two electrodes having approximately 15 volts potential difference between them, and it is therefore reasonable to assume that the grid will try to assume a potential somewhere near these values. Any attempt to force it to a different potential will most certainly lead to a great amount of grid current that may be destructive to its structure. This is the reason for placing a resistor between the grid and the source from which the control voltage is obtained so that the grid may find its own level during the discharge without large grid-current values.

Let us assume now that in the preceding example we took this precaution and that we adjusted the *source* of grid voltage to -30 volts. The

tube is still conducting, however, owing to the fact that the grid has been less than 20 volts negative during the earlier part of the experiment. If we should now open the anode circuit, the current through the tube will naturally drop to zero, and the grid will assume the level dictated by the source to which it is connected, *i.e.*, it will become -30 volts with respect to the cathode. If the anode circuit is closed again, the tube will remain in a nonconducting state and will be ready for the repetition of our original experiment if this should be desired. As will be seen later, in many practical circuit arrangements it is of importance to know for how long the anode circuit must be opened to permit the grid to regain control. Opening the anode circuit will, of course, reduce the current instantaneously to zero. The ionized gas in the tube, however, cannot instantaneously recombine with free electrons to form neutral molecules. Consequently, if the anode voltage should be applied after an extremely short period, the tube may fire again, even if the grid should have been made negative in the meantime. This time, which is, as already stated, very important for certain problems, is called the "deionization" time. It depends to a large extent on the type of grid structure of the tube and is always considerably in excess of the ionization time; typical values for it are from 100 to 1,000 μsec.

21-20. Rating of Gaseous Tubes.—By now the reader should have enough of an idea of how gaseous tubes operate so that it may be appropriate to discuss the methods of rating them. Since gaseous tubes, as has been shown, are essentially switches, it may be well to point out how an ordinary switch is rated. The two important values for a switch are the current that it can safely carry while it is closed, and the voltage that it is capable of withstanding while in the open position. For a switch we demand also that it is capable of interrupting the rated current; in other words, when a switch is rated 250 volts, 3 amp, we not only expect it to carry 3 amp while in the closed position and to withstand 250 volts in the open position, but also demand that it is capable of interrupting this current while operating in a circuit of 250 volts. A gaseous tube, as we have seen, cannot interrupt the current after it has once been established; therefore we are always depending on some other means of interrupting the current flow. Although the inability of the tube to interrupt a current seems like a handicap, it was pointed out that in certain circuit applications this may be a very desirable feature. When gaseous tubes are used in ac circuits, however, and most of them are found in just such circuits, the inability of the grid to interrupt the flow of current suddenly becomes utterly insignificant. After all, these gaseous tubes, besides being switches, are at the same time rectifiers; consequently, if they are used in an ac circuit, the current on its own accord falls to zero, and the tube will extinguish. It has then a full half cycle of time to deionize, which is usually greatly in excess of the actual deionization time, and firing will not take

place at the beginning of the next half cycle if the grid has become sufficiently negative at this moment. With these facts in mind, the discussion of the various ratings should become clear.

The *maximum peak forward voltage* is the maximum voltage that the tube can withstand (while it is nonconducting) in the direction in which it is supposed to carry current. This is a rating that does not exist for a straight rectifier since in a rectifier tube current will flow as soon as the anode becomes positive with respect to the cathode by a relatively small amount.

The *maximum peak inverse voltage* states how negative the anode may be made with respect to the cathode without causing a breakdown and current flow in a direction opposite to that for which the tube is designed. This value depends not only on the structure or geometry of the tube, the gas pressure, and the electrode material—quantities which admittedly have an influence on the peak inverse voltage—but also on the current that the tube is carrying in the forward direction and on the frequency of the anode supply voltage. The reason for this is that the speed with which deionization takes place is of importance. When the tube has been carrying a large current in the forward direction, there is naturally more ionized gas in it so that breakdown in the opposite direction is more likely. It is furthermore obvious that this danger will also be increased if the voltage in the opposite direction builds up with a high speed, *i.e.*, if it is of high frequency.

The *maximum instantaneous anode current* is the highest instantaneous *periodic* current that the tube can stand under normal operating conditions. This limit is dictated by the heating of the tube and by positive ion bombardment that may damage the cathode.

The *maximum surge current* is another rating that the designer of gaseous-tube circuits must take into consideration. The circuit should be designed in such a way that even under conditions of failure, such as a short circuit in the equipment, the current that will pass through the tube will not exceed this value. In other words, this is not a condition that can be permitted to occur under normal operating conditions, and if the tube is subjected to it repeatedly, its life will be considerably shortened.

The most important rating for the designer of electronic circuits is the *maximum average anode current*. This value is based on the heating of the tube due to the power loss occurring in the arc. This current can be measured by a dc meter in series with the anode or the cathode lead. If the operating cycle is fast enough, it is simply equal to the steady reading of such a meter. Since the cathodes of gaseous tubes have usually a fairly large mass, however, the averaging period is usually several seconds and varies from tube to tube. If a tube is rated, for instance, 2.5 amp maximum average anode current and if the averaging period is given as 15 sec, then as far as this rating is concerned, the tube could carry 7.5 amp for

every 5 sec out of every 15 sec, or 15 amp for 2.5 sec out of every 15 sec, or 37.5 amp for 1 sec out of every 15 sec. If the manufacturer specified, however, that the maximum instantaneous anode current must not exceed 15 amp, then the last-mentioned operation would, of course, exceed the rating of the tube. As a matter of fact, if the tube should be used in an ac circuit, carrying a sinusoidal current during alternate half cycles, then the average anode current, as indicated by a dc meter, evidently must never exceed approximately 5 amp since under this type of operation the amplitude of the half waves will be π times the average as indicated by the dc meter. A dc meter reading of 5 amp would therefore be equivalent to approximately 15.7 maximum instantaneous anode current.

The grid-current ratings are given in terms of the maximum instantaneous grid current and the maximum average grid current; the integration period is the same as for the anode current. The grid does not take any current, of course, when it is negative with respect to the cathode and if the tube is in the nonconducting state. In the circuits in which such tubes are used, a condition often prevails where the anode is negative for one half cycle while the grid is positive during this period. Under such a condition grid current would, of course, flow even if no current can flow in the anode circuit. This is a reason, additional to the one mentioned in an earlier paragraph, for including a resistor in series with the grid such that the grid current will be limited to the values permitted by the manufacturer.

The tube drop, or the arc drop, is the voltage existing between anode and cathode during the time the tube carries its normal current. As stated previously, this drop depends on the nature of the gas, and its usual value is from 15 to 25 volts.

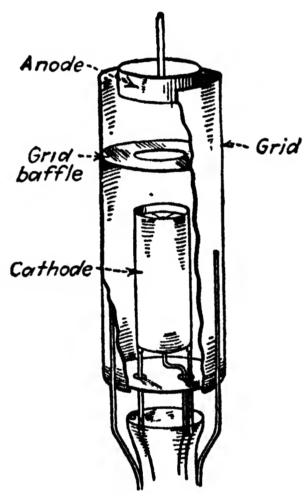

Fig. 21-7.—Structural details of a grid-controlled gaseous tube. (*From "Theory and Applications of Electron Tubes," by H. J. Reich.*)

21-21. Structural Differences between Vacuum and Gas Tubes.—The electrode structure of a thyratron differs considerably from that of the ordinary vacuum tube. The anode is usually a graphite disk while the cathode is of the heat-shielded construction, as discussed in an earlier chapter. The biggest difference, however, is found in the construction of the grid, which really can hardly be called a grid any more. Figure 21-7 shows the cross section of an ordinary thyratron. It is seen that the grid is a circular disk with a hole in the center, surrounded by a cylinder; this construction assures that the electric field near the cathode in the nonconducting state is determined practically entirely by the potential of the grid with respect to the cathode.

21-22. Grid Current of Thyratrons.—The most important characteristic of a vacuum tube is the fact that it permits the control of current flowing in the plate circuit with the expenditure of a very small amount of electrical energy in the grid circuit. The gaseous tube is a switch, operating by a change of control voltage applied to its grid; evidently, its usefulness and its fields of application become the larger, the smaller the current that the grid takes before the tube becomes conducting. With the grid in a thyratron having an area so much larger than the grid in a vacuum tube and with the grid structure submerged in the arc during the time of conduction, the danger of having cathode material deposited on it is much greater than in a vacuum tube. Preconduction grid current in an ordinary thyratron can therefore be expected to be considerably larger than the grid current usually encountered in vacuum tubes, which may lead to trouble if very high resistances are included in the grid circuit.

21-23. Negative and Positive Thyratrons.—In some circuit applications the need for a negative bias in the case of an ordinary thyratron is more of a disadvantage than a small amount of grid current, *i.e.*, a small amount of wattage, for control purposes would be. If the grid is provided with several baffles instead of the one shown in Fig. 21-7, it is possible to obtain a tube that will not fire until the voltage on the control grid is made positive. Such tubes are called "positive grid thyratrons," whereas the tubes discussed previously are referred to as "negative grid thyratrons." When the grid becomes positive with respect to the cathode, it naturally takes considerably more current than when it is negative. Positive tubes have other advantages, however, besides not requiring any negative bias, as has already been mentioned. They usually, although not always, have a shorter deionization time than the negative tubes. In many circuit applications, especially those involving higher frequencies and the use of tubes as inverters, the deionization time is of the utmost importance and is often the limiting factor of the design.

21-24. Shield-grid Thyratrons.—The desire to reduce the grid current and with it the amount of power required for control purposes has led to the development of the shield-grid thyratron. Figure 21-8 shows the cross section through such a tube. It is evident that it is the equivalent of the tetrode or screen-grid tube familiar to us from the discussion of vacuum tubes. As a matter of fact, the low-power thyratrons of types 884 and 2050 are often referred to as "gas triode" and "gas tetrode," respectively. Examination of Fig. 21-8 indicates that the control grid is now well outside the arc stream that passes from cathode to anode through the two holes in the shield grid. Such tubes have very low preconduction grid current and are therefore preferred in circuits having high impedances. The characteristics of a typical shield-grid thyratron are shown in Fig. 21-9, which discloses another possible use of the shield grid itself. It is possible to convert the tube into either a positive or a negative tube at

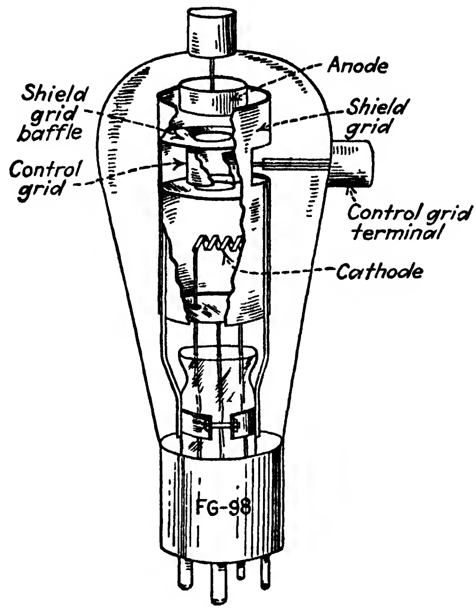

FIG. 21-8.—Structural details of a shield-grid gaseous tube. (*From "Theory and Applications of Electron Tubes," by H. J. Reich.*)

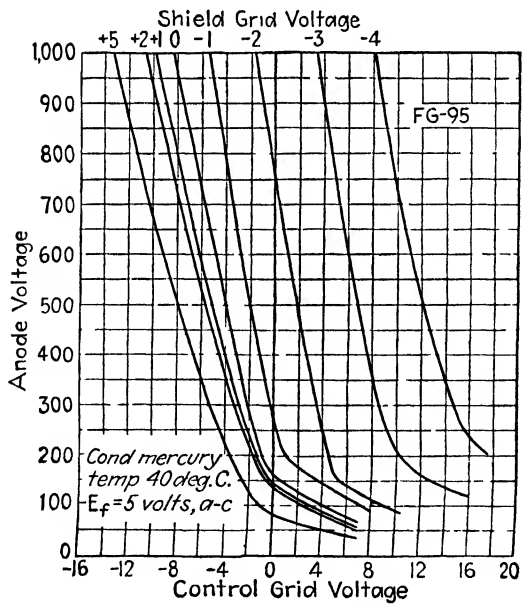

FIG. 21-9.—The control characteristics of a shield-grid tube depend on the voltage applied to the shield grid. (*From "Theory and Applications of Electron Tubes," by H. J. Reich.*)

will simply by changing the bias on the shield grid. With the shield grid connected directly to the cathode, *i.e.*, with its potential at zero volt, the tube is essentially a negative tube requiring about −8 volts on the control grid to cause firing with an anode voltage of 1,000 volts; if the shield grid is made approximately 3 volts negative, on the other hand, it is seen that the tube becomes a positive tube, requiring about 4 volts positive on the control grid to make it fire with an anode voltage of 1,000 volts.

21–25. Cold-cathode Grid-controlled Tubes, or Grid-glow Tubes.[8]— Cold-cathode glow-discharge tubes may also be equipped with a third electrode or grid to control the start of the discharge. They are called "grid-glow" tubes. The relatively high voltage across the discharge naturally limits the amount of current that they can carry. Nevertheless, the fact that they do not require any filament current makes them quite often a desirable tool of the electronic engineer, especially where only occasional operation of a relay or other control device is desired. Figure 21–10 shows the cross section through a typical grid-glow tube. It will be noted that, just as in the voltage-regulator tubes, the cathode is of large area, while the anode is very small compared to the cathode; the anode is surrounded by a small cylinder which acts as a grid. Ignition takes place in a manner somewhat differing from that of the hot-cathode type of thyratron; breakdown occurs at first between grid and cathode and then transfers to the anode, provided the latter is more positive than the grid. The grid-to-cathode ignition potential of the grid-glow tube is approximately 270 volts, while the extinction potential is 170 volts. In order for the discharge to transfer to the anode, the latter must be positive with respect to the grid by an amount depending on the current flowing in the grid circuit. If a grid current of 10 μa has been established, for instance, the anode must be approximately 300 volts positive with respect to the grid before transfer takes place. With a grid current of 50 μa, on the other hand, transfer will take place if the anode is only 20 to 30 volts positive with respect to the grid.

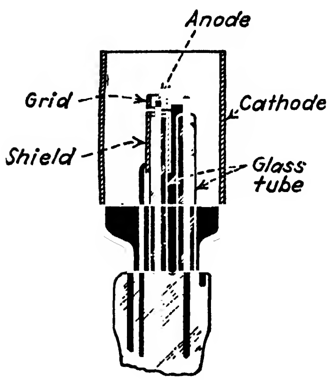

FIG. 21–10.—Structural details of a grid-controlled cold-cathode gaseous tube. (*From "Theory and Applications of Electron Tubes," by H. J. Reich.*)

21–26. Mercury-pool Type of Tubes.—The current-carrying ability of a thyratron is essentially determined by the cathode. High current surges, even if lasting only a very short time, can seriously damage the cathode. The average current of even the largest thyratron, the FG 41, is only approximately 12 amp; the peak inverse voltage of this tube, on the other hand, is quite high, approximately 15,000 volts. A pair of these tubes

can handle an appreciable amount of power (approximately 200 kw), but it is seen that they can do so only if the voltage of the circuit is high; in other words, a thyratron is essentially a high-voltage low-current device. For really large amounts of power another member of the gaseous discharge type of tube has attained much importance; as a matter of fact, it has been in use as a rectifier much longer than any of the other types so far discussed. This tube is the mercury-arc rectifier, which uses a pool of liquid mercury as the cathode. Such a cathode can carry tremendous amounts of current compared to the other types; during discharge, some of the mercury vaporizes because of the heat of the arc but is condensed again in the cooler part of the tube and made to return to the cathode pool. Since there are no other gases included in the envelope, the mercury cannot enter into any chemical combination, and it is therefore evident that such a cathode is self-renewing and practically indestructible. The mercury-arc rectifier with its high current-carrying ability can be employed in relatively low-voltage circuits such as 230 or 440 volts ac.

21–27. Starting of the Arc in a Mercury-arc Tube.[5–6]—A cold mercury-pool-type cathode naturally does not emit electrons, and it would require a very high voltage to cause breakdown from anode to cathode. Ever since mercury-arc rectifiers were used it has been necessary to have some auxiliary means to establish what is known as a "cathode spot" on the mercury. This has been accomplished by drawing an arc between an auxiliary electrode and the pool of mercury. In the early glass tubes a mercury bridge was established between the cathode pool and a small auxiliary pool of mercury located in a well in the side of the tube. When it was desired to start operation of the main circuit a suitable dc source was connected over a resistance to these two pools, the mercury in which touched in the starting position. Tilting of the tube caused this mercury bridge to break, with the result that an arc was drawn between the two pools. Whenever the main anode was then positive with respect to the cathode pool, current would flow in the main circuit.

21–28. Multiphase Mercury-arc Rectifiers.—It is the usual practice to obtain large amounts of dc power from a three- or six-phase ac system by the use of as many individual rectifying elements as there are phases; the individual rectifiers all feed into a common dc load in a manner entirely analogous to the full-wave rectifier system which was analyzed in detail in Chap. VIII and which, as was shown there, could be considered as a biphase system. Until a few years ago, the usual high-power mercury-arc rectifier consisted of a steel tank, several feet in diameter, carrying in its bottom a pool of mercury which served as a cathode to a number of anodes arranged in a circle in the cover of the tank. Such a construction seems quite logical, but recently the trend has been toward a single anode rectifier in preference over a multianode type, since a number of advantages, such as lower arc drop and consequently higher efficiency, simpler mainte-

nance, and others are in favor of this construction. Figure 21–11 shows a unit consisting of six single-anode tanks, which is used to convert alternating current furnished by a six-phase system into direct current. The cross section through an individual tank is shown in Fig. 21–12. In order to make such a tank perform in the same manner as a hot-cathode-type tube, where there is a supply of electrons always available, it is necessary to maintain at all times an auxiliary arc between the mercury-pool cathode

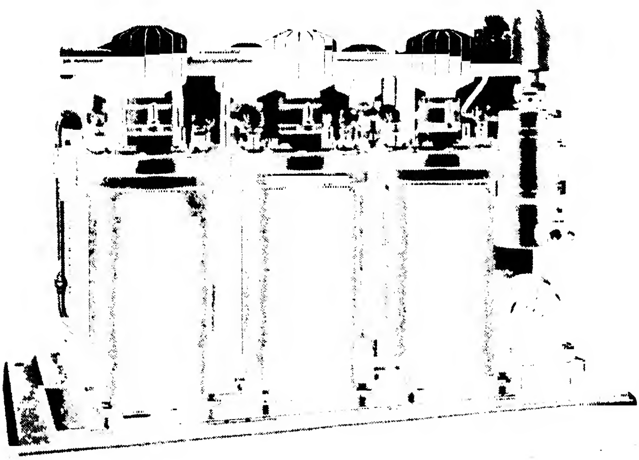

FIG. 21–11.—A multiphase rectifier consisting of six single-anode tank units for handling large amounts of power. (*Courtesy Allis-Chalmers Manufacturing Company.*)

and a special excitation anode. This auxiliary or pilot arc is started automatically by a jet of mercury which momentarily short-circuits the excitation anode to the cathode. The current required for excitation is from 7 to 8 amp, supplied from a dc source of about 35 volts; the power required for excitation is, therefore, of the same order of magnitude as that required for the heating of the filaments in a hot-cathode-type tube capable of furnishing the same amount of power.

Figure 21–12 shows that a basket-shaped grid structure surrounds the anode, which makes it possible to exercise grid control of the current in the same manner as would be the case with a hot-cathode-type tube. When an arrangement of six of these tubes is used in connection with a six-phase ac system, each rectifying unit carries current only during one-sixth of a cycle when the phase to which its anode is connected has a higher positive potential than the other five anodes. The current is therefore seen to transfer successively from one rectifying element to the other. Similarly to any gaseous tube, the grid can prevent the anode from taking over the

current when it becomes positive with respect to the cathode, but it cannot interrupt the current once it has been established. If the grid is made sufficiently negative, however, during the period of nonconduction of the tube, the respective anode will fail to take over the current which will then be carried by the anode that is conducting at this instant, to the end of

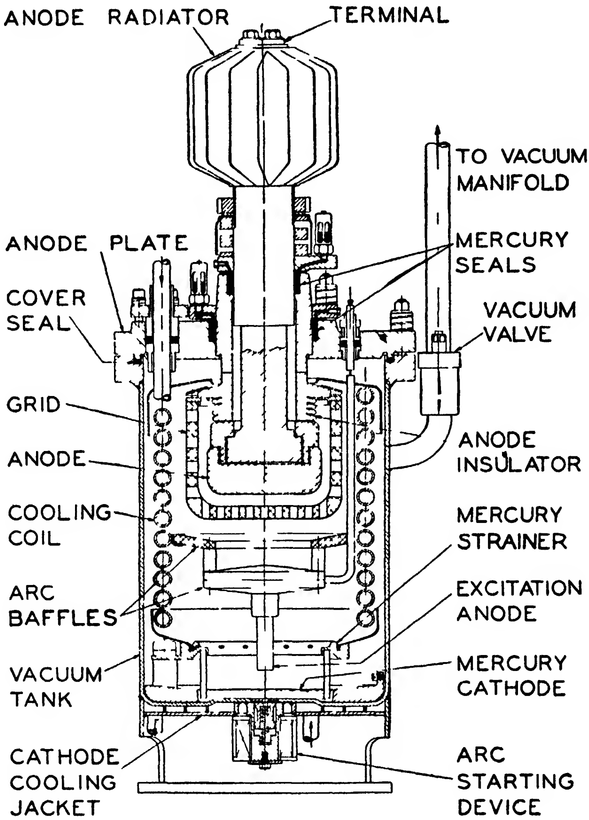

FIG. 21–12.—Cross section through the Excitron, a single-anode mercury-pool-type rectifying unit. Note the basket-shaped grid and the excitation anode. (*Courtesy Allis-Chalmers Manufacturing Company.*)

the half cycle, when conduction in the load circuit will then stop. If it is therefore desired to interrupt the load current of a mercury-arc rectifier in case of overload, or for any other reason, it is only necessary to make all the grids negative; current flow will then persist at the most for one half cycle only, that is, until the current of the anode that is just conducting passes through zero.

In the case of a multiphase mercury-arc rectifier consisting of an assembly of individual tanks of the type described above, the load current may also be interrupted by opening of the auxiliary dc excitation circuit.

In this case too the load current will then flow only for one half cycle at the most but, since all other cathodes (with the exception of the one that was just carrying the load current) have become nonemissive, the current cannot be taken over by any of the other anodes.

21-29. Ignitron.[7-10]—The inability to start an arc in a mercury-pool cathode type of tube by means of grid control was a serious handicap and therefore received the attention of many workers in this field. An acceptable

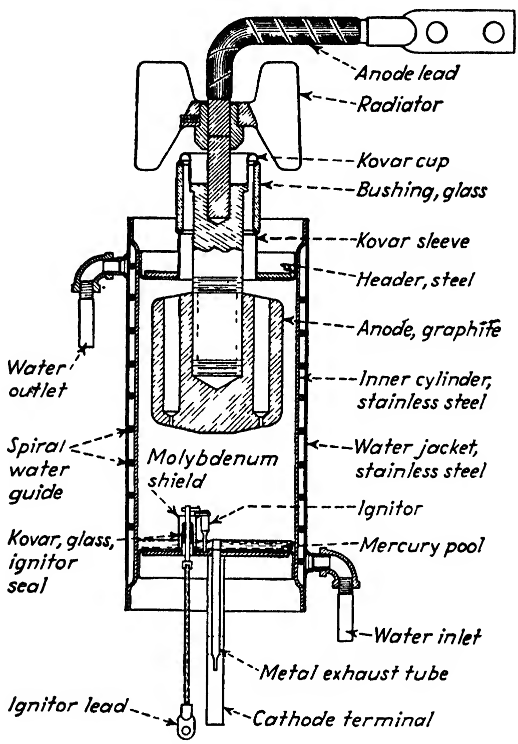

FIG. 21-13.—In an ignitron the main arc is started by sending an igniting current from the igniter to the mercury pool. (*Courtesy Westinghouse Electric Corporation.*)

solution was found in the shape of the ignitron, first proposed by Westinghouse engineers. Breakdown or firing of the tube is based on a principle radically different from that found in the other gaseous tubes. It was discovered that by letting a rod of semiconducting material, such as carborundum, dip into the mercury pool and by passing a current from this rod to the mercury, minute arcs formed at the point of contact between this rod and the mercury and that the main arc from anode to cathode then followed. The rod is usually referred to as the "igniter," which gave the tube its name. From the foregoing, it is evident that the firing of an ignitron requires the expenditure of a definite and appreciable amount of en-

ergy, in contrast to the thyratron where firing is essentially a matter of electrostatic control. However, this admitted disadvantage is more than made up by the high current-carrying ability and overload capacity that the ignitron possesses. Figure 21–13 shows the cross section through an ignitron tube. It is seen that the tube is constructed with two concentric steel cylinders forming the walls, which permits the circulation of water between the two cylinders and thus provides a convenient method of carrying away the heat developed by the arc within the tube. The current required through the igniter electrode to cause breakdown of the main path varies between 3 and 20 amp, and the voltage required to drive this current through the igniter is approximately 50 volts. This looks like a large amount of power, but the time required for ignition is very short, usually between 20 and 100 μsec. As would be expected, the time required for breakdown is the shorter, the higher the ignition current is. A discussion of the various methods of obtaining the current required by the igniter electrode will be deferred to Chap. XXII.

Although the ignitron represents the most common method of causing a mercury-pool type of tube to fire, it is not the only solution to this problem. Thus, electrostatic breakdown can be obtained by using a glass tube and placing a metallic band around the outside of the tube in such a way that it is level with the pool of mercury on the inside. The application of a very high voltage to this band causes a discharge to take place on the inside of the glass and leads to breakdown of the tube. Another method [9] employing the same principle places an electrode housed in a small glass tube on top of the mercury pool. Here too the application of a high voltage to this auxiliary electrode insulated from the mercury pool causes a discharge to take place that then leads to the breakdown of the main path. It can be confidently expected that much thought and research will be done on the subject of starting an arc in a mercury-pool type of tube and that only time will tell whether the present-day ignitron is the final solution.

PROBLEMS

21–1. Why does a gaseous tube carry much more current than a vacuum tube and with much lower voltage?

21–2. Why do the cathodes of gaseous tubes have a higher emission per watt of filament power?

21–3. A dc power supply using a 5Y3 rectifier tube with choke input (see Fig. 21–14 for characteristics) and a center-tapped transformer with an over-all voltage of 700 volts is to be combined with two VR-105 tubes to provide a source of 210 volts regulated direct voltage. The specifications for the VR-105 will be found in Sec. 21–12. Design the circuit for a condition where

a. The load draws a current varying between 0 and 25 ma.

b. The load draws a current varying between 30 and 50 ma.

In each case, design the circuit so that the VR tubes are operating at the mid-point of their operating range when the load current is midway between its extreme limits.

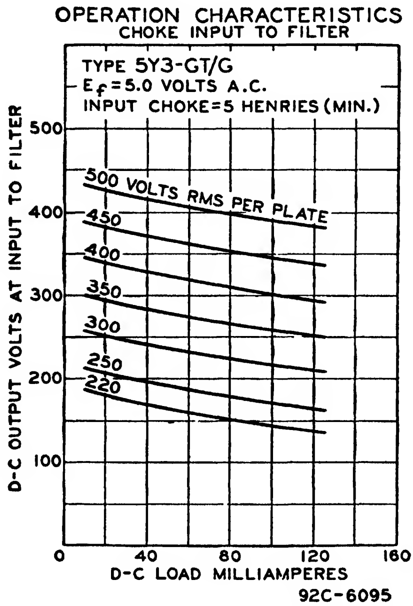

FIG. 21-14.—Characteristics of a 5Y3 tube. *(Courtesy Radio Corporation of America.)*

c. Assuming that the dc output of the rectifier itself is roughly proportional to the alternating line voltage applied to the primary of the transformer, how many per cent could the line voltage drop in case (*b*) before the VR tubes under the most unfavorable load condition receive less than the required minimum current?

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CHAPTER XXII

FUNDAMENTAL CONTROL CIRCUITS FOR GASEOUS TUBES

22-1. Field of Application of Gaseous Tubes.—Owing to the fact that their current-carrying ability is much greater than that of vacuum tubes, gaseous tubes have found wide application for industrial control purposes. It would be far beyond the scope of this book to discuss specific applications of these tubes; however, since the principles underlying their control are essentially the same in all applications, they will be discussed in detail. The reader should then be in a position to apply them to the analysis of any given specific control.

We have seen that the current flowing through a grid-controlled gaseous tube, such as a thyratron, could not be interrupted by a control voltage applied to the grid, but that it was necessary either to interrupt the anode circuit or to bring the anode voltage to zero by some other means. Although this feature makes the use of thyratrons in dc circuits almost impossible—except where it is desired to keep the current flowing even after the control impulse has ceased to exist—it must be remembered that the thyratron is also a rectifier tube. If the tube is used in an ac circuit, the current will automatically go to zero, and extinction of the tube will take place. During the half cycle that follows, the tube deionizes and the grid will regain control. By making the grid voltage sufficiently negative, the tube will therefore be prevented from firing when the anode voltage becomes positive again.

22-2. Two Basic Connections for Gaseous Tubes.[1]—Thyratrons—or for that matter ignitrons—may be employed in two basic control circuits. In the first group, use is made not only of their control characteristics but also of the fact that they are rectifiers at the same time. The tubes, their number ranging from one to six or even more, are arranged in such a way that the current flowing through them will pass through the load in the same direction. These circuits have found wide application for the operation and control of dc motors from an ac supply system.

It is evident that the rectifier circuits discussed in Chaps. VII and VIII will automatically become controlled rectifier circuits if we replace the conventional diode with a grid-controlled rectifier, such as a thyratron. The basic circuit applying to a full-wave rectifier is shown in Fig. 22-1. A center-tapped transformer is connected to two gaseous tubes in a manner exactly identical to that of a full-wave rectifier. But while with ordinary rectifiers the current in the load could not be controlled except by reduc-

ing the transformer voltage, it is obvious that with grid-controlled rectifiers taking the place of the ordinary rectifiers, it will be possible to do so. If we make the two grids sufficiently negative with respect to their cathodes, for instance, no current will flow in the load at all. The methods available for the control of the grid will be discussed a little later in this chapter. The load in the circuit shown in Fig. 22–1 may be any dc load; thus it may be a dc motor, or it may be a dc relay, in which case the two thyratrons would be of a small type.

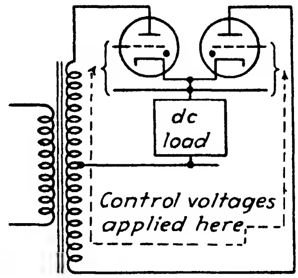

FIG. 22–1.—When two gaseous tubes are connected as shown here, they serve two purposes simultaneously: (1) being rectifiers, they permit the operation of a dc load from an alternating-voltage-supply system; (2) by making use of their control characteristics the amount of direct current may be controlled.

In the second basic group it is not desired to convert the alternating current, but the tubes are meant to control an ac load. To this group belongs an application that probably accounts for more than 90 per cent of all the applications of this type. This is the control of resistance welding machines that, as we know, must operate from alternating current since they employ a transformer. The basic circuit applying to this group is shown in Fig. 22–2. The connection of the two tubes shown in this diagram is often referred to as "back-to-back" connection. The solution of the problem shown in this figure is evidently entirely logical. Since a gaseous tube can conduct current in only one direction, it is only necessary to arrange two of them in such a way that each will take over the conduction of current during one half cycle. This is exactly what is accomplished by the circuit shown in Fig. 22–2.

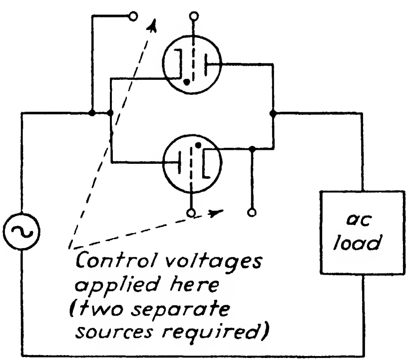

FIG. 22–2.—When the load is an ac load (such as a transformer), the two gaseous tubes may be connected in a back-to-back connection. If both tubes act identically, a pure alternating current flows in the load.

22–3. Comparison of Control Voltage Requirements for the Two Basic Connections.—As far as the individual tubes are concerned, there is no difference between the two circuits shown in Figs. 22–1 and 22–2; from a practical point, however, the circuit shown in the former is somewhat easier to handle than the other. It will be noted that in the circuit shown in Fig. 22–1 the cathodes of the two tubes are connected together; if the tubes are hot-cathode-type tubes, such as thyratrons, the two filaments can be operated from the same transformer. Furthermore, the control voltages that must be applied

between grids and cathodes can have a common point. Examination of Fig. 22–2 shows that this circuit does not enjoy the advantages just mentioned. If the tubes are kept from firing, *i.e.*, if they act like open switches, it is clear that full line voltage will exist between the two cathodes. The filament current can therefore not be supplied by a common transformer but must be supplied by two separate windings insulated from each other for a voltage equal to the line voltage. The two control voltages that must be applied to the two grids must also be obtained from entirely separate sources. Although this does not change in any way the fundamentals involved in the control of the tubes themselves in either of the circuits, it is evident that it is easier to design and construct a circuit as shown in Fig. 22–1.

In both circuits shown in Figs. 22–1 and 22–2 the symbols indicating a heated cathode could be replaced by one indicating a mercury-pool-type cathode, and the grid would then be replaced by the symbol of an igniter dipping into the mercury. Although the operation of the igniter in the case of an ignitron differs from that of a grid in the case of a thyratron, the two circuits will operate with ignitrons as well as with thyratrons, as far as the load is concerned.

22–4. Construction of Control Locus.—We shall now investigate the various methods of applying a control voltage to the grid of the tubes. Consider the circuit shown in Fig. 22–1 and assume, at first, that the load is a pure resistance. Let the two tubes employed in this circuit be two FG 81's, the characteristic of which is shown in Fig. 21–5. Since these tubes cannot hold back more than approximately 140 volts between anode and cathode, the voltage delivered by the center-tapped transformer between the center tap and the end of each winding must not exceed 140 volts, which is equivalent to an rms voltage of approximately 100 volts. The characteristic shown in Fig. 21–5 discloses that with a negative voltage of 4 volts applied between cathode and grid, the tube will not fire when the anode voltage reaches 140 volts. Consequently, if in Fig. 22–1 we should connect the two grids together and connect a 4.5-volt battery between them and the cathodes, the tubes will never fire, and the current through the load will be zero. What will happen if we make this direct voltage applied to the two grids variable and reduce it to 3, 2, or 1 volt? In order to answer this question the characteristic as given in Fig. 21–5 is used to construct what is known as the "control locus." The procedure is shown in Fig. 22–3. To the right of the vertical axis is plotted one half cycle of the anode supply voltage, *i.e.*, of the voltage delivered by one-half the center-tapped winding of the supply transformer. The scale is the same as that used for the tube characteristic itself. Now take any point on this curve, such as A. At the time represented by this point the transformer voltage has reached a value slightly in excess of 60 volts. Now draw a horizontal line through A until it intersects with the tube charac-

teristic; this gives us point *B*. Going vertically down from *B* tells us that if the tube is to be kept from firing at this particular instant, the grid

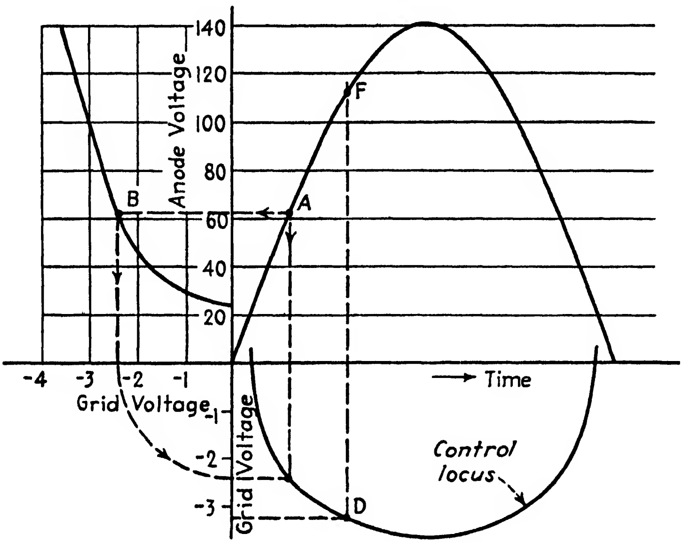

FIG. 22 3.—The method of constructing the control locus of a gaseous tube.

must be at least 2.4 volts negative with respect to the cathode. This value is plotted on a vertical line passing through *A*, using the region below the *X* axis for this purpose, which will, of course, then have a different scale from the region above it. If we repeat this process for a number of points on the half sine wave representing the transformer voltage, we obtain below the *X* axis a curve for the grid voltage that is called the "control locus" of the tube. It is evident that as long as the grid voltage at any given instant is below (*i.e.*, more negative than) this control locus, the tube will not fire and will therefore represent an open switch. Now let us go back to the circuit shown in Fig. 22–1. The circuit is redrawn in a somewhat more convenient form in Fig. 22–4; at the same time, a method of applying a negative voltage to the grid of the two tubes is shown. If now the arm of the potential divider is adjusted so that the grids are 4 volts negative with respect to the cathodes, the tubes will never fire, as has already been explained. Let us reduce the negative grid voltage to −3.2 volts. Drawing a horizontal line through this value of grid voltage in Fig. 22–3, we find it to intersect the control locus at point *D*. This means that although the

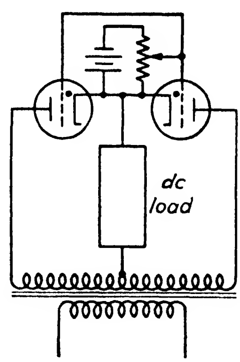

FIG. 22–4.—The firing point of a tube may be controlled by the application of a dc bias. This method does not permit the delaying of the firing point beyond the instant when the anode voltage has reached its maximum.

grid is still able to prevent the tube from firing during the earlier part of the cycle while the voltage from the transformer is low, it is not able to prevent the tube from firing any more when D has been reached. At this instant the anode has reached a voltage given by point F, and breakdown of the tube will now occur. Since the grid has no control after current flow has once started, the tube will now be conducting for the rest of the half cycle. It is quite evident that we can never reduce the time of conduction to less than one quarter cycle, except, of course, by cutting the tube off completely.

By reducing the negative grid voltage applied to the two grids further, the firing point of the tube can be further advanced in the half cycle. When the grid voltage is zero, each tube will conduct during practically a complete half cycle. A dc meter placed in series with the load will therefore record a maximum value when the grid voltage is zero, and this value would be exactly the same as if we were dealing with two ordinary rectifier tubes instead of two grid-controlled tubes. By making the grid increasingly negative, the dc meter will read less and less until it reaches a value just one-half the maximum value. An attempt to make the grid more negative will lead to a sudden cessation of current altogether. The reader will find it an excellent mental exercise to find what an rms meter would do if connected in series with the load.

22-5. Principle of Phase-shift Control.[2]—It is quite natural that engineers were not satisfied with the results obtainable by using a variable dc bias. A method permitting the delay of the firing instant *beyond* the halfway mark of the positive half cycle of the anode voltage was first proposed by the French engineer Toulon. Although the basic principle of this method is still the one in use today, further refinements have been applied to it. In Fig. 22-5 is shown the positive half wave of the anode voltage and the control locus. The figure is therefore essentially the same as Fig. 22-3, except that the scale of the grid voltages has been reduced. Now let us apply to the grid an alternating voltage with an amplitude considerably in excess of the maximum negative voltage required to prevent the tube from firing, even at the instant when the anode has reached its maximum. Let this control voltage be 180 deg out of phase with the anode voltage; in other words, let the grid potential reach its negative maximum at

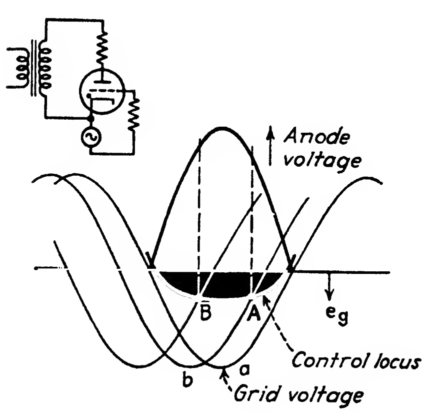

FIG. 22-5.—If an alternating voltage of constant magnitude but of varying phase is used for control purposes, the firing point may be delayed any desired amount.

the moment when the anode voltage reaches its positive maximum. This is indicated in Fig. 22-5 by the line *a*. It is seen that the actual grid voltage is at any given instant more negative than the critical grid voltage, as given by the control locus, and the tube will therefore not fire at all. The grid will, of course, become positive during the following half cycle; since the anode voltage will then be negative, however, firing cannot take place. (In order to prevent excessive values of grid current during this period, it is desirable to place a resistor in series with the grid that will limit the grid current during the positive half wave of the grid voltage.) Now let us assume that we can change the phase of the grid voltage with respect to the anode voltage. Line *a* had shown the grid voltage 180 deg out of phase with the anode voltage. We could evidently say with equal right that the grid voltage is either retarded or advanced 180 deg with respect to the anode voltage. Now let the grid voltage be shifted to the position indicated by line *b*. This is obviously an advance from the position shown by line *a*, and the most logical description is that the grid voltage is now retarded with respect to the anode voltage by an angle less than 180 deg. This line crosses the control locus of the tube at point *A*, which is seen to make the tube fire during the second part of the positive half wave of the anode voltage. By a further advance of the grid voltage, the tube can be fired at any desired point of the positive half cycle of the anode voltage. When the grid voltage is in phase with the anode voltage, firing will take place practically at the beginning of the half cycle, and the tube will act like an ordinary rectifier. If we can devise a circuit that will permit us to retard the phase of the grid voltage from a condition where it is in phase with the anode voltage to a condition where it is 180 deg out of phase, we shall be able to control the firing point of the gaseous tube so that the average current can be cut down from full value to zero. This method of control of gaseous tubes is known as "phase-shift" control.

22-6. Circuits Providing Phase Shift.[3]—The basic principle of practically all circuits providing the desired shift of phase is shown in Fig. 22-6. Let a series combination of an inductance and a variable resistance be connected across a center-tapped source of alternating current. For convenience this source is shown as a center-tapped transformer, but two equal resistances or inductances connected across any source of alternating voltage will, of course, provide such a center tap. Consider, for instance, point *B* as the zero, or reference level; the voltages of points *O* and *A* with respect to *B* are, of course, in phase since they come from the same transformer winding, and the voltage E_{AB} is twice the voltage of E_{OB}. This relation is shown in the vector diagram accompanying the circuit diagram.

The voltage E_{AB}, produced by the total winding of the transformer, will cause a current to flow through the series combination of resistance

and inductance; the current will lag the voltage by an amount depending on the ratio of the inductive reactance to the resistance. In the vector diagram the current I is shown lagging the voltage by an angle φ. The voltage across the resistor R will be in phase with the current, and the voltage across the inductance will be 90 deg ahead of the current. The vector sum of these two voltages must, of course, be equal to the applied voltage, *i.e.*, to the potential difference of A with respect to B. The line BD represents the voltage across the resistance, and DA represents the voltage across the inductance, these two components being at right angles to each other regardless of the amount of the resistance R or the inductance L. If we

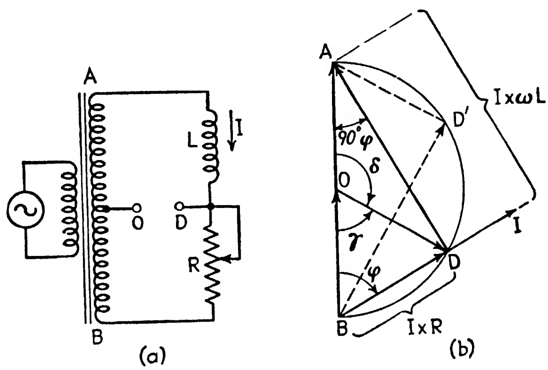

FIG. 22 6.—A standard phase-shift circuit and its vector diagram. Variation of R (or of L) does not change the magnitude of the voltage between points O and D but changes its phase. Note that the potential of point D with respect to O lags the potential of point A.

increase the value of the resistance, for instance, the voltage across it will increase, and the voltage across the inductance will decrease. The right-angle relationship is not disturbed, however, and we may thus obtain a condition as indicated by point D'. According to a fundamental and well-known law of geometry, D will therefore describe in the vector diagram a half circle with the vector BA as the diameter, when the resistance R is changed from zero to infinity. Now, starting from our reference point B, we can reach the level of D at any instant by two paths: we can go along the resistance R or along half the center-tapped winding to the level of O and then jump the gap from O to D. Both of these paths must bring us to the same level. Expressed electrically, the voltage of D with respect to B, given at any instant by the projection of the vector BD, must be equal the sum of the voltage of O with respect to B, plus the voltage of D with respect to O. But the voltage of O with respect to the reference point B is given by the vector BO in the vector diagram, which means that the vector OD represents the voltage of D with respect to O. This voltage is seen to have a value of one-half the total voltage across the center-tapped winding, regardless of the size of the resistance R (or for that matter, the inductance L) but is seen to go through a phase change

of 180 deg, when the resistance R is changed from zero to infinity. When the resistance is short-circuited, D will be simply connected to B and will therefore have a voltage with respect to O which is 180 deg out of phase with the voltage that A has with respect to O. With R equal to infinity, on the other hand, D will effectively be connected to A since there will be no voltage across the inductance with no current flowing through it, and its voltage will then be in phase with the voltage of A with respect to O. This means that we may connect O to the cathode of the gaseous tube that we want to control and connect D to its grid. The primary of the transformer shown in Fig. 22–6 must be connected to the voltage that furnishes the anode voltage of the gaseous tube, and the polarity must be such that A becomes positive when the anode of the gaseous tube becomes positive; in other words, the voltage E_{AO} must be in phase with the anode voltage.

22–7. Calculation of Phase Shift Obtained with a Resistance-inductance Combination.[4]—The amount of phase shift obtained by means of this circuit for given values of inductance and resistance can easily be calculated with the aid of Fig. 22–6. The vector diagram shown in Fig. 22–6 indicates that the voltage OD is retarded with respect to the voltage OA by an angle δ. The reader should have no difficulty in understanding the following equations:

$$\delta = 180° - \gamma \tag{22–1}$$

$$\gamma = 2(90° - \varphi) \tag{22–2}$$

and therefore

$$\delta = 2\varphi \tag{22–3}$$

The angle φ is given by the well-known relation:

$$\tan \varphi = \frac{\omega L}{R} = \frac{2\pi f L}{R} \tag{22–4}$$

For $R = 0$, $\tan \varphi$ becomes infinite and φ is therefore 90 deg; the angle of retardation δ will then be 180 deg according to Eq. (22–3). If it is desired to retard the grid voltage by 90 deg, then the angle φ must be equal to 45 deg, which will be accomplished when the inductive reactance $2\pi fL$ is equal to the resistance R.

It is, of course, impossible to provide a resistor that is continuously variable from zero to infinity. Let us investigate the range of the retardation angle if the maximum value of the resistance R is equal to ten times the inductive reactance of the coil L. Equation (22–4) gives us the angle φ as about 11 deg so that the angle δ will be approximately 22 deg. Drawing a sine wave 22 deg retarded with respect to the anode voltage, as indicated for other values of phase displacement in Fig. 22–5, and determining the intersection of this curve with the control locus would then give us the earliest moment at which the tube could be made to fire.

22-8. Necessity of Observing Correct Connection of Phase-shifting Elements.—In connection with Fig. 22-5 it should be noted that only the retardation of the grid voltage from the in-phase condition with the anode voltage or the advance from the 180-deg out-of-phase condition will result in the gradual control of the firing point described in the preceding paragraphs. The reader should convince himself that if we advance the grid voltage from the in-phase condition (or retard it from the 180-deg out-of-phase condition), the grid will always be positive at the instant when the anode voltage becomes positive. Under this condition the tube would fire at all times at the beginning of the cycle, and, consequently, no control could be obtained with such an arrangement. Such a condition would arise if in Fig. 22-6 we were to exchange the position of the inductance L and the variable resistor R. At first glance this statement sounds rather strange because the current flowing through a series combination of inductance and resistance certainly will not change either in size or in phase if the two elements are interchanged. However, owing to the fact that the inductance L will be connected to point B and the resistor R will be connected to A, the vector diagram of the circuit assumes the shape shown in Fig. 22-7. The current I, of course, still lags the voltage by the angle shown in Fig. 22-6, and the voltages across the resistor and the inductance are the same as those shown in Fig. 22-6, but the junction D between the two elements now lies on the other side of the line BA. Vector BD, representing the voltage across the inductance, is equal in size as well as in phase to vector DA of Fig. 22-6, and vector DA of Fig. 22-7 is equal to vector BD of Fig. 22-6. But vector OD, instead of being retarded with respect to vector OA, is now advanced with respect to the latter. In actual control circuits a number of transformers are usually employed, and it is necessary to pay careful attention to their polarities before a correct analysis can be made.

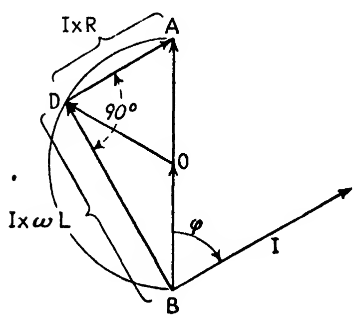

FIG. 22-7.—If the inductance L and the resistance R of Fig. 22-6 are exchanged, the current as well as the voltage appearing across L and R will not change, of course; but the potential of point D with respect to point O is now seen to be advanced against the potential of point A with respect to O.

22-9. Resistance-capacitance Phase-shifting Combination.—The performance of the circuit shown in Fig. 22-6 is obviously based on the quadrature relation of the two voltages across the inductance and the resistance, respectively. It is evident that similar results can be obtained with a capacitor-resistor combination. The circuit must, of course, again be arranged in such a way that *retardation of the grid voltage from its in-phase condition with the anode voltage* will be obtained. The proper connection

to achieve this result with a capacitor-resistor combination is shown in Fig. 22–8, which also shows the vector diagram of the combination. The current through the resistor-capacitor combination will now lead the applied voltage, as shown in the vector diagram; the voltage across the resistance is in phase with this current, and the voltage across the capacitor will lag the current. The vector *OD* is seen to be retarded with respect to the vector *OA*, which is the correct and necessary relation to obtain grid control. It should be noted that the circuits shown in Figs. 22–6 and 22–8 produce opposite results with a change of resistance. In Fig. 22–6 zero resistance caused the grid voltage to be 180 deg out of phase with the anode

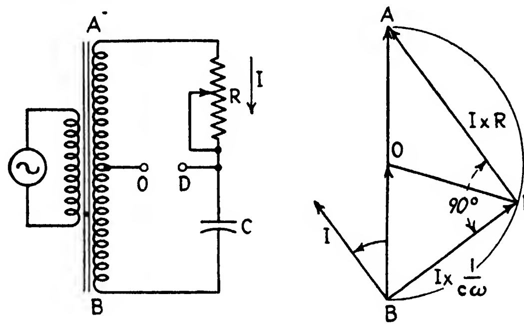

FIG. 22–8.—Phase shifting can also be obtained by the use of a resistor-capacitor combination. If the same phase relation is wanted as in Fig. 22 6, the two circuit elements must occupy exchanged positions, however.

voltage and so prevented the tube from firing. An increase of resistance then permitted the tube to fire at progressively earlier instants of the cycle until with infinite resistance it would fire at the beginning of the half cycle. With the circuit shown in Fig. 22–8, on the other hand, zero resistance will cause the grid voltage to be in phase with the anode voltage and thus let the tube fire right at the beginning of the cycle. An increase of the resistance will then lead to a decrease of the average current flowing through the tube.

22–10. Control of Phase Shift by Changing the Inductance or Capacitance of the Phase-shift Combination.—In the discussion of the circuits shown in Figs. 22–6 and 22–8 the resistor was assumed to be the variable circuit element. It will be realized, of course, that phase shift may just as well be obtained by giving the resistor a fixed value and changing the inductance or the capacitance. In many control applications involving the phase-shift method of control, the inductance consists of a coil with a movable laminated plunger. The inductance of the coil will, of course, increase the deeper the plunger is inserted into the coil, and the coil may then be used as the variable element in the phase-shift circuit shown in Fig. 22–6.

In some control circuits combining vacuum tubes with gas tubes, it is desired to convert a change of magnitude of a small direct current, such as

flowing in the plate circuit of a vacuum tube, into a phase-shifting alternating voltage for the control of the gas tubes. At present this problem is solved by the use of saturating reactors. A saturating reactor is an iron-core inductance with two or more windings, through one of which direct current is flowing. The iron core begins to saturate when the direct current reaches a certain value. One could also say that the amount of direct current determines the permeability of the core. When an alternating voltage is applied to the other winding, the current that will flow will depend on the reactance, which in turn depends on the permeability of the iron core. (The windings are usually in sections and so arranged that the reactor will not act as a transformer, feeding an alternating voltage into the source of the direct current.)

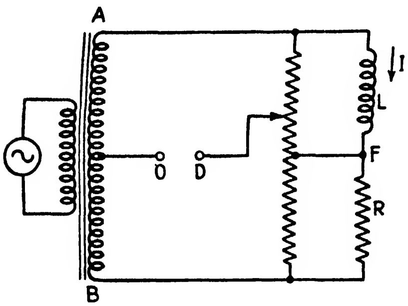

FIG. 22-9.—If it is desired to obtain a complete 180-deg phase shift, the circuit shown here may be used.

22-11. Production of Complete 180-deg Phase Shift.—Neither the circuit shown in Fig. 22-6 nor the one in Fig. 22-8 is capable of giving a complete 180-deg phase shift of the grid voltage because this would require the variable element—whether the resistance or the quadrature element—to have a range from zero to infinity. Where it is of importance to have such a complete control, a circuit as shown in Fig. 22-9 (used in Westinghouse welding controls) may be used to advantage. An inductance L and a resistor R, of such value that the inductive reactance x_L is equal to the resistance R, are placed across the total voltage of a center-tapped transformer. A center-tapped voltage divider is then placed across the two elements, as shown in Fig. 22-9. When the sliding arm of the potential divider is moved from one end to the other of the resistor, the phase of the grid voltage varies through 180 deg. The voltage does not remain constant, as can easily be seen from a study of the vector diagram shown in Fig. 22-10, which shows various possible positions and values of the vector OD. This variation in amplitude is usually of no consequence in the control of gaseous tubes.

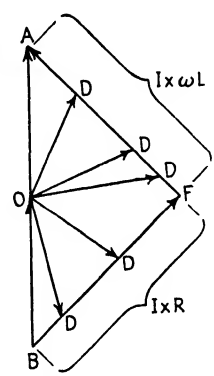

FIG. 22-10.—The vector diagram of the voltages furnished by the circuit shown in Fig. 22-9. The amplitude of the phase-shifting voltage does not remain constant, but this is usually of no consequence in the control of gaseous tubes.

22-12. Control of a Pair of Tubes with Phase-shift Circuits.—The discussion of the circuits shown in Figs. 22-6 and 22-8 considered the control of only one tube. If two tubes must be controlled in either one of the con-

nections shown in Fig. 22–1 or 22–2, the anode voltages of the two tubes are necessarily 180 deg out of phase, and the two grid voltages required for their control must therefore also be 180 deg out of phase with each other. The voltage *OD* furnished by the phase-shift circuits discussed is usually applied to the primary of a transformer. There are two secondary windings provided, each of which furnishes the control voltage for one of the tubes. In the case of the circuit shown in Fig. 22–1, these two windings may constitute simply a single center-tapped winding, the center tap being connected to the junction of the two cathodes. In the case of the circuit shown in Fig. 22–2, the two secondary windings must be separate and insulated from each other for a voltage equal to the anode supply voltage of the main circuit.

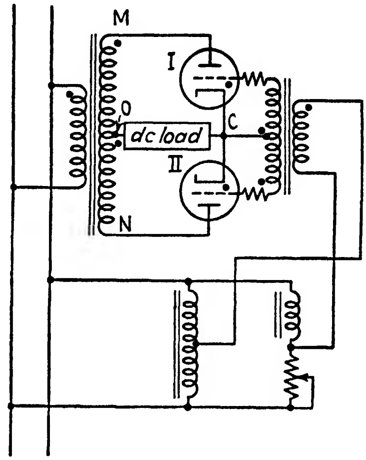

FIG. 22–11.—Phase-shift control applied to the two grids of two gaseous tubes for the control of the current flowing in a dc load.

Figure 22–11 shows a complete circuit embodying the principles just discussed. The two gaseous tubes are arranged in such a way as to furnish a controlled direct current. The reader should have no difficulty in analyzing this circuit. The center-tapped voltage required for the phase-shifting network is obtained by connecting a center-tapped reactor or choke across the total voltage; such a reactor could of course also be considered as an autotransformer with a 2:1 ratio of the voltages. Note the placing of the dots on all the transformer windings. With the aid of these dots indicating which of the transformer terminals have the same instantaneous polarity, the reader should be in a position to state definitely whether the connection of the grid transformer to the phase-shifting network, as shown in Fig. 22–11, is correct; in other words, whether a change of the resistance value of *R* from the short circuit to the open position will result in the required retardation of grid voltage with respect to the anode voltage, or whether the connection of this transformer will have to be reversed. Remember that with an inductance-resistance phase-shifting circuit the grid voltage must be 180 deg out of phase with the plate voltage when the resistor is short-circuited, as discussed in connection with Fig. 22–6. Will this be the case in the circuit shown in Fig. 22–11?

22–13. "Matching" of Thyratrons to Low-voltage Circuits.[5, 6]—Thyratrons are switches, as has already been stated repeatedly. They are essentially high-voltage low-current devices; even the largest one of the thyratron family, the FG 41, can handle only an average current of approximately 12 amp. But this tube can handle circuits operating at volt-

ages up to 7,000 or 8,000 volts. Suppose now that it is desired to control a welding transformer rated at 440 volts and 200 amp by means of two thyratrons. This would call for a connection as shown in Fig. 22–2; but, evidently, the current is beyond the rating of the two tubes, whereas the voltage is way below the ability of the tube. One could properly say that the switch formed by the combination of the two tubes is not "matched" to the load. The very word "matched" suggests a possible solution. We could step up the 440 volts at first by means of a transformer to 8,800 volts, a 20:1 ratio, which would bring the current down to 10 amp; we could then have the manufacturer wind the welding transformer for 8,800

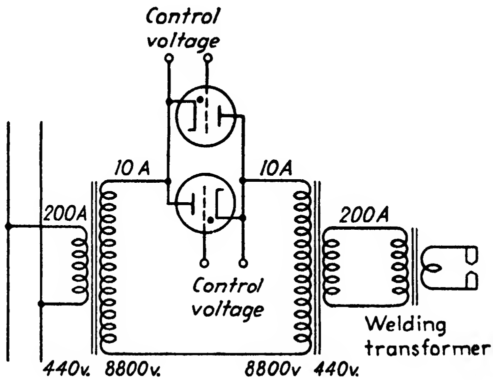

FIG. 22–12.—Hot-cathode gaseous tubes are essentially high-voltage low-current devices. If they are to be used for the control of loads operated from relatively low-voltage circuits, it is necessary to introduce two additional transformers.

volts instead of 440 and then use the two thyratrons as a switch in this circuit. For many very good reasons the manufacturer would refuse to do such a thing, however, which means that we shall have to buy a second transformer with which to step down the 8,800 volts to 440 again. The complete circuit will then look as shown in Fig. 22–12. In order to make use of the thyratrons, we shall have to purchase two transformers of the full rating of the welding transformer, an expensive procedure indeed! A solution to the problem requiring only one transformer is shown in Fig. 22–13. This circuit was used extensively for the operation of welding transformers by means of thyratron panels before the ignitron tube became the standard equipment for this service. The principle involved is so generally useful, however, and can be used if it is desired to control low-voltage circuits by means of the smaller thyratrons, that the reader should be familiar with the circuit. Again a 440- to 8,800-volt transformer is used, but this time the transformer is simply placed in series with the welding transformer or other load that it is desired to control. As long as the secondary winding of this transformer is open, the transformer will act like a reactor, and at the very most only the relatively small magnet-

izing current of it can flow through the load. The tubes connected to the secondary side will have to withstand the open-circuit voltage of 8,800 volts. On the other hand, when the tubes are made conducting, they short-circuit the transformer, and it will then offer only a negligible impedance to the flow of current, this remaining impedance being determined by the stray reactances of the transformer. In vacuum-tube circuits we have seen how a transformer may be used to match any given load to an amplifier tube, and the reader will realize that in the present case the transformer is used to match our special switch to the load. In other

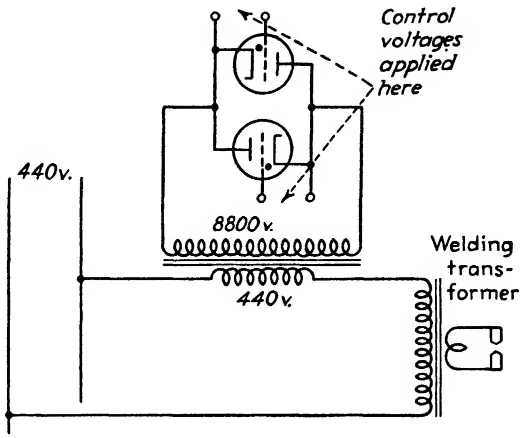

Fig. 22-13.—Results practically identical with those obtainable with the circuit shown in Fig. 22-12 may be obtained by matching the high-voltage low-current thyratrons to the load with the aid of a matching transformer.

words, the switch is being fooled into believing that it is closing and opening a circuit operating at a voltage of 8,800 volts and a current of 10 amp instead of the actual circuit that is operating at 440 volts and 200 amp.

22-14. Use of Peaked Voltages for the Control of the Firing Point.—In the phase-shift method of control of gaseous tubes, the instant at which the actual grid voltage crosses the control locus of the tube and thus causes firing is essentially determined only by the phase of the grid voltage. It will be appreciated, however, that variations in the amplitude of the applied grid voltage will also have an effect on the instant of firing. For this reason, later types of this control have been refined, making the firing point more independent of any possible fluctuation in the line voltage. The gaseous tube or tubes are now usually supplied with a direct bias voltage, obtained by means of a small dry-type rectifier; this direct voltage is many times as large as would be needed to prevent the tube from firing. Superimposed on this steady negative voltage is then a series of sharp impulses of positive polarity. This condition is illustrated in Fig. 22-14. The steady dc bias, marked E_{dc}, is approximately four times as large as the maximum negative voltage required to prevent the tube from

firing. A sharp positive impulse is then used to fire the tube at the desired instant in the cycle. A control operating on this principle obviously poses two problems: (1) we must obtain a sharply peaked impulse and (2) we must be able to shift the phase of this impulse with respect to the anode voltage. The phase shift is again obtained by a network like those shown in Figs. 22-6 and 22-8. Of course, the output voltage of these two circuits, *i.e.*, the voltage appearing between points D and O, is sinusoidal, but this output may be applied to a series combination of a capacitor, a transformer, the core of which saturates, and a resistor. By the proper choice of the circuit constants, it is possible to convert the sinusoidal voltage delivered by the phase-shifting network into a sharply peaked impulse, which may then be placed in series with the direct bias voltage. When the reader in the course of his work runs across the diagram of a welding control, for instance, and finds there some peculiar resistance, capacitance, and inductance values, he may be quite sure that they were arrived at by experiment rather than by calculation and that their purpose is to give a sharply peaked impulse.

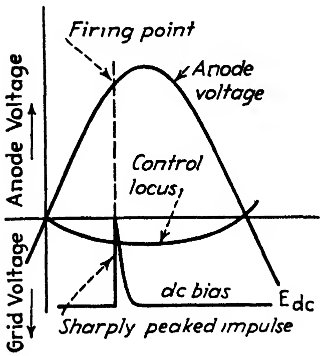

FIG. 22-14.—To avoid the influence of the *magnitude* of the anode voltage or grid voltage on the firing point, firing is usually accomplished by means of sharply peaked impulse.

22-15. Control Circuits for Ignitrons.—If the control consists of ignitron tubes instead of thyratrons, the problem is somewhat different because an ignitron has, of course, no such thing as a control locus. In order to fire an ignitron, it is necessary to pass a current through the igniter rod to the mercury-pool cathode. In the more refined ignitron controls the current through the igniter rod is in turn controlled by a thyratron tube so that, fundamentally, the control circuit has to perform in exactly the same manner as it would if the main power tubes were also thyratrons. In other words, at the desired instant in the cycle the auxiliary thyratron is made to fire, and the current flowing through it will pass through the igniter rod of the ignitron and thus cause the current to flow in the main circuit.

With a pair of ignitrons, which act so much like a contactor that a panel of two such tubes is often referred to as an "ignitron contactor," a very simple control may be obtained if it is not required to control the firing of the tube at a particular instant of the cycle. Figure 22-15*a* shows the circuit drawn in the usual way, whereas Fig. 22-15*b* shows it in a way that may make it easier to understand its performance. The current necessary for the igniter is obtained from the main supply system. The action of the circuit is as follows. With control switch S open, no current can flow in either one of the igniters and the circuit will therefore be open. If S is

closed at an instant when terminal A of the line is positive, the direction of current flow will be through copper oxide rectifier D_1, switch S, rectifier D_3, igniter of ignitron I, and through the load to terminal B of the line. Tube I will therefore fire, and the current will be taken over by its anode. If terminal B of the line is positive at the instant of switch closure, the direction of current flow will be through the load, the rectifier D_2, switch S, rectifier D_4, and through the igniter of ignitron II to terminal A of the line which will cause tube II to fire. It is evident that load current will keep on flowing as long as S remains closed. But the instant it is opened the load current will complete only the half cycle flowing at this instant. Theoretically, the rectifiers D_1 to D_4 are not required, but it

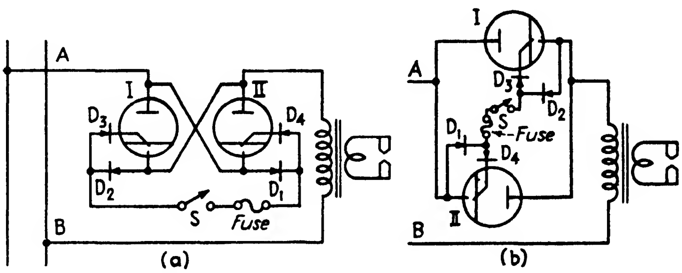

FIG. 22-15.—Two ignitrons connected in a back-to-back connection form an ignitron contactor, the control of which may be obtained with a simple switch or relay.

has been found to prolong the life of the igniter rod if current flow through it is reduced to a minimum and especially if it is not subjected to flow of current from the mercury pool to the igniter rod. The circuit shown in Fig. 22-15b shows the two rectifiers D_1 and D_2 polarized in such a way as to provide a shunt around the igniter path when the current flows in the undesirable direction. The control circuit should be carefully fused because if for any reason an ignitron should fail to fire, it is seen that the full-load current would attempt to flow through the control circuit leading to the destruction of the igniter electrode as well as of the control switch S. It is, of course, true that even under normal operation the load current *starts* to flow through the control circuit, but it will transfer to the main path the instant it has reached the few amperes required for ignition of the tube. Switch S may be a small control relay. It will be noted that it never has to open a circuit with either very much voltage or very much current. While either one of the ignitrons is conducting, S is seen to be connected only across a voltage equal to the arc drop of the ignitron. This amounts ordinarily to approximately only 15 volts so that the opening of the contact occurs without any sparking whatsoever, especially since there is no inductive load, like a coil, in the circuit. Actual interruption of the main current then takes place on its own accord when it passes through zero.

22-16. Resistance-welding Controls Using Ignitrons; "Heat" Control.—Although the circuit shown in Fig. 22-15 is desirable on account of its extreme simplicity, the speed with which it can be operated depends on the speed of the mechanical switch or relay *S*. A well-designed and -constructed mechanical relay is capable of a surprising speed; satisfactory operation of this circuit has been obtained in the case of a seam-welding machine at a rate of 200 to 300 operations per minute, with the life of the relay running into many million operations. The relay itself was operated by means of a multivibrator circuit described in Chap. XX. It will be appreciated, however, that this type of control can give no other results than a fast-operating mechanical contactor would give. It is true that the absence of moving parts and of contact wear makes it a more desirable arrangement than a mechanical contactor, but, electrically, the results will not differ from those obtained with a mechanical arrangement. Furthermore, many welding operations, especially on thin aluminum, may require very short periods of welding current; thus, it is not uncommon to have the welding current flowing, for example, during two cycles and then have it interrupted for three cycles. This is equivalent to 12 operations per second, or 720 operations per minute, which can hardly be achieved with a mechanical control relay. In this case the ignition must, in turn, be accomplished by means of a tube control. Not only are modern tube controls for welding machines expected to provide duty cycles as just outlined, but during that part of the duty cycle in which the tubes are conducting, the firing point can be delayed within each half cycle, thus giving what is termed "heat control." The fundamental arrangement employing thyratron tubes for the firing of the ignitron tubes is shown in Fig. 22-16. The thyratrons are simply placed in parallel with the ignitrons so that their firing will provide current through the igniters and thus fire the ignitrons themselves. The control voltage will then have to be applied between the cathode and the grid of the two thyratron tubes in a way exactly similar to that shown in Figs. 22-12 and 22-13.

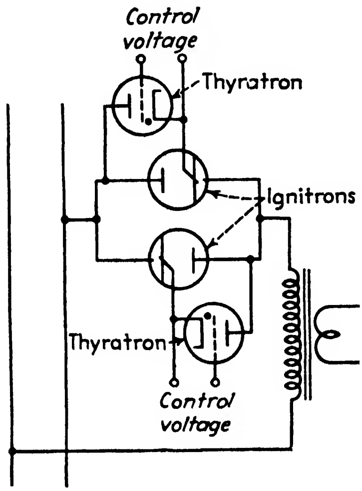

FIG. 22-16.—If high-speed control is demanded, the firing of the ignitrons is accomplished with thyratrons. The problem is then simply to fire the two hot-cathode gaseous tubes at the desired instant of the cycle.

To discuss the multiplicity of circuits used in present-day welding-machine controls would fill a separate book and, since such a book has been written (G. M. Chute, "Electronic Control of Resistance Welding"),

the interested reader will find there a complete discussion of the various types of control.

22-17. Influence of the Type of Load on the Performance of Gaseous Tubes; Capacitive Load; Dc Motor as Load.[7-9]—In the discussion of the performance of gaseous tubes not much has been said about the character of the load. Thus, in the analysis of the circuit shown in Fig. 22-11, the load was considered as purely resistive. This means that at the instant of firing the current will jump immediately to a value given by the transformer voltage at that instant, divided by the resistance, and it will continue to flow until the supply voltage becomes zero. When discussing ordinary rectifier circuits, we saw that altogether different results were obtained, depending on whether the load was resistive, whether it was bypassed by a capacitor, or whether an inductance was in series with it. In a similar way, the analysis of grid-controlled gaseous-tube circuits becomes considerably more complicated when the load circuit contains capacitance or inductance. An exact analysis of these circuits is beyond the scope of this book, but a few general remarks may help the reader if he wishes to study this subject. Suppose that the dc load shown in Fig. 22-11 is a capacitor. Obviously it will charge to the peak value of the transformer voltage if either one of the tubes is permitted to fire during the first half of the positive anode-voltage cycle. After having been charged once to this value, no conduction through either tube will take place because the anodes would no longer become positive with respect to the cathode. The diagrams shown in Figs. 22-3 and 22-5 are valid only when the anode-to-cathode voltage is equal to the supply voltage before firing takes place. This is evidently the case only when the load is purely resistive. With a capacitive load, on the other hand, the anode does not become positive with respect to the cathode until the transformer voltage has reached a value equal to the voltage existing across the capacitor. This is a condition exactly similar to the one existing in the case of a capacitance-input-type rectifier, and the reader should refer to the remarks made on that subject.

In the preceding paragraphs we have discussed the expected performance of a capacitive type of load. What is the characteristic of such a load? A capacitor is a device across which a voltage will build up when current flows through it and which will retain this voltage even after the current flow ceases. Now consider the armature of a dc motor. It represents a short circuit as long as it is standing still, just as a discharged capacitor represents a short circuit. Letting a direct current flow through it will cause it to accelerate, and a voltage will appear across its terminals, just as across a capacitor. When the current flow is interrupted, it will keep on rotating if it does not have to drive a load or if the friction is negligible; the voltage will therefore remain across its terminals, just as in the case of a capacitor. Connecting a resistance across the armature will make it act like a generator furnishing electrical energy at the expense of

its momentum, and the voltage will therefore decrease. Note again the exact equivalence of this performance with that of a charged capacitor. This equivalence goes so far that formulas have been developed permitting the calculation of the capacitance equivalent to that exhibited by the armature of a dc motor. Actual values depend on the moment of inertia of the armature and its speed because these two factors evidently determine the amount of energy that can be stored in the armature, typical values being in the order of several farads. Therefore, if the load shown in Fig. 22–11 is a dc motor, the performance of the circuit will be very nearly identical with that of a resistive load shunted by a very large capacitor. High peak currents can therefore be expected, although the small amount of inductance naturally associated with the dc armature will prevent the current from reaching values equal to those encountered in the case of a pure capacitance.

22–18. Gaseous Tubes with an Inductive Load, Dc Connection.—In Secs. 7–7 and 8–14, the performance of a half-wave and a full-wave rectifier, respectively, with an inductive load was discussed. Suppose that the dc load shown in Fig. 22–11 was highly inductive. It will then take several cycles before a final steady state is reached, but after it has once been reached, the fluctuations from cycle to cycle will be very small. In order to gain an insight into the performance of such a circuit, it is permissible to consider the load as consisting of a resistance and an inductance of very large value—in the extreme, an infinite inductance—in series with the resistance. After the current has reached its steady state, it can then be considered as a pure direct current. The current through the two gaseous tubes must then necessarily consist of a series of alternate square waves. It will be remembered that this was also the case for a full-wave rectifier circuit. The inductance will furnish whatever voltage is necessary to keep the current at a steady value in spite of the fact that the supply voltage, *i.e.*, the transformer voltage, varies during a cycle.

In the case of a full-wave rectifier working into a highly inductive load, we have seen that the current switchover from one tube to the other occurred when the supply voltage went through zero, *i.e.*, when the anode of the previously nonconducting tube became more positive than the anode of the tube still conducting. But when an ordinary rectifier tube is replaced by a grid-controlled rectifier tube, then the grid voltage will determine the instant when the second tube becomes conducting. In the case of a full-wave rectifier circuit using uncontrolled rectifiers, the common cathode of the two rectifying tubes will never become negative with respect to the center tap of the supply transformer (assuming the two rectifying tubes as perfect). With a pair of grid-controlled rectifiers, on the other hand, the common cathode of the tubes may, through the action of the inductance, become negative with respect to the center tap of the transformer because the nonconducting tube with its anode positive with respect to

its cathode may be prevented from taking over the current since its grid is still negative enough to prevent firing. In Fig. 22-17 are shown the potentials of points M and N of the circuit shown in Fig. 22-11; the center tap of the transformer is taken as reference point O. It is assumed that the steady state has been reached, *i.e.*, that an almost pure direct current is flowing through the load, owing to the action of the inductance. Let us start our consideration shortly after the time t_1. M is positive and tube I is conducting. If for the purpose of this analysis the two tubes

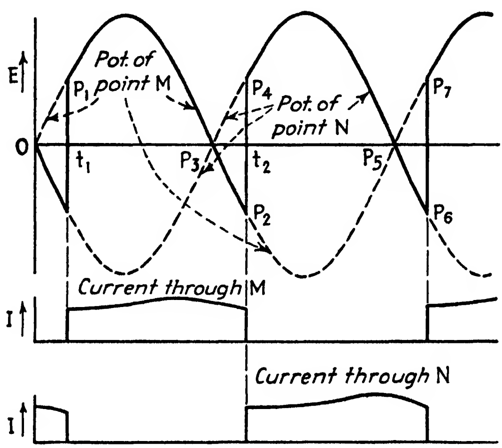

FIG. 22-17.—If the load shown in Fig. 22-11 is highly inductive, an essentially constant current will flow through it. This figure shows the tube currents as well as the potentials of the anodes and cathodes.

are considered as perfect rectifying devices, there will be no voltage between the anode and cathode of the conducting tube and, consequently, point C is now at the same potential as M. Points M and C rise in potential and then fall together to the zero level. With a highly inductive load this does not mean that the current also falls to zero since the inductance, by allowing a small current change, furnishes a voltage to keep the current going. The instant M and with it C have reached the zero level, point N, which had been negative with respect to the reference point up to now, has also reached the zero level. This is point P_3 of Fig. 22-17. An instant later M will be negative, and N will be positive. If the two tubes were simply ordinary rectifiers, N would immediately take over the current. But if the two tubes are grid-controlled rectifiers and tube II is prevented from firing at this instant, M and with it C will continue to swing negative, the inductance furnishing the exact amount of negative voltage to keep the cathodes C just "under" M. Suppose that at the instant t_2 the grid voltage on tube II is such as to permit this tube to fire; the cathodes of the two tubes are then negative as indicated by point P_2 in Fig. 22-17, and

the anode-to-cathode voltage of tube II is therefore given by the vertical distance P_2P_4. If tube II is permitted to fire at this instant, the cathodes of the two tubes will suddenly jump from P_2 to P_4. The potential of the cathodes will then follow the line P_4, P_5, P_6 at which instant tube I will take over again, and potential of the cathodes will suddenly jump from P_6 to P_7.

As a glance at Fig. 22–11 shows, the voltage of the two cathodes with respect to the reference point O is at the same time the voltage existing across the load. A dc voltmeter connected across the load will therefore show the average of the heavy solid line in Fig. 22–17. It will be noted that this voltage is negative between P_3 and P_2. If the firing point is delayed 90 deg, the negative area will become equal in size to the positive area, and the average or direct current will become zero.

This analysis indicates that in the case of a highly inductive load a delay in the firing point will reduce the direct current flowing in the load approximately twice as much as if no inductance is in series with the resistance of the load. Although this discussion is rather sketchy and is not meant to treat the subject completely, it should be sufficient to give the reader fair warning to consult more exhaustive treatises on it when he is faced with a problem of this kind.

22–19. Gaseous Tubes with Inductive Load; Back-to-back connection in Ac Circuits.[10–12]—When two gaseous tubes are used in a back-to-back connection for the control of an ac load supplied from an ac system, as shown in Figs. 22–13, 22–15, and 22–16 and if this load is not purely resistive, some rather interesting phenomena take place. In contrast to the dc circuit shown in Fig. 22–11 where the presence of a large inductance kept the current flowing at all times, the current will consist of a series of transients in the case of the ac load. In order to understand what is taking place, consider the circuit shown in Fig. 22–16 or 22–15 and assume for a moment that the two tubes are perfect rectifiers and are not grid-controlled. Under such conditions the current can always pass freely in whatever direction it wishes to flow; therefore, current through the load will be exactly the same as if the two tubes were absent. If the load is inductive, as is the case in most practical applications involving welding machines, the current will lag the voltage by an angle sometimes referred to as the "power-factor" angle. This condition is indicated in Fig. 22–18, where the current i passes through zero at the point P_1, at which time the supply voltage is already considerably positive. The reader should be clear about the very important fact that at this time the current ceases to flow through one tube and begins to flow through the other; in other words, transfer from one tube to the other will *not* occur when the voltage passes through zero but when the *current* passes through zero, which is seen to depend entirely on the power factor of the load. Suppose now that the two tubes are grid-controlled rectifiers but that the grid voltage is such as

to permit them to fire at the instant P_1, for instance, by supplying a sharp peak positive impulse as shown in Fig. 22–14. Clearly the conditions would then not differ in any way from the case where the two tubes were perfect rectifiers. We immediately learn a very important fact. If two ignitrons or thyratrons are used for the control of a welding machine, for instance, and if it is desired to have the maximum amount of current flowing, sometimes also called "full heat," the firing impulse must be delivered, not at the instant when the voltage passes through zero but *at an instant dictated by the power factor of the circuit.* Suppose that the firing impulse had been delivered every half cycle at exactly the right moment, *i.e.*, at point P_1, so that the full current is flowing in the circuit. Now let it be assumed that the firing point is delayed for the next cycle. Consequently, when the current has fallen to zero through one of the tubes, the other tube does not take over at P_1, and a voltage given by the length P_1Q_1 appears across the tube since with no current through the load the voltage across any kind of switch will be equal to the supply voltage. If the firing is delayed until the time given by point P_2, the current will build up and reach zero again in a loop located completely within the half wave resulting when no control is exercised. This diminished loop is not just a reduced part of a sine wave; neither is it symmetrical with respect to the two zero points. It is exactly equal to the transient that would occur when this circuit—without any rectifiers—is closed at P_2. The theory of transients tells us that it consists of the steady-state current that would flow in the circuit at that instant—in other words, the "full-heat" current—and superimposed on it a transient current falling to zero along an exponential curve. This exponential transient is of such magnitude and polarity as to result in actual zero current at the instant of closure when superimposed on the steady-state current, which would be flowing at that instant. A further discussion of this subject, although of much interest, is beyond the scope of this book. A similar negative loop of current will be passed by the second tube during the half cycle of opposite polarity. It is of interest to note that when a back-to-back-connected pair of gaseous tubes is used to control the flow of alternating current in an inductive load, the current will consist of a number of disconnected loops. As explained above, these loops are not of sinusoidal wave shape but are a combination

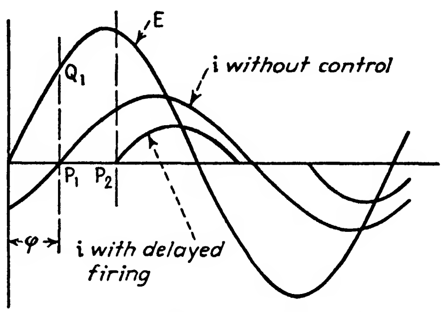

FIG. 22–18.—With two gaseous tubes in a back-to-back connection and no grid control exercised, the current will transfer from one tube to the other, *not* at the instant when the voltage passes through zero, but when the *current passes through zero.* Delay of the firing point has results as shown above.

of a sine wave and an exponential curve. It is evident that the exact determination of the power delivered by the source to the load becomes very difficult. It is hoped that, with the few remarks presented here, the reader will have at least some start for a better understanding of the literature dealing in greater detail with this subject.

22–20. Basic Method to Extinguish a Gas Tube in a Dc Circuit.—Earlier in this chapter it was stated that gaseous tubes can be used in connection with dc circuits only if the *making* of the circuit is of importance, but if it is not necessary to interrupt the current again. We cannot interrupt the current in a gaseous tube by grid control but have either to open the anode

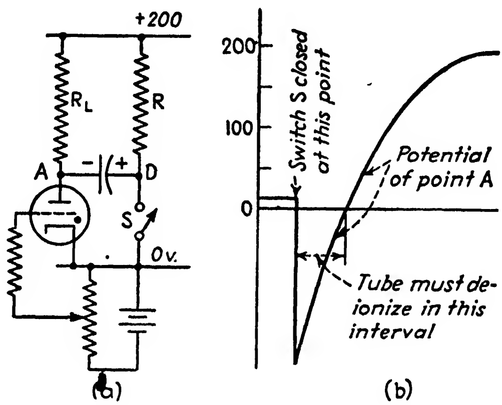

FIG. 22–19.—When a gaseous tube is conducting in a dc circuit, the direct current may be interrupted by making the anode of the gaseous tube negative with respect to the cathode for a period in excess of the deionization time.

circuit for an instant or to make the anode negative with respect to the cathode. This action naturally leads to cessation of current flow through the tube. This latter method has opened a new field of application for gaseous tubes and may become of great importance in the future. Consider the circuit shown in Fig. 22–19*a*. Suppose that the tube is permitted to fire by reducing the negative grid voltage below the critical value. A current will then flow through the load R_L across which a voltage equal to the supply voltage minus the tube drop will appear. If the tube drop is, for instance, 15 volts, a voltage of 185 volts will appear across the load. Parallel to the load is connected a resistance R in series with a capacitor. It is evident that this capacitor will charge with a polarity as shown to a voltage equal to the voltage appearing across the load. In this particular example, the voltage appearing across the capacitor will therefore be 185 volts, with the negative terminal of the capacitor at point *A* and the positive terminal at *D*. Let us assume that the potential divider determining the control voltage on the grid has again been moved to a more negative value, which, of course, will not interrupt the anode

current but will prevent the tube from firing again if the anode current could be interrupted for a period in excess of the deionization time. Now, consider what happens when switch S is closed. This will connect the positive terminal of the capacitor to the cathode and, since the capacitor is charged to 185 volts, A will momentarily become 185 volts negative with respect to the cathode. This will interrupt the current through the gaseous tube, but A cannot, of course, stay negative with respect to the cathode, since the capacitor will at first discharge and then recharge in the opposite direction until A has reached the 200-volt level. A will therefore sweep up from a level of -185 to a level of $+200$ along an exponential curve, as repeatedly discussed in connection with capacitor charging. The excursion of the potential of A after closure of S is shown in Fig. 22–19*b*. The speed with which A sweeps up from its most negative point is determined by the time constant of the capacitor and the load resistance R_L. If the tube has deionized at the time when A becomes positive with respect to the cathode, ignition will no longer take place, and interruption of the current through the gaseous tube has been accomplished. It should be noted that interruption has been accomplished not by *opening* a switch, but by *closing* one.

22–21. Use of a Second Gas Tube to Extinguish Current through the First.—Although the circuit shown in Fig. 22–19 may be of use in some applications, its dependence on the operation of a mechanical switch is evidently a serious handicap and prevents its application to high-speed problems. Since a gaseous tube is a high-speed switch, the suggestion to replace the switch S in Fig. 22–19*a* by another gaseous tube is quite obvious. This leads to the circuit shown in Fig. 22–20. The circuit is shown as perfectly symmetrical but it should, of course, be realized that only one of the loads R_L is the actual one, while the other is merely a dummy load, serving the same function as the resistance R in the circuit shown in Fig. 22–19*a*. Assume that the grids of both tubes were at first negative enough to prevent the firing of the tubes; now, let tube I, for instance, fire by reduction of the negative grid voltage, but let it be assumed that the grid voltage is immediately readjusted again to the negative value. The capacitor C will now charge to 185 volts with point A negative and B positive. If we now permit tube II to fire, B will come

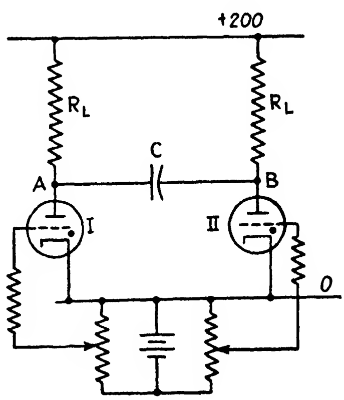

FIG. 22–20.—When one of the two tubes is conducting in the above circuit, it may be extinguished by making the other tube conducting, because such an action will make the anode of the previously conducting tube negative with respect to its cathode for a short time.

down to a level of 15 volts, which in turn will make A negative with respect to the cathode by 170 volts. This will, of course, extinguish tube I, and if the time during which its anode is negative with respect to the cathode exceeds the deionization time, it will not reignite and A will therefore sweep up to a level of 200 volts. The capacitor will now be charged again to 185 volts, but with a polarity opposite to that of the previous state, *i.e.*, A will be positive and B negative. If now the grid of tube II has been made negative again and tube I is permitted to fire, the same chain of events takes place, with the two tubes interchanging their roles. Through the action of the capacitor C, the firing of one tube extinguishes or "blows out" the other tube. The capacitor C is also referred to as the "commutating" capacitor. It should be noted that if the firing of one tube does not succeed in extinguishing the other one for any reason, so that both tubes are conducting at the same time, control over the circuit will have been lost because no manipulation of the grid voltages will permit the interruption of the current.

22-22. Principle of Gaseous-tube Frequency Meters.—If in the circuit shown in Fig. 22-20 the control of the two grids is accomplished by means of a center-tapped transformer, as shown in Fig. 22-21, the two gaseous tubes will fire alternately with every half cycle of the applied alternating voltage. This circuit is the basis of some very practical frequency-measuring devices. With every cycle of the control voltage a switchover from one tube to the other and back takes place, but the currents established in the two tube circuits are evidently independent of the amplitude of the applied alternating voltage, as long as it is sufficient to cause the tubes to fire at all. Thus, the commutating capacitor gets a reversal of charge with every half cycle of the applied alternating voltage. If the circuit is designed in such a way that the time constant of this capacitor in conjunction with the two resistors R_L is small compared to one half cycle of the applied alternating voltage, then every cycle of the latter will cause a definite amount of charge to pass back and forth through the commutating capacitor. If a full-wave rectifier-type instrument is included in series with the commutating capacitor, its indication will therefore be proportional to the frequency applied to the grid of the two gaseous tubes. This is, of course, not the only way in which the circuit can be used as a frequency meter, but it indicates in general how such a result can be achieved.

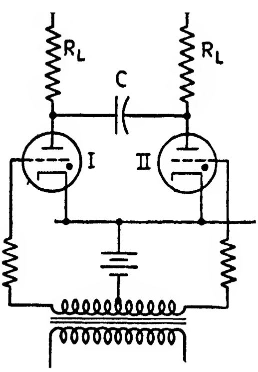

FIG. 22-21.—In the circuit shown here, tubes I and II will carry current alternately during half cycles of the applied alternating control voltage.

22-23. Modification of Circuit by Placing Resistors in Cathode Leads.—Figure 22-22 shows a circuit where the load resistors are placed in the

cathode leads instead of the plate leads. The performance of this circuit is essentially identical with that of Fig. 22–21. Assume, for instance, that tube I is conducting, while tube II is nonconducting. The commutating capacitor will then be charged with a polarity making point *A* positive and *B* negative. The voltage across it will again be equal to the supply voltage minus the arc drop occurring across tube I. When tube II is permitted to fire, *B* will suddenly jump from zero level to within a few volts (the arc drop) of the positive end of the voltage supply. Since the commutating capacitor cannot change its voltage instantaneously, the potential of *A* will jump above that of the anode, which will stop the current flow through tube I. *A* will, of course, come down to the zero potential along an exponential curve but, if it remains positive with respect to the anode of tube I long enough to permit deionization of this tube, the tube will not fire again. The reader will note that the swing of the potential shown in Fig. 22–19*b* will also apply to the circuit shown in Fig. 22–22 by simply turning Fig. 22–19*b* upside down.

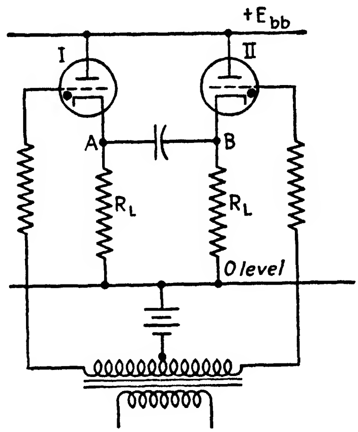

FIG. 22–22.—Figure 22–21 may be modified in the manner shown here by placing the load resistors in the cathode leads.

22–24. High-speed Counting Circuits.[13, 14]—A variation of the circuit shown in Fig. 22–20 is also used for the purpose of high-speed counting. Electrically operated counters available on the market usually have an upper limit of approximately 600 operations per minute, or approximately 10 counts per second. In the mass production of many small articles the production rate is considerably in excess of this speed, and in certain scientific measurements phenomena occurring at much higher speeds have to be counted. Evidently if it is possible to count only every fourth, or eighth, or tenth object, a mechanical counter could be used for counting at a speed equal to four times, or eight times, or even ten times its maximum counting speed. Suppose that in Fig. 22–23 two gaseous tubes are biased negatively by means of the fixed voltage E_c to prevent them from firing. Let it be further assumed that one tube has been made conducting by a temporary reduction of the negative grid voltage. The grid of the conducting tube, say, tube I, is submerged in the plasma, and current will flow through its grid resistor R_g. Now, let a voltage be applied suddenly between points *O* and *F* with such a polarity as to make *F* positive with respect to *O*. It is of no importance how long this voltage lasts as long as it is applied suddenly or, expressed in different words, with a steep front. Since the two capacitors *C* cannot change their charge suddenly,

this positive impulse is trying to make the grid of both tubes more positive. But tube I is conducting already; therefore the positive grid impulse is of no consequence. Tube II, on the other hand, will now fire. Owing to the action of the commutating capacitor, as explained in conjunction with the circuit described in the preceding paragraph, the anode of tube I becomes negative with respect to its cathode and therefore extinguishes. If the time constants of the capacitor C and grid resistors R_g are sufficiently small, the grid of tube I will be at the potential of the bias battery when its anode becomes positive with respect to the cathode, regardless of whether the control voltage applied to point F is still positive or has become zero again. The next positive impulse applied to point F will now fire tube I and extinguish tube II. Consequently, if the load in one of the plate circuits, say, tube II, is a counter, this counter will operate only with every other impulse of the control voltage. The designer of such circuits may sometimes experience difficulties by overlooking the fact that an electromagnetic counter, besides having a dc coil resistance, also represents a fairly high inductance. It was pointed out in Sec. 21-18 that an inductive load in the anode circuit prevents the rise of current and that a tripping impulse, of a duration sufficient to permit ionization to take place in the case of a resistive load, may fail to "fire" the tube when appreciable inductance is associated with the resistance of the load. The reader is referred to the section just mentioned for a remedy for such a situation.

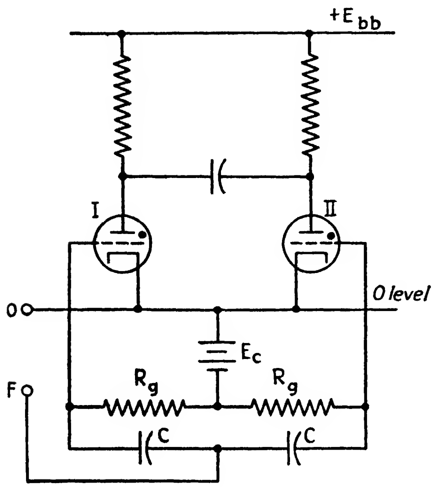

FIG. 22-23.—The application of a sudden positive impulse to point F will cause the firing of the nonconducting tube and the extinction of the conducting tube. With every impulse, a transfer of current from one tube to the other therefore takes place.

Instead of placing the counter in one of the plate circuits, we may place a resistor in both and apply the voltage developed across one of them (through capacitive coupling) to the input of another circuit built exactly like that shown in Fig. 22-23. In this circuit there will then again be two tubes, and it is seen that one of them will now operate only on every fourth impulse delivered to the input of the first circuit. Every additional stage of this kind will reduce the counting rate once more by two, so that with four pairs of tubes, for instance, it is possible to scale the counting rate down in the ratio 16:1.

22-25. Inverters.[15-18]—In the circuits shown in Figs. 22-20 to 22-23 the loads in the two plate circuits consisted of resistors, which were entirely

independent of each other. Figure 22-24 shows a circuit that promises to become very important. In it the separate loads in the two plate circuits are replaced by a center-tapped transformer, the center tap of which is connected through an inductance to the positive terminal of the direct supply voltage. If we apply to the two grids of the gaseous tubes an alternating voltage obtained from a center-tapped transformer, the tubes will fire alternately on every half cycle of the applied voltage. This action could be likened to two switches alternately connecting the ends of a center-tapped transformer to the negative end of the direct voltage, with a frequency dictated by the frequency of the alternating voltage applied to the two grids. The commutating capacitor is again required for the purpose of extinguishing one tube when the other fires. In contrast to the circuits discussed previously where the current in the plate circuit of the tube was limited by a resistor and where no harm would come to the tube if by any chance both of them should be conducting simultaneously, such a failure would be disastrous in the case of the circuit shown in Fig. 22-24. Since the ohmic resistance of the winding of the transformer and of the choke is not sufficient to limit the current, destruction of the tube will follow if the circuit is not interrupted by a circuit breaker or fuse. The circuit is obviously capable of converting direct current into alternating current, provided a small source of the alternating current with the desired frequency is available. Since devices changing alternating current into direct current are known as "converters," devices that do the opposite have been given the name "inverters." The circuit shown in Fig. 22-24 is known as "separately excited parallel inverter." The correct size of the commutating capacitor is a very involved problem, depending on the type of load connected to the secondary of the transformer. If the load is highly inductive, this capacitor must be larger than in the case of a resistive load, since the reactive volt amperes required by the load can, of course, not come from the dc system, which is incapable of furnishing reactive power.

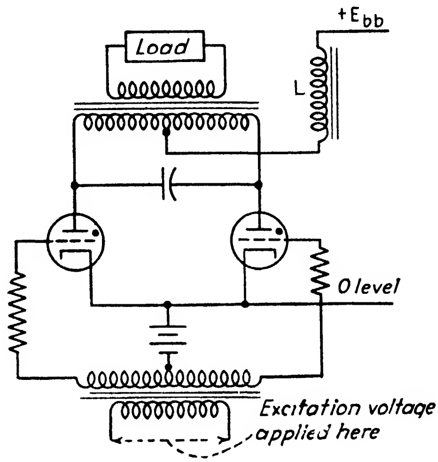

FIG. 22-24.—A parallel inverter. The two resistors of the preceding figures are replaced by a center-tapped transformer, with the result that a pure alternating current flows in the load connected to the secondary of this transformer.

22-26. Self-excitation of Inverters.—The inverter shown in Fig. 22-24 requires an alternating voltage of the desired frequency for its excitation.

The energy to be furnished is, of course, extremely small since it has to operate only the grids. It is, however, possible to make the inverter self-excited. When the load connected to the secondary of the transformer (or across the primary of the transformer) has a definite frequency of its own, such as would be the case if the load consisted of a resonant circuit, then the primary of the excitation transformer shown in Fig. 22–24 can be connected across the load itself instead of across a separate source of alternating voltage as in Fig. 22–24. A very careful study of the phase relationships is required, however, in the case of such a circuit. Furthermore, to start oscillations in such a circuit presents a considerably more difficult problem than that of an oscillator employing vacuum tubes.

22–27. Series Inverter.—The parallel inverter derives its name from the fact that the two gaseous tubes are connected in parallel. Another circuit permitting the conversion of direct current into alternating current is shown in Fig. 22–25. In it the two gaseous tubes are seen to be connected in series instead of parallel. Suppose that tube I is conducting while tube II is nonconducting. Under this condition, point A will be essentially at zero potential (neglecting the tube drop). Now let tube II fire. Since the capacitor cannot change its charge suddenly nor can the current change suddenly owing to the presence of the inductance, A will remain at the zero level for an instant. Since point B is, however, effectively connected to $+E_{bb}$, the transformer action of the center-tapped inductance L will make point D negative and therefore cause interruption of the current through tube I. The potential of A will therefore go up to essentially $+E_{bb}$. Alternate firing of the two tubes therefore effectively connects A alternately to the negative and to the positive end of the direct supply voltage. This will cause an alternating current to flow through the capacitor and the load. It will be noted that, if ever the two tubes are made conducting at the same time, a short circuit will result over the inductance L. Another disadvantage of this circuit is found in the fact that the two cathodes are not at the same potential so that the necessary voltages for the control of the two grids will have to be isolated from each other. Figure 22–25 shows two separate bias voltages. If the circuit is to be operated by an alternating voltage, two separate windings will have to be used in series with the direct bias voltages.

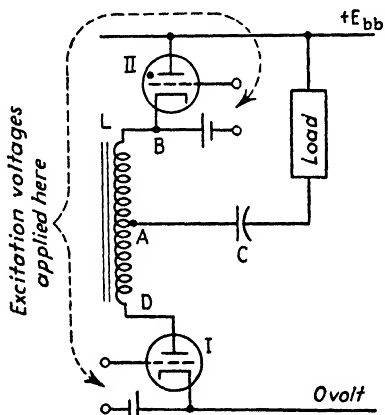

FIG. 22-25.—The connection of a series inverter.

22-28. Comparison of Vacuum-tube Oscillators and Gaseous-tube Inverters.—The output available from a gaseous-tube inverter is far greater than that from a vacuum-tube oscillator for the same initial investment. This does not mean, however, that the gaseous-tube inverter can replace the vacuum-tube oscillator in all cases. The inverter has a frequency limit determined by the deionization time of the gas tube. At present, this limit seems to be 2,000 cps and, although it can be expected that it may be raised in the future, it will hardly reach frequencies in the megacycle range, such as are obtained easily with vacuum-tube oscillators. A frequency of 1,000 cps or so is, however, particularly suitable when it is desired to melt material without contamination, or when pieces have to be heated all the way through for forging or annealing. On the other hand, when the heating is to be confined to the outer layers (as in surface hardening), or when the pieces to be heated are of small diameter, much higher frequencies must be used. The two types of equipment are therefore not competitive, and both of them can be expected to find wide use in industry.

PROBLEMS

22-1. The control characteristics of an argon-filled triode are shown in Fig. 21-5. The tube is used in a circuit as shown in Fig. 22-26. The voltage supplied by the secondary of the transformer is of sinusoidal wave shape and has an amplitude of 100 volts.

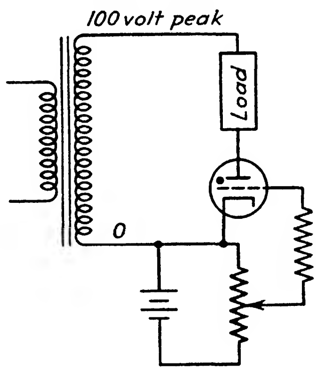

Fig. 22-26.—Circuit diagram for Prob. 22-1.

Plot a wave showing the voltage that would have to be applied to the grid at any instant to prevent the tube from *just* firing, *i.e.*, plot the control locus.

22-2. The control characteristics of an FG 17 at 40°C condensed-mercury temperature are shown in the accompanying table.

Volts e_c	0	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10
Volts e_b	50	60	90	280	550	960	1,400	1,840	2,280	2,720	3,160

Two tubes are connected in a full-wave rectifier circuit to a resistive load of 1,000 ohms, as shown in Fig. 22-27.

a. To what value would e_c have to be adjusted to prevent any current flow at all?
b. With e_c adjusted to -7 volts, what direct current will flow in the load? (Neglect tube drop.)
c. What wattage will be developed in the load?

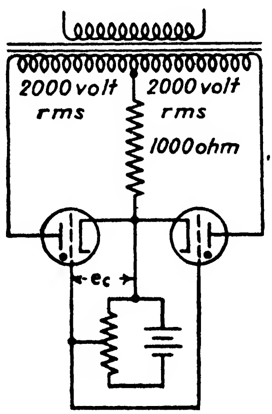

FIG. 22-27.—Full-wave rectifier circuit of Prob. 22-2.

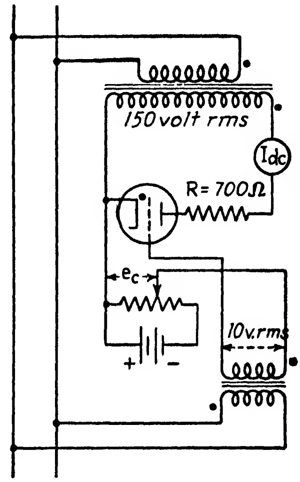

FIG. 22-28.—Half-wave rectifier circuit of Prob. 22-3.

22-3. The control characteristic of the 884 and 885 gas triodes is a straight line passing through the two points given by the following data:

Grid Voltage	Anode Voltage
−5	41
−30	299

Other characteristic values are (for frequencies below 75 cps)

Tube drop	16 volts
Peak voltage (between any two electrodes)	350 volts max
Peak anode current*	300 ma
Average anode current (averaged over a period not more than 30 sec)	75 ma

The resistance of the grid resistor should not be less than 1,000 ohms per maximum instantaneous volt applied to the grid; values in excess of 500,000 ohms may cause circuit instability.

A tube of this type is connected in a circuit as shown in Fig. 22-28, with $e_c = -30$ volts.

What direct current will flow in the load under the conditions indicated?

22-4. In the circuit shown in Fig. 22-29, T_1 is a step-up transformer with a ratio of 1:1.5 from primary to secondary. T_2 has a center-tapped secondary, and the ratio of the primary to the *whole* secondary is 2:1. The value of the control inductance can be changed by changing the position of the movable plunger within the coil.

a. In order to obtain control with this circuit, how must A and B be connected to X and W? (In other words, must A be connected to X, and B to W, or vice versa?) What current will flow if the inductance happens to have a value of 5 henrys? What current will flow if the value of L changes to 2 henrys?

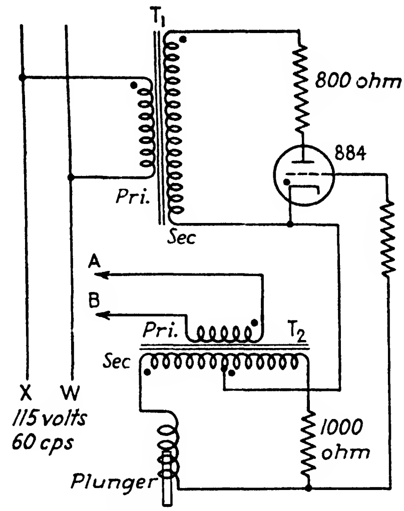

FIG. 22-29.—Phase-shift control circuit of Prob. 22-4.

22-5. The characteristic of an FG 57 thyratron at 40°C condensed-mercury temperature is as follows:

Volts e_c	0	−1	−2	−3	−4	−5	−6	−7	−8
Volts e_b	60	70	130	275	425	570	715	860	1,000

Two tubes are connected in a circuit as shown in Fig. 22-30. Transformer T_1 has a ratio of 1:2 from primary to whole secondary. The load resistance is 150 ohms; the value of L is so high that the ripple of the load current may be neglected. The tube drop may also be neglected.

The control transformer T_2 has a ratio of 5:1 from primary to the whole secondary. Its impedance is high enough so that it does not constitute a load on the phase-shifting network consisting of C and the variable resistor R.

a. In order to obtain control by varying R, how must the terminals A and B of the transformer T_1 be connected to the line, *i.e.*, to conductors X and W?
b. What direct current will flow in the load, if R is adjusted to a value of 300 ohms?
c. To obtain an increase in current, must R be increased or decreased?

22-6. An FG 57 thyratron is connected to a direct voltage as shown in Fig. 22-31. The negative bias of −4.5 volts is sufficient to prevent the tube from firing (see characteristics given in Prob. 22-5). The tube has been made conducting, however, by the application of a positive pulse to the grid, delivered by the transformer T. It is desired to extinguish the tube by closing the switch S. The arc drop of the tube is given as 20

volts, and the deionization time as approximately 1,000 μsec. Assuming that the danger of reignition in a tube not yet deionized exists when the anode becomes positive with respect to the cathode by an amount equal to the arc drop, how large would the capacitor C have to be at least to achieve the desired result?

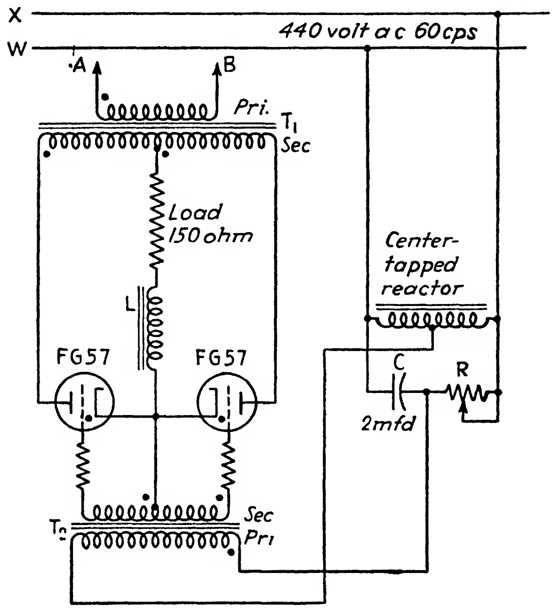

FIG. 22-30.—Full-wave rectifier circuit with phase control (see Prob. 22 5).

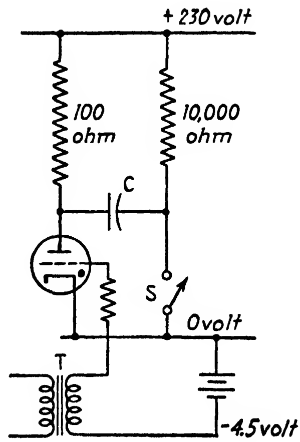

FIG. 22 31.—Extinction of thyratron in a dc circuit (see Prob. 22 6).

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CHAPTER XXIII

PHOTOELECTRIC TUBES

23-1. Photoemission and History of Photoelectric Cells.—In the electronic tubes discussed so far, emission of electrons was obtained by raising the temperature of the emitting body (thermionic emission) or by bombarding a metal with electrons (secondary emission). Section 6-5 gave a summary of all methods causing emission of electrons, and among those listed was photoemission. The present chapter deals with tubes in which emission is obtained by letting light strike a specially prepared surface. Such tubes are called "photoelectric tubes" or "phototubes" and have attained great importance to the electronic engineer.

Nothing has captured popular imagination more than the electronic applications involving the photoelectric tube, sometimes also referred to as the "electric eye." It is therefore surprising to learn that they are among the oldest members of the family of electron tubes. The photoelectric effect was discovered as early as 1887, and physicists of that time constructed photoelectric tubes almost identical with those available to us today. But the current obtainable from photoelectric cells is so small that very sensitive galvanometers are required for its measurement and observation and that direct relay operation from it is out of the question. The photoelectric cell could therefore not become of practical importance until it could team up with the amplifier tube, which was not invented until much later. Even then the photoelectric tube needed the stimulus of an industry, the requirements of which ran into many thousands of these cells before manufacturing problems were solved and the cell became a practical commercial article. This industry was the motion-picture industry, which, with the introduction of sound on film, required a reliable and reasonably priced photoelectric cell.

In 1887, Heinrich Hertz in his experiments with electric oscillations and waves observed that a spark gap, when illuminated by sunlight or by the light of another spark, would break down at a lower voltage than if it was in the dark. Further investigation disclosed that it was only necessary to illuminate the negative electrode of the gap. It was soon discovered that this phenomenon is due to the fact that light striking a metal surface liberates electrons from it. It takes ultraviolet light, however, to produce electrons from ordinary metals, which explains why Hertz observed the effect only in the case of sunlight or of light from another electric spark, which we know to be rich in ultraviolet light. Just as in the case of ther-

mionic emission, where the pure metals with a high work function require higher temperatures before they become efficient emitters of electrons, whereas the oxide-coated cathodes with their lower work function emit electrons at a much lower temperature, so in the case of photoelectric tubes an electrode coated with a low-work-function material will emit electrons when struck by light of higher wave length. This will be discussed later in more detail.

23–2. Types of Light-responsive Devices.—Although in this book we are mainly interested in electron tubes, a discussion of photoelectric cells operating on electronic principles would not be complete without a comparison of their performance with other light-sensitive devices. For this reason it seems desirable to discuss briefly the other devices capable of responding to light.

Light-sensitive cells may be divided into three classes, as follows:

1. Photoemissive cells.
2. Photoconductive cells.
3. Photovoltaic cells.

Each of these classes has its respective field. Here we shall pay most of our attention to the cells belonging in Class 1. Their discussion will therefore be deferred until a few remarks about Classes 2 and 3 have been made.

23–3. Photoconductive Cells.—The best known example of the photoconductive cell is the selenium cell. In 1872, it was discovered that selenium had a high resistance when dark but that its resistance dropped to one-tenth of this value when it became illuminated. It should be noted that this is a straightforward change in resistance and that no rectification is involved. In other words, the cell is equivalent to a metallic conductor that changes its resistance when illuminated. The manufacture of these cells is a rather difficult one, if consistent results are desired. In order to expose as much of the material as possible to the light, the selenium is usually deposited in a very thin layer between two wires wound parallel to each other on a glass plate or it is deposited between two grids, as shown in Fig. 23–1. It is seen that in this manner a short path of large cross section can be obtained. The General Electric selenium cell type FJ 31 is formed in vacuum to avoid instability. Its dark resistance is 6 megohms, which changes to approximately 0.75 megohm when exposed to light. These values should not be considered, however, as typical (except the ratio of dark to light resistance) because voltage and current values depend entirely on the construction of

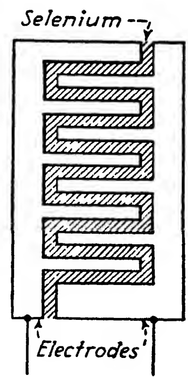

Fig. 23-1.—In a selenium cell, a conducting path of large cross section but short length is usually desired. This can be accomplished by depositing the selenium between two grid-shaped electrodes.

the cells. For instance, cells have been constructed capable of passing a current of ½ amp. The speed of response of a selenium cell is considerably slower than that of the other types of photocells. In view of the easy availability of the other types, the selenium cell has not found much practical application at the present time.

23-4. Photovoltaic Cells.—The photovoltaic cells differ from the cells in the other two classes by the fact that they develop a voltage when light is falling on them. In 1839, Becquerel discovered that two electrodes immersed in an electrolyte furnished a voltage when one of them was more illuminated than the other. Pure metals give a voltage of only a few microvolts. When the electrodes are covered with certain oxides, sulphides, or halides, many millivolts may be obtained from the cell when it is illuminated. This type of photocell is referred to as "wet cell." Several years ago a cell of this type was commercially offered under the name of "Rayfoto" cell. The cathode consisted of cuprous oxide, while the anode was of lead. The electrolyte was lead nitrate. This cell had a fairly high dark voltage because the electrodes were dissimilar metals. In another combination both electrodes were covered with cuprous oxide, and the dark voltage of this cell was naturally zero. It is of interest to note that this is the combination used by Becquerel in 1839. At present this type of cell is not being manufactured.

Of considerably more importance are the dry-type photovoltaic cells, also called "barrier-layer" photocells, which found widespread use in meters for the measurement of illumination and in exposure meters for photographic purposes. In 1924, Dr. Grondahl of the Union Switch and Signal Company discovered that a standard copper oxide rectifier disk produced a voltage when illuminated. Since that time additional compounds have been discovered furnishing the same effect, notably the iron selenide cell. Improvements in design and manufacture have made this type of cell quite stable and capable of furnishing 100 μa per lumen or even more. The phenomenon of the production of a current is assumed to be due to the emission of electrons from the compound into the metal itself. It is of interest to note that the current generated by the illumination passes through the border between the compound and the base metal in a direction opposite to that in which it would flow if the assembly is used as a rectifier. The two makes best known commercially are the Westinghouse cell, known under the name of "Photox" cell, and the "Photronic" cell of the Weston Electrical Instrument Company. The former is a cuprous oxide cell, while the latter is an iron selenide cell.

The problem of making contact with the two elements of the cell without interfering with the light reaching the active border between the compound and the metal has been solved in two different ways. In one design, called the "back-effect" cell, the cuprous oxide or iron selenide is deposited upon the metal in the form of a thin layer. Contact to the

compound is made either by a ring pressed against it near the outside edge, or by a thin film of metal deposited on the edge. The light passes through the compound itself and reaches the active border between compound and metal where it generates the electromotive force. It should be noted that in this case the light has to penetrate the layer of compound itself, and since the latter usually absorbs the shorter wave lengths of the light, the sensitivity of the cell is usually near the red end of the spectrum.

In the second type a semitransparent conducting layer is placed over the compound. In this case, two opposing voltages are generated: one at the front of the cell between the semitransparent layer and the compound,

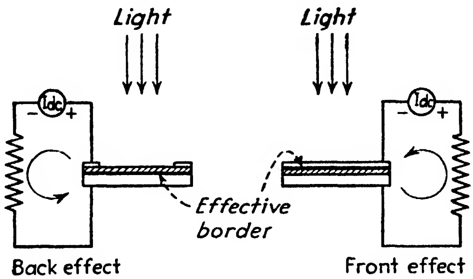

FIG. 23-2.—In the barrier-layer-type photocell, a voltage is generated at the border between the light-sensitive material and the metal on which it is deposited. Depending on the method of making contact with the light-sensitive material, the generated current will flow in opposite directions.

the other at the border between the compound and the base. The voltage produced at the front border is naturally higher and therefore determines the flow of current. Cells based on this principle are also referred to as "front-effect" cells. The two types are illustrated in Fig. 23-2.

23-5. Wave Nature of Light.—Before discussing the response and the characteristics of the various types of photoelectric devices, we shall turn our attention to the measurement and characteristics of light in general. It will, of course, be impossible to discuss the nature of light in detail, but we should be familiar at least with certain fundamental definitions that are required if we wish to interpret the data given by the manufacturer in connection with photoelectric cells.

The antenna of a broadcasting station sends out energy by means of radio waves; these waves consist of interlinking electric and magnetic fields, and the fields propagate themselves at the rate of 300,000 km per sec. The wave length of these radio waves, *i.e.*, the distance between two points where the electric or the magnetic field is just at a maximum, ranges from several hundred yards to a few inches. The human body has no mechanism with which to detect such waves, and we therefore have to build radio receiving sets to do this for us. A heated body sends out waves of exactly the same nature, except that their lengths are considerably smaller; if the radiation is of sufficient density, we can detect it by the fact

that the part of our body struck by this radiation will heat up. Light is an electromagnetic radiation of exactly the same character but with wave lengths still shorter than heat radiation. Nature has endowed us with an organ capable of detecting radiation with wave lengths ranging between, approximately, 0.4 and 0.7 μ, where 1 μ is the millionth part of 1 meter. In some books the range is given in millimicrons; with this unit of length the range of radiation visible to us extends from 400 to 700 mμ. A third unit of length often employed in the discussion of light is the angstrom unit, which is again ten times smaller than the millimicron so that the range of radiation visible to us covers the range between 4,000 and 7,000 angstrom units. Since 1 m is approximately 40 in., 1 μ, being the millionth part of a meter is equal to 40 μin.; consequently, the wave lengths of the visible spectrum are between approximately 16 and 28 μin.

The wave length of light determines its color. A radiation with a wave length of 700 mμ is a deep red and a radiation with a wave length of 400 mμ is a purple; between these two colors are found all the colors of the rainbow. Figure 23-3 shows the color sensation evoked in the human eye by radiation of various wave lengths. It will be noted on this figure that there is no such thing as white light; white light is rather the presence of radiation of all the different wave lengths. In the well-known classical experiment, sunlight is made to pass through a prism that has the faculty of bending light rays of different wave lengths a different amount, and the result is a decomposition of the beam of white light into its components. The spectrum containing all the colors of the rainbow then results.

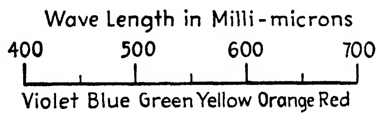

FIG. 23-3.—The human eye perceives radiation of different wave lengths as different colors.

23-6. Measurement of the Intensity of Radiation.—How can the intensity of radiation be measured? This can be accomplished by converting radiant power to some other form and then measuring the effect. Whenever radiant power strikes an object, part of it will be absorbed and another part will be reflected; the absorbed part usually is converted into heat and thus causes a rise of temperature of the absorbing body. When the surface of the body is made black, as can be done by covering it with soot, it is possible to make it absorb almost all of the radiant energy striking it. This method of detecting radiating power has not been changed since the earliest experiments when Herschel, in 1800, placed a thermometer in the spectrum obtained by passing sunlight through a prism and found that the thermometer not only was heated in the visible part of the spectrum but also showed a temperature rise beyond the red part of the spectrum where the eye could detect no radiation whatsoever. It is only in the sensitivity of the instruments that our present-day methods vary from those of earlier days. If radiant power is permitted to fall on a

blackened target and the resulting temperature rise is measured electrically by means of a bolometer or a vacuum thermocouple, a temperature rise as small as one one-millionth of a degree can be detected.

The amount of radiant power present at a given point can evidently be stated by giving the energy received by 1 sq cm per sec. For the electrical engineer the unit for the rate of energy production is the watt; therefore, radiation may be and should be given by stating the watts per square centimeter which an irradiated surface receives. On a clear day with the sun at the zenith, the power received is approximately 0.12 watt per sq cm. But only about 40 per cent of this radiation is in the visible part of the spectrum, about 55 per cent is in the infrared, and the rest is in the ultraviolet region. The ordinary incandescent lamp is a very efficient converter for changing electrical energy into radiant energy; more than 95 per cent of the electrical energy appears as radiant energy, the rest of it being carried away by conduction and convection. This statement does not, however, make the incandescent lamp an efficient light source. More than 85 per cent of the radiation is in the infrared part of the spectrum with less than 10 per cent (usually only from 3 to 5 per cent) appearing as radiation with a wave length visible to the human eye.

23-7. Human Eye as a Detector of Radiation.—Even within the range of visible radiation the human eye does not respond equally well to radiations of various wave lengths. By this statement is meant that equal amounts of radiation of different wave lengths will not be considered equal by an observer. Although it is, of course, extremely difficult for an individual to state when two areas, one illuminated by green light, the other one illuminated by red light, for example, are equally bright, nevertheless by collecting data from thousands of observers it has been possible to construct and plot what is known as the "visibility curve." The standard visibility curve, shown in Fig. 23-4, is the combined average of many observers and should therefore not be taken as applying to any given individual. It shows that radiation of a wave length of approximately 550 mμ (a yellowish green) will produce the highest effect as far as the human eye is concerned. Radiation with a wave length of 490 mμ, for instance, produces only 20 per cent of the effect of the first named radiation. Let us express this in another way. Suppose that we illuminate a newspaper with radiation of 490 mμ wave length, and that in order to be able to read the paper the average observer requires a certain intensity of radiation, or watts per square inch. On the other hand, if the same newspaper is

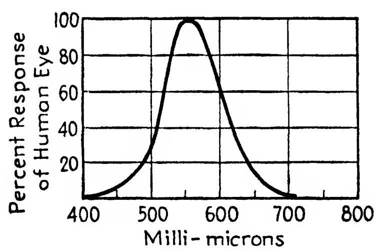

FIG. 23-4.—The standard visibility curve of the human eye. Radiation with a wave length of 555 mμ produces the strongest effect on the human eye.

illuminated with a radiation of 550 mμ, the power per square inch necessary to make the paper readable can be reduced to one-fifth of the former value. The most efficient illumination would therefore be obtained from a light source that would convert all energy into radiation with a wave length of approximately 550 mμ. Whether such a light source would be a pleasing one is not under discussion here. Generally speaking, a light source is the more efficient in producing visible radiation the nearer its maximum output is to the 550-mμ wave length, and the less radiation is produced at higher or lower wave lengths. It is of interest to note that the radiation produced by a firefly is particularly efficient in this respect. Most of the radiation occurs at a wave length of 560 mμ, with none produced below 500 and above 650 mμ. Since it is hardly probable that Mother Nature has endowed the firefly with a blinking system just to be particularly visible to human beings, but more likely for biological reasons having to do with the ease with which two fireflies find each other, the assumption is justified that the eyes of these insects have about the same response curve as ours.

This rather lengthy discussion of the nature of light has only one purpose: to make clear that what we term "light," *i.e.*, what stimulates our optical nerve, is only a very narrow band of a general phenomenon termed "radiation." Photocells of any kind are also detectors of radiation, but it surely would be a colossal coincidence if they happened to have the same response curve as our eyes. In other words, we must not be too much surprised when a certain photocell says that there is a lot of light when we see none, and vice versa. This simply means that the cell is responsive to a radiation of a wave length that does not stimulate the human eye. It also shows the extreme difficulty of properly rating photocells or other light-sensitive devices.

23-8. Candle Power; Foot-candles; Lumens.—We shall now discuss the various terms used in the measurement and description of light. In this field we meet candle power, foot-candles, and lumens. There are other quantities of interest to the illuminating engineer, but the electrical engineer will get along on the three just mentioned. Before we go into detail, it may be well to give a short summary of these terms. Candle power is a measure of the ability of a light source to produce a total amount of light. This has nothing to do with how bright the source may appear. A source with a large surface, but not very bright, may produce more light, and therefore have more candle power than a source of small but very bright surface (like an electric arc, for instance). The lumen is a measure of total light; it may refer to either a light source or a surface receiving light. It has nothing to do with the intensity of the light. A large surface being dimly illuminated may receive more lumens than a small spot illuminated brilliantly. Since candle power is a measure of the ability to produce light, there must evidently be a relation between the

candle power of a source and the lumens produced by it. This will be discussed presently. The foot-candle, or also lumens per square foot is a measure of the intensity of illumination. To read a newspaper or to perform any particular task, the intensity of illumination on the spot where we want to work is of importance. Theoretically, we can read a paper by illuminating only a small part, say, 1 sq in. of it. This may not be very practical, but it shows that it is not the lumens that are important in this case, but the intensity, or the lumens per square foot. Note that all these definitions deal only with radiation in the visible region.

The unit of light-producing ability is possessed by the international candle, a flame carefully specified; it is also represented by the average of some incandescent lamps preserved at the Bureau of Standards. The unit of light, or light flux, the lumen, is the amount of light that a source of 1 cp —assumed as a point source—sends into a unit solid angle, the apex of which coincides with the light source. A solid angle is nothing but a cone of any cross section whatsoever. A unit solid angle is a cone which, when intersected with a sphere of 1 ft (or 1 in., or 1 mile) radius, whose center coincides with the apex of the cone, cuts out on this sphere an area of 1 sq ft (or 1 sq in. or 1 sq mile). Since a sphere of 1 ft radius has a surface of $4 \times \pi \times 1^2 = 12.56$ sq ft, the solid angle represented by the whole sphere is 12.56; consequently the total lumens coming from a source of 1 cp must be 12.56 lumens, which is the relation already intimated. The amount of light contained in any cone with the light source at its apex evidently is the same no matter how far we go away from the source. The intensity of illumination that exists at a distance of 1 ft from a source of 1 cp is the unit of illumination or light intensity. It is called "one foot-candle." Since at a distance of 1 ft from a unit light source the amount of light falling on 1 sq ft is by definition 1 lumen, the intensity of illumination is clearly 1 lumen per sq ft. The two terms "foot-candle" and "lumens per square foot" are therefore interchangeable and describe exactly the same physical quantity. Since the surface of a sphere increases in proportion to the square of the radius, whereas the amount of light received by the inside surface of the sphere from a light source disposed at its center remains constant, it is clear that the intensity of illumination decreases with the square of the distance from a point light source.

All original illumination measurements were made by comparing the intensities of illumination of two fields, or white surfaces, one of them illuminated by a source of known strength, the other by the unknown light source. The human eye served to indicate when the two fields seem to be equally bright. This may be a very practical method if we are after the illuminating quality of the unknown light source. But as a means of predicting the response of a photoelectric cell to a particular "light," it is all but useless. The fact that two sheets of paper illuminated by two differ-

ent light sources seem to be equally bright (not only that, but also seem to be of equal color) does not permit us to predict that two photocells disposed in front of them will give identical response, even if the response curves of the two cells are alike.

23-9. Light Output of Standard Incandescent Lamps.—The carbon filament lamp required from 3 to 4 watts to produce 1 cp, or 12.56 lumens. The modern Mazda lamp requires only 1 watt to produce 1 cp. This is an average value, with the higher wattage lamps giving more than this value and the lower ones falling short of it. The manufacturer gives, for instance, the values for standard Mazda lamps as shown in the accompanying table.

Wattage	Initial lumens	Candle power or lumens/4π
25	260	20.5
60	835	66
100	1,600	127
300	5,850	465
500	9,850	780

At a distance of 1 ft from a 60-watt Mazda lamp, if it were truly a point source, we would have an intensity of 66 lumens per sq ft, or 66 foot-candles. As it is, the intensity will be less at certain spots (such as in line with the base) and higher in others; the average, however, is 66 lumens per sq ft. How much light (*i.e.*, how many lumens) would fall on a 2 in. diameter circle at a distance of 5 ft from a 100-watt lamp? Since the candle power of the lamp is 127, the intensity of illumination at 5 ft distance will be $127/5^2$, or approximately 5.1 foot-candles, or 5.1 lumens per sq ft. A square foot has 144 sq in. The lumens per square inch would therefore be $5.1/144 = 0.0354$ lumen per sq in. On the 2 in. diameter circle, which has an area of 3.14 sq in., we shall therefore have $0.0354 \times 3.14 = 0.111$ lumen.

23-10. Rating and Spectral Response of Barrier-layer Type of Photocells.—With these definitions in mind, we can approach the study of the data given by various manufacturers for photoelectric devices. Two methods of rating are possible, as far as the illumination is concerned. The self-generating cells of the barrier-layer type are usually rated by giving their electrical output as a function of the light intensity prevailing on the spot where they are disposed. Their whole area must then be illuminated, of course. For the photoemissive-type cells, on the other hand, the manufacturer seems to prefer giving their electrical output as a function of the lumens falling on the cathode. It is to be hoped that a more uniform method will be evolved.

Figure 23–5 shows the spectral response of the Photronic cell. The curve indicates that the cell has its maximum response somewhat more toward the red part of the spectrum and that it "sees" beyond the limits

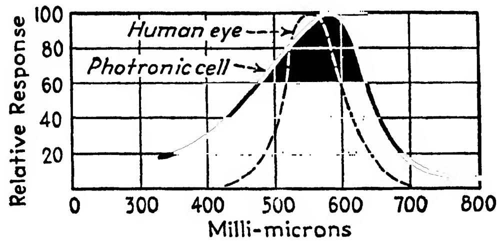

FIG. 23–5.—The Photronic cell has a response curve which approximates that of the human eye. Note, however, that it "sees" radiation to which the human eye does not respond. (*From "Electron Tubes in Industry," by K. Henney.*)

of human vision on both sides of the spectrum. By means to be discussed later, the response curve can be made more nearly equal to that of the human eye, if so desired. Figure 23–6 shows the open-circuit voltage produced by the two most popular types of barrier-layer cells as a function of the intensity of illumination. It is apparent that no linearity exists between the two values. If the resistance of the cells were a constant value, then the current flowing in any load connected to the cell would show the same general characteristics. This is not the case, however. As Fig. 23–7 discloses, when the cell is practically short-circuited (R = 3 ohms), the current is quite linearly related to the light intensity. Under such a condition the voltage across a load of such low resistance is, of course, very low. When nothing more than a *measurement* of light is desired so that the cell has to operate only a meter, the handicap just outlined is of little consequence, and it is in this field that the barrier-layer cell has found wide application. Its small size and the fact that no batteries are required make it a very convenient device for an illumination meter, or an exposure meter.

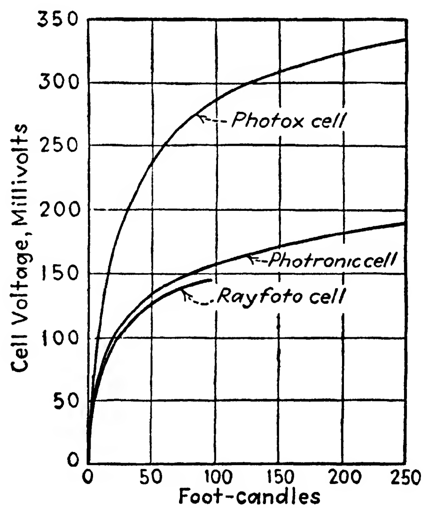

FIG. 23–6.—The open-circuit voltage of two well-known barrier-layer types of cell is seen to be far from linearly related to the intensity of illumination. The characteristic of a wet cell is also shown. (*From "Theory and Applications of Electron Tubes," by H. J. Reich.*)

Relay operation can be had from these cells, but the relays must be

extremely sensitive and are usually built just like a meter, with a contact mounted on the pointer. Amplification of the output of these cells is never attempted. Any amplification would require a dc supply for its operation, and when a direct voltage of 100 volts or more is available, the photoemissive type of cell is the logical choice since it is capable of furnishing much higher voltages than the fraction of a volt delivered by the barrier-layer cell, and its linearity is also much better.

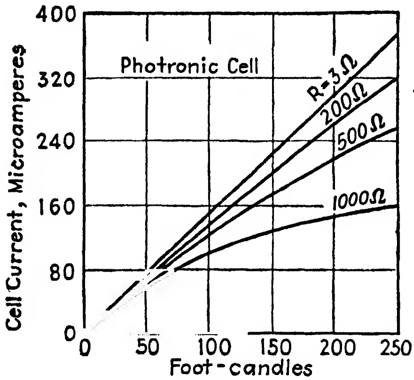

FIG. 23-7.—The current produced by a Photronic cell is reasonably proportional to the intensity of illumination only for very low values of load resistor. (*From "Theory and Applications of Electron Tubes," by H. J. Reich.*)

From the curves it is apparent that these cells require a fairly large amount of light for their operation. Intensities of 20 to 50 foot-candles are values of illumination level as found in factories and offices. Coupled with the impracticability of amplification, the cells are evidently not suitable for the measurement of very small amounts of light.

23-11. Fundamental Considerations Pertaining to Photoemissive-type Cells.—The photoemissive cells are by far the most important members of the family of photoelectric devices. As stated at the beginning of this discussion, the emission of electrons from metal when struck by light was discovered very early. It has also been stated that metal with a high work function will not emit electrons except if struck by light with a very short wave length. Einstein, in 1905, formulated the laws governing the emission of electrons under the influence of light. These laws are simply as follows:

1. The amount of electrons emitted from a given cathode is strictly proportional to the light intensity, *i.e.*, to the lumens falling on it.

2. Whether or not electrons are emitted at all and the speed with which they emerge depend only on the material of the cathode and the wave length of the light striking it. Thus a given cathode material may be illuminated with the most powerful red light without emitting a single electron while a weak blue light coming from a distant star succeeds in causing emission.

A photoemissive type of cell is then nothing but a diode with the familiar thermionic cathode replaced by a photoemissive cathode. Thermionic diodes, it will be remembered, operate either under space charge or under temperature saturation conditions, although in practice they operate most of the time under space-charge limitations. In the case of the photocell, the current available from the cathode is so small that it does not take

very much voltage between anode and cathode to reach what would be the equivalent of temperature saturation in the case of the thermionic diode. In other words, a relatively small voltage applied between the two electrodes will take all emitted electrons away from the cathode. In the present-day photocell the cathode is usually a half cylinder, the inside surface of which is covered with the emitting material, as shown in Fig. 23-8. The preparation of these cathodes is a complicated process on which much research has been done. The procedure of preparing the cathode has much to do with the sensitivity and the stability of the finished cell, and the material used determines essentially its spectral response. The metals commonly used for the preparation of the cathode are lithium, sodium, potassium, rubidium, and cesium, with lithium being more sensitive near the blue end of the spectrum, while cesium has its maximum response nearer the red end. By using a combination of metals, the spectral response may be controlled over wide limits. The anode may have any shape, as long as it does not cut off the light. In early types of phototubes, the inside walls were silvered and served as an anode; in the present type of cell, the anode is usually simply a straight wire concentric with the cathode.

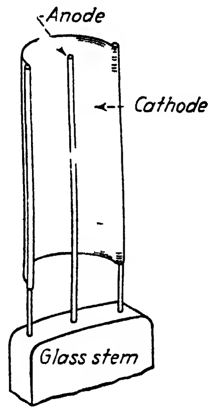

FIG. 23-8.—The standard photoemissive type of phototube consists of a semicylindrical cathode and a rod as the anode.

23-12. Characteristic Curves and Rating of a Vacuum-type Phototube. As stated in the beginning of the preceding paragraph, a vacuum-type photocell is a device working essentially under "light-saturation" condition. Figure 23-9 shows the characteristics of one of the most popular vacuum types of photocell, the 917. Observe that even under the maximum light condition shown in the characteristics (*i.e.*, 0.5 lumen), saturation is reached with about 40 volts on the anode; with 0.1 lumen this state is reached with about 15 volts. The manufacturer tells us that we can put several hundred volts on such a tube. One might ask what good it will do to put so high a voltage on the tube if any increase over 40 volts is not going to produce any more current. This argument would be justified indeed, if all we wanted to do was to operate a meter; then 40 volts would serve just as well as 200. But if we place a load in series with the tube, it is evident that the conditions are different. With a 200-volt supply voltage, for instance, it would be possible to develop as much as 160 volts across a properly chosen load resistor and still have 40 volts left for the tube. The performance of such a photocell reminds one strongly of the performance of a pentode, where a large voltage swing across a load resistor can be obtained with a small change in grid voltage. With the photocell a light change takes the place of the grid-voltage change. The

process of drawing a load line into such a family of curves is exactly identical with the same process in the case of any other tube. The voltages obtainable from such tubes are truly fantastic compared to those of the barrier-layer cells. The fact that the voltage appears across a very high resistance, since the current output of the cell is low, is no handicap whatsoever, since the vacuum tube is ideally suited for the amplification of such voltages.

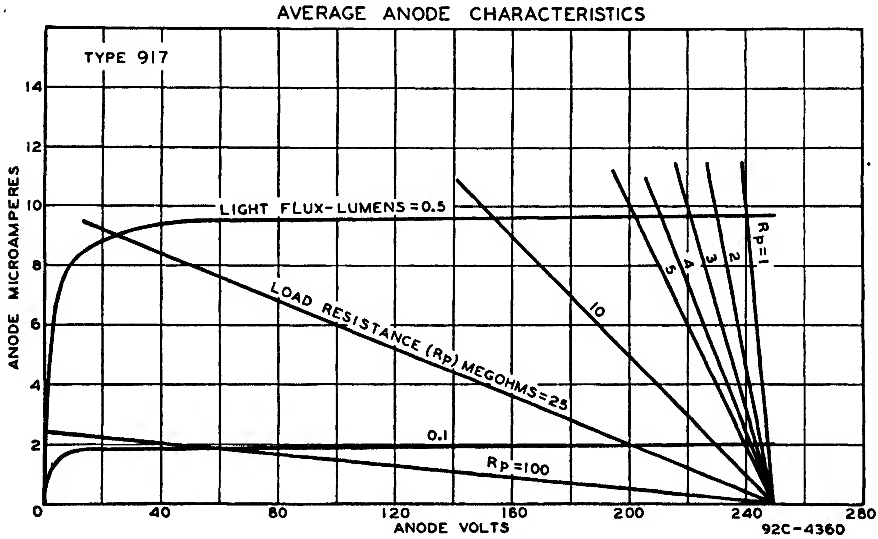

FIG. 23-9.—Characteristics of a type 917 phototube. For voltages in excess of approximately 30 volts, the current through the tube no longer rises appreciably. This indicates that the current is not limited by space charge, but that all emitted electrons are taken to the anode. (*Courtesy Radio Corporation of America.*)

The ratings of the 917 given by the manufacturer are shown in the accompanying table. It will be noted that the manufacturer gives the light

917 HIGH-VACUUM PHOTOTUBE—MAXIMUM RATINGS AND CHARACTERISTICS

Anode supply voltage (dc or peak ac)	500 max volts
Anode current *	30 max μa
Ambient temperature	100°C max
Luminous sensitivity †	20 μa per lumen
Sensitivity at 8,000 angstroms	0.0020 μa per μw
Dc resistance of load:	
With anode supply voltage of 250 volts	1 min megohm
With anode supply voltage of 500 volts	10 min megohms

* On the basis of the use of a sensitive cathode area ½ in. in diameter.
† Subject to variations; value given is average value. Taken with a Mazda projection lamp operating at a filament color temperature of 2870°K.

in lumens. Engineers want accurate and, at the same time, easily applicable information. As in so many other cases, it is impossible to eat

your cake and also have it. A correct and proper way of giving the response of a photocell, as repeatedly stated, would be to give its response curve, stating the microamperes per microwatt of radiation as a function of the wave length of the radiation. To predict the performance of the cell with a given light source, the engineer would then have to know the total radiation of the given light source as well as the spectral distribution of it. It is easy to imagine what howls would arise from the engineering profession if such a complicated process were involved. The manufacturer in desperation therefore specifies not only the lumens but also the light source from which they must be coming. The light must be obtained from a Mazda projection lamp, the filament of which must be operated with a temperature of 2870°K. (This is somewhat higher than that with which the ordinary Mazda bulb is operating.) Since the light source generally happens to be a Mazda bulb, this method of rating does not lead to any trouble in most cases. But the ridiculous part about it is, that more than 90 per cent of the radiation of the Mazda bulb is invisible and produces no lumens (which are solely a measure of *visible* light) whatsoever, and that the cell itself quite often has most of its response in the infrared part of the spectrum. Therefore a light source other than a Mazda lamp producing perhaps five times as many lumens, *i.e.*, actual light, but having no radiation in the infrared part of the spectrum may give only a fraction of the current produced by the much "weaker" Mazda bulb. This matter is discussed in much detail here so that the reader will not be shocked too much if in his work he finds that his calculations deviate a few thousand per cent from the actual performance. The moral: allow plenty of margin in the design of photoelectric circuits for just such occasions.

23–13. Characteristics and Ratings of Gas-type Phototubes.—The introduction of gas into the thermionic diode removed the space-charge effect and thus permitted a larger current with the same voltage. The phototube does not operate under space-charge conditions, as we have seen. From this fact one would be tempted to predict that the introduction of gas into such a tube would not have much influence. This is a mistake, however. The introduction of gas does increase the current over what it would be without gas, but the reason for this is somewhat different from that in the case of a thermionic diode. It will be remembered that a diode is usually operating under space-charge conditions, which means that more electrons are emitted than can be taken away by the electric field. In the case of a phototube the number of electrons emitted from the cathode is a function of the light falling on it, so that, whatever we may do to increase the current, we still want to have the final output current a function of the light. In Chap. XXI we saw that, under correct voltage- and gas-pressure conditions, the traveling electron or ion may demolish a neutral gas molecule (ionize it). This results in the creation of two new carriers. In the case of the thermionic diode, the slow-moving ions thus produced

served the purpose of canceling the space charge, and the main benefit resulting from the gas was not so much the creation of additional charge carriers but the ability to make fuller use of the electrons emitted from the thermionic cathode. In the case of the phototube, on the other hand, the creation of the new charge carriers is the principal benefit arising from the presence of the gas. The voltage is made high enough so that an electron emitting from the cathode has the opportunity to acquire several times

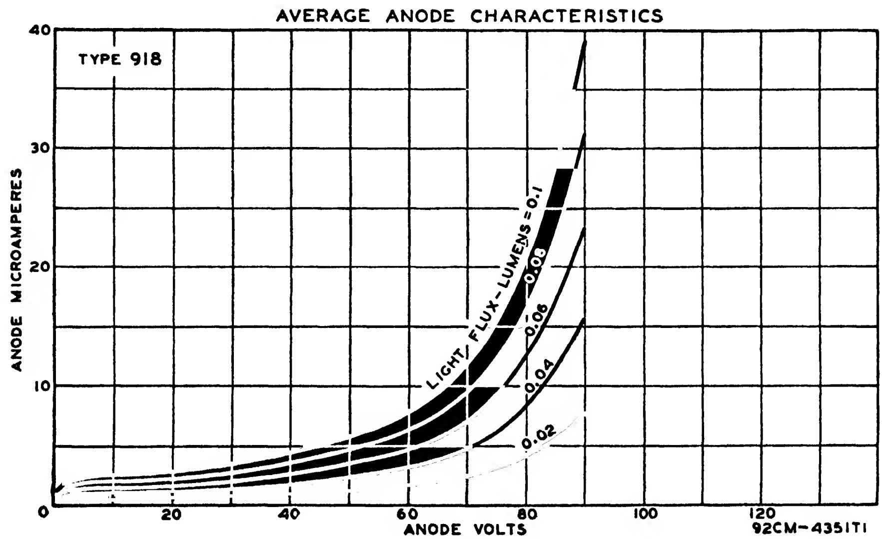

FIG. 23-10.—Characteristics of a type 918 gas-filled phototube. Larger currents are obtained with a gas-filled tube than with a vacuum tube because ionization of the gas provides additional charge carriers. (*Courtesy Radio Corporation of America.*)

enough speed to ionize a gas molecule. Suppose that the gas used in the tube has an ionization voltage of 20 volts. With an anode voltage less than this it can be expected that the gas-filled phototube will give the same current as the vacuum type, and indeed it does. As the voltage is increased slightly beyond this value, electrons having almost reached the anode will have acquired enough speed for the demolition act. Some of them will have a chance to hit a gas molecule, and not only will such an electron then have a companion electron, but an ion will in due time arrive at the cathode. If all electrons had this chance, the current would suddenly increase threefold. As the voltage is increased, more and more electrons will have this chance and, when the voltage is finally raised above 40 volts, an electron that might have had the luck to strike a molecule about halfway between cathode and anode would again gain sufficient speed just before reaching the anode to ionize another molecule. It can therefore be expected that in such a tube the current will increase with an increase in anode voltage. That this is actually the case is shown in Fig. 23-10.

The ratio of the actual current to the current without gas in the tube is called the "gas amplification factor." It usually has a value of from 8 to 10. One might think that this process could be carried farther and farther without limit. But the reader will realize that this discharge must under all conditions remain nonself-sustaining (see Sec. 21-5). In other words, when the original source of electrons, in this case the photoemissive cathode, ceases to furnish electrons, the current must stop. Although

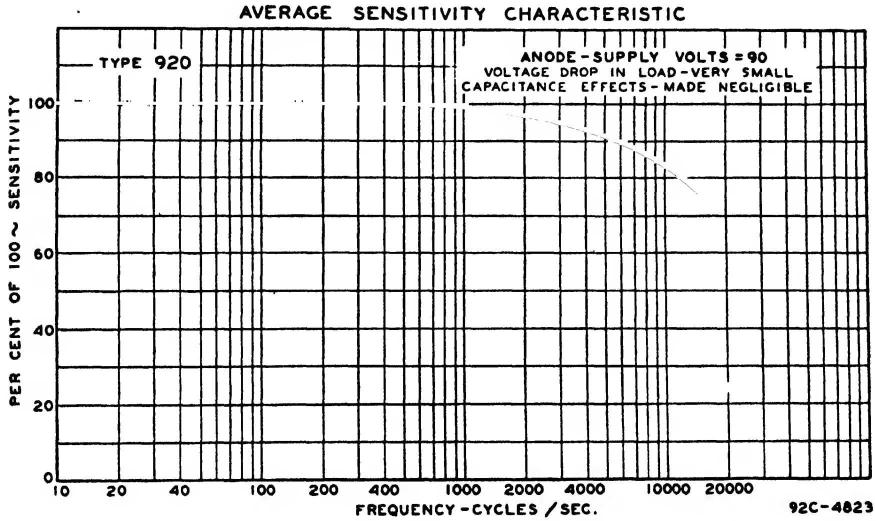

FIG. 23-11.—Whereas a vacuum-type phototube inherently has no frequency limitation, a gas-filled phototube shows a decrease of response when the frequency of the light variation exceeds certain values. This is due to the fact that ionization of the gas takes a definite time. *(Courtesy Radio Corporation of America.)*

some of the ions arriving at the cathode may succeed in knocking an additional electron out of it, this phenomenon must not take place on the average; otherwise the discharge becomes self-maintaining. This is why the maximum voltage permissible on a gas tube is much lower than on a vacuum type. Anode voltages of gas-filled phototubes are usually limited to 90 volts; voltages in excess of this value will lead to a self-sustained glow discharge, which will quickly ruin the cathode.

23-14. Choice of the Tube Type for Various Problems.—When we try to decide whether to use a gas tube or a vacuum tube for a particular problem, the following considerations may be of help. The linearity between current and amount of light is not so good for the gas tube as for the vacuum type. Furthermore, the fact that the current depends on the anode voltage is a decided handicap, if the cell is to be used for accurate measurement of the amount of light or radiation. Therefore, wherever reproducibility of results and stability of the operating point are of prime importance, the use of a vacuum-type cell is preferable, in spite of the con-

siderably lower sensitivity. If the problem deals, on the other hand, only with "On" and "Off" operation of a light relay, for instance, such as may arise when it is desired to count objects as they come along on a conveyer and interrupt a light beam, then the higher sensitivity of the gas tube is a decided advantage and should not be passed up. Another difference between the two types that is of importance for certain problems is the speed of response. There is no observable time lag between the instant when light strikes the cathode and when electrons are emitted. But there is a time delay in the process of ionization such as takes place in a gas tube. The speed of response of a vacuum-type cell is limited only by the time constant of the circuit consisting of the tube capacities and the load resistor; in the case of a gas tube, on the other hand, it takes time before the current can build up to its final value. This difference is ordinarily of little significance for the industrial engineer but, where a photocell is used for the reproduction of sound from film, for instance, or in any other problem involving the speed of response, the reduction of response for high frequencies is decidedly undesirable. Figure 23-11 shows how the response of a typical gas cell falls off with the frequency of the light variations.

23-15. Multiplier Phototubes.—The average sensitivity of a vacuum-type photocell is in the order of 20 μa per lumen; for very low values of light flux the current will therefore be a fraction of 1 μa. In order to make use of such small current, we either have to use resistors of very high value in series with the tube (in order to develop a fair amount of voltage across the resistor), or we have to use several stages of amplification. The first method gives a poor frequency response, while the second suffers from instability, especially when direct voltages must be amplified. These were the reasons for attacking the problem from an entirely different direction. In the so-called "multiplier phototubes" use is made of the phenomenon of secondary emission to increase the current output of the tube many thousandfold.

In Sec. 6-5, it was explained that an electron striking a surface with a certain speed may knock out of the surface several new electrons. This principle is employed in the multiplier phototubes, a schematic diagram of which is shown in Fig. 23-12. The electrons emitted from the cathode, when light strikes it, travel to the first anode in exactly the same manner as in an ordinary photocell. This electrode is made of material particularly suitable for secondary emission. Consequently, there will be more electrons emitted from it than are arriving from the first cathode. In a sense, therefore, this anode becomes at the same time a cathode, which is why all the intermediate electrodes in a multiplier phototube are referred to as "dynodes." This increased number of electrons emitting from the first dynode will travel toward the second dynode, which is at a potential positive with respect to the first dynode, where secondary emission again takes place. In the commercially available cells this process is repeated

in nine stages, resulting in a current amplification from 30,000 to 200,000, depending on the voltage applied to the individual stages. The voltage per stage is usually from 75 to 100 volts. Dynode 9, the last, acts as the

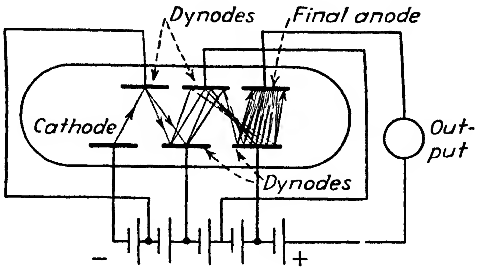

FIG. 23 12.—The principle of the multiplier phototube. A single electron emitting from the light-sensitive cathode causes many thousands of electrons to arrive at the final anode, a result due to secondary emission.

cathode for the final anode in the tube which collects the final output current.

The first question that will come to anybody's mind is why the electrons emitted from the original cathode do not travel directly to the final anode, which is after all the most positive electrode in the tube. This would indeed be the case if the tubes were actually constructed as shown schematically in Fig. 23–12. Much research and development work had to be done before the proper shape and arrangement of the various electrodes were determined, so that the electrons would actually follow the correct

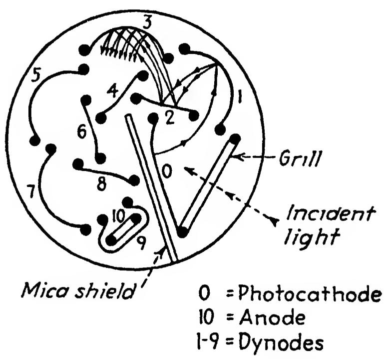

FIG. 23 13.—A cross section through a typical multiplier phototube. The shape and position of the electrodes are such as to force the secondary electrons into the proper paths. (*Courtesy Radio Corporation of America.*)

paths between the various electrodes. Figure 23–13 shows the cross section through a type 931A multiplier phototube with the paths and multiplying action of the electrons indicated. Circuit diagrams to be used in connection with multiplier phototubes will be discussed in a later chapter.

PROBLEMS

23-1. What is the intensity of illumination in foot-candles at a distance of 15 ft from a 100-watt Mazda lamp?

23-2. How many lumens will a disk 3 in. in diameter receive in Prob. 23-1 (*i.e.*, 15 ft from the 100-watt lamp and facing the lamp, of course)?

SUGGESTED ADDITIONAL READING

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CHAPTER XXIV

OPTICAL SYSTEMS; LIGHT SOURCES; FILTERS FOR PHOTOELECTRIC TUBES

24–1. Results Obtainable from a Light Source without Additional Optical System.—For the solution of most problems involving photoelectric cells, more than a simple light source and a photocell is required. Suppose that calculations for a particular photocell and amplifier tube to be used in connection with it (as will be discussed in more detail in Chap. XXV) indicate that a light flux of 0.2 lumen is required on the cell. Let the projected area of the cathode be 0.9 sq in. How far away can we go from a 100-watt Mazda bulb? or from a 21-cp headlight lamp (a very popular light source for photoelectric cells)? If we want to catch 0.2 lumen on an area of 0.9 sq in., the intensity of illumination, *i.e.*, the foot-candles or lumens per square foot, existing at the place of the photoelectric cell must be $144 \times 0.2/0.9 = 32$ lumens per sq ft. Since a 100-watt lamp has an average candle power of 127 (which means that at 1 ft distance from it the average illumination is 127 lumens per sq ft) and since the illumination falls off as the square of the distance, an intensity of 32 foot-candles will exist at a distance of $\sqrt{127/32} = 1.99$ ft. This represents the maximum distance at which we can place the photocell in order to obtain 0.2 lumen on the cathode. In the case of the 21-cp automobile-headlight bulb, we have to come closer than 1 ft, namely, $\sqrt{21/32} = 0.81$ ft or about 10 in. In some problems it may be possible to keep the separation between the photocell and its light source down to the values just mentioned, but in others the beam may have to be several feet long. Although it would, of course, be theoretically possible to achieve operation either by increasing the strength of the light source or by increasing the amplification following the photocell, so that a much smaller intensity would be sufficient, it is much more desirable to increase the efficiency of the light beam by a proper optical system. Very inexpensive lenses, such as are found in reading glasses or magnifiers, will do wonders in this respect.

24–2. Effect of a Lens in Front of the Light Source.—In order to appreciate the improvement obtainable by placing a lens in front of a light source, consider at first the conditions existing when a photocell is placed at a distance of 6 ft from a light source radiating with approximately equal intensity in all directions. A sphere with 6-ft radius has a surface of $4 \times 6^2 \times \pi = 452$ sq ft, or 65,200 sq in. If the photocell has a cathode

area of 1 sq in., we see that in such an arrangement only about 1/65,000 of the light produced is used for the operation of the cell; or, putting it in a more striking way, we have to provide 65,000 times as much light as is actually required for the operation of the cell. One certainly would have to go far to find anything as inefficient as this! As already stated, this situation can fortunately be improved very considerably by the use of lenses. Lenses are devices that collect rays of light and bend them. As far as the engineer designing optical systems for photoelectric cells is concerned, there are only two values pertaining to a lens that are required to predict its performance in an optical system: the diameter of the lens and its focal length, which is the distance between the center of the lens and the point at which a parallel beam of light striking the lens comes to a focus, as indicated in Fig. 24–1. Stated in reverse: if we place a point source of light at a distance equal to the focal length from the center of the lens, then all the light rays striking the lens from this source will be bent to emerge on the other side of the lens as a parallel beam of light. By reversing the direction of the arrows in Fig. 24–1, this figure will then apply to the second case. Now suppose we place a lens of 3 in. diameter and 6 in. focal length at a distance of 6 in. from a 21-cp automobile-headlight lamp. In the first approximation this source can be considered as a point source of light. The intensity of illumination existing at a distance of 6 in., or ½ ft, from a 21-cp source is $21/(\frac{1}{2})^2 = 21 \times 4 = 84$ foot-candles, or 84 lumens per sq ft; this is also equal to 0.583 lumen per sq in. If the light source were truly a point source and if the lens were perfect, a parallel beam 3 in. in diameter with an intensity of 0.58 lumen per sq in., or $0.58 \times 144 = 84$ lumens per sq ft, would result. This intensity would remain constant no matter how far we were to go away from the lens. Without a lens the intensity at a distance of 6 ft from a 21-cp source would be $21/36 = 0.58$ lumen per sq ft. With the lens, the intensity is consequently 144 times as high, theoretically at least. Even making allowance for the facts that the light source is not a point, that the lens is not perfect, and that its glass absorbs a good deal of the efficient radiation, we can expect at least twenty to thirty times as much light as without a lens.

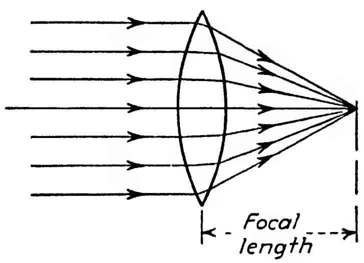

FIG. 24 1.—A lens is an optical device to collect a parallel beam of light and to bring it to a focus at one point The distance of this point from the center of the lens is the focal length of the lens.

24–3. Effect of a Lens in Front of the Phototube.—This is not all the improvement that can be obtained by lenses. The 3 in. diameter lens has an area of $3^2 \times \pi/4 = 7.1$ sq in. Since the intensity of illumination at 6 in. from the light source has been found to be 0.58 lumen per sq in.,

the lens will catch an amount of light equal to 7.1 × 0.58 = 4.1 lumens and the light beam emerging from the lens will therefore contain this amount of light no matter how far away we go. If we place a photocell with a cathode area of only 0.9 sq in. in this beam, we let most of the light go by again. However, by letting our beam strike a 3 in. diameter or larger lens, the light rays will converge and come to a focus at one point, and by placing the photocell at this point (or better, slightly ahead or behind so that one small spot on the cathode will not do all the emitting and become overloaded), we can get all the 4.1 lumens of the beam on the cathode of the photoelectric cell. This is now a theoretical improvement of 1,000:1 over the arrangement without any lens! Even if absorption of the glass and other factors may cause a reduction of this figure to one-fifth of the theoretical value, for instance, we would still have about 200 times as much light on the tube as without any lenses. Most of the present-day photoelectric-cell installations make use of lenses at both the source and the cell. It may be argued that the above line of reasoning leads to the obviously absurd conclusion that we could move our photoelectric cell as far away as we wanted without any loss of light. But it must be remembered that we started our analysis with perfect and ideal conditions such as cannot be had in practice.

24-4. Effect of a Light Source of Finite Dimensions.—By far the most important reason for an actual optical system to fall short of the performance predicted on the basis of the preceding paragraph is the fact that the light source is not a point source; indeed the farther a light source deviates from this ideal, the less satisfactory will be the performance of a lens system. It is for this reason that the automobile-headlight lamp is so popular, because it evidently comes much closer to the ideal of a point source than a 110-volt light bulb for instance, especially one with frosted glass. To make a reasonably intelligent guess of what we may expect with a light source of finite dimensions, we have to make use of some simple optical laws. In the above example we saw how the first lens produced a parallel beam of light while the second one collected it again at one spot. If the light source were actually a point source, as assumed in the example, we would not need a second lens. A lens not only is able to make a parallel beam of light out of a divergent beam coming from a point source, but, by moving it a little farther away from the point source, it makes a convergent beam out of the divergent one. When we focus a camera, for instance, we move the lens until the divergent beam of light rays coming from any point of the subject is converted into a convergent beam coming to a focus at exactly the place where the ground glass or the film is disposed. Therefore, in the above example we did not need the second lens but could simply have brought the beam produced by the first one to a focus on the cell 6 ft away. But what will happen now, if our light source is no longer a point source but is contained—let us say—in a square with sides ⅛ in. long?

(This assumption is reasonably true for the filament of a headlight lamp.) Suppose we hold a white sheet of paper in front of the photocell, as close as possible to it. When the lens is moved back and forth, there will suddenly appear on the sheet a reasonably sharp picture of the light source, in this case a picture of the filament of the lamp. The quality and sharpness of this picture will depend on the quality of the lens, but even with an inexpensive lens the picture of the filament will be easily recognized. We are now interested in the size of this picture. The laws of optics tell us that the linear dimensions of the object and of the picture of it are proportional to the distances of the object and picture from the lens. Thus if,

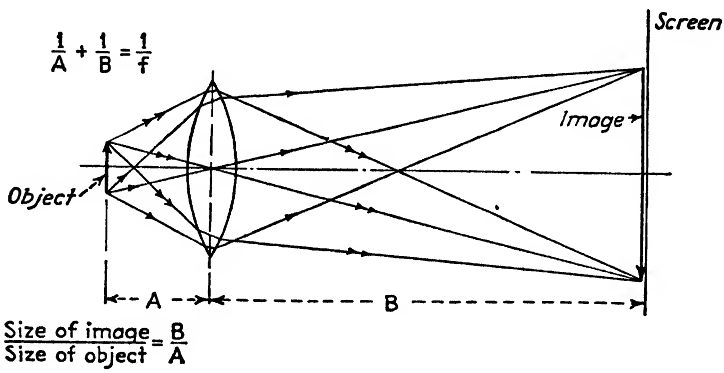

FIG. 24-2.—Owing to the focusing action of a lens, the image of an object can be made to appear on a screen. The various distances and the magnitudes of images and objects are given by the equations in the figure.

in the above example, the sheet is 6 ft, or 72 in. from the lens, while the light source is approximately 6 in. from it, the size of the picture on the sheet is twelve times the size of the light source. For simplicity's sake, we assumed the source as a square 0.2 by 0.2 in. The size of the picture thrown on the sheet would therefore be a square with sides 2.4 in. long. The distance of the lens from the light source at the instant when a sharp picture is produced on the screen will always be more than the focal length of the lens. In Fig. 24-2 let an object at a distance A from a lens produce an image at the distance B from the lens. The relation among A, B, and the focal length f of the lens is given by

$$\frac{1}{A} + \frac{1}{B} = \frac{1}{f} \tag{24-1}$$

and, furthermore,

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{B}{A}$$

For a lens with a focal length of 6 in. and a distance of 72 in. between the lens and the sheet, application of Eq. (24-1) will give a value of 6.55 in.

for the distance between the lens and the light source. Since any calculations pertaining to photoelectric cells are at best only rough approximations, the assumption that a lens will be at a distance from the light source equal to its focal length will not lead to any serious error except when the distance between the photocell (or sheet) and the lens in turn is of the same order of magnitude as the focal length of the lens; in such a case, the exact distances needed for the calculation of the size of the image may be obtained with the aid of Eq. (24–1).

In the above example we saw that a light source of square shape with 0.2 in. length of its sides would appear at the screen as a square of approximately $2\frac{1}{2}$ in. The total amount of light contained in this image of the light source will be equal to the amount of light falling on the lens. This was calculated earlier as 4.1 lumens. Allowance should be made for the fact that the lens will absorb some of the light. Depending on the kind of glass used in its manufacture, the absorption in the infrared part of the spectrum may be considerable and a loss of from 25 to 50 per cent may take place in the effective radiation. The area of the picture thrown on the screen will be approximately 6 sq in., and if we place a photoelectric cell with a cathode area of approximately 1 sq in. in place of the screen, we evidently cannot catch more than approximately one-sixth of the total light contained in the image. By placing a second lens, however, at this spot with an area so large that the whole image of the light source will fall on the lens, we shall be able to collect all the light contained in the image for the operation of the photocell.

Let us now assume that the screen is moved 12 ft away from the lens. The picture of the light source will now be twice as large as it was in the case of the 6-ft distance and the image of the light source will be a square of approximately 5 in., with the area of the image evidently four times as large as before. It is obvious that the area of the image of the light source will increase with the square of the distance between the screen and the lens. The total amount of light contained in the picture will remain constant, however, since it is determined only by the amount of light flux caught by the lens. The intensity of illumination at the place of the image will therefore decrease with the square of the distance, and it is therefore seen that no contradiction exists between the case of a lens and the case of a simple light source. If it is required, for instance, that the beam traverse a distance of 50 ft, the image of the light source produced by a 6-in. focal length lens would be approximately 20 in. square; if it is desired to collect all the light flux contained in this beam, it would take a lens of approximately 30 in. diameter, which would obviously be rather impracticable.

24–5. Use of Parabolic Reflectors for Concentrating the Light Beam.—Another method of concentrating a large portion of the light emitted from a small light source into a narrow beam is by means of properly shaped reflectors; as a matter of fact, this method is even more efficient than the

use of lenses. An outstanding example of the application of this principle is found in the optical system embodied in automobile headlights. Everyone is familiar with the fact that these lights throw a sharp and powerful beam ahead of the car, but few people realize the very high efficiency of the system. Figure 24-3 shows the intensity of illumination in lumens per square foot at various distances from an automobile-headlight lamp. We see from these curves that such a lamp equipped with a 50-cp bulb will produce an illumination of 50 lumens per sq ft at a distance of 35 ft from the lamp; since without the aid of an optical system a 50-cp light source would produce an illumination of 50 lumens per sq ft at a distance of only

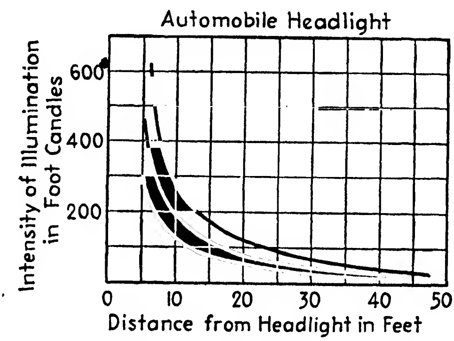

FIG. 24-3.—An automobile headlight is a highly efficient optical system to concentrate the light from a small source into a beam. The upper curve is for a 50 candle power, the next one for a 32 candle power, and the lower one for a 21 candle power bulb. (*From "Electron Tubes in Industry," by K. Henney.*)

1 ft from the light source, it is seen that the optical system results in an increase of 35^2 or approximately 1,200 fold. Suppose we place a collecting lens of 4 in. diameter in the beam. Such a lens has an area of approximately 12 sq in., or $\frac{1}{12}$ sq ft. With an intensity of illumination of 50 lumens per sq ft, the lens will collect approximately 4 lumens! This represents about forty to fifty times as much light as is ordinarily required for the operation of a photocell.

24-6. Lens System as a Means for Excluding Unwanted Light.—Optical systems consisting of one or two lenses not only serve the purpose of making more efficient use of light and thus reduce the required size of a light source, or the amplification following the photocell, but they can also be used to make sure that a cell operates only from the light source intended to operate it, and not from any stray light. Suppose that a photocell is to be operated from a light source 20 to 30 ft away. Regardless of whether a lens is used in front of the light source or not, the light rays arriving at the photocell can be considered as very nearly parallel. If we let them strike a lens, they will be converted into a converging beam, which comes to a focus and forms a very small spot of light at a distance equal to the focal length of the lens, as shown in Fig. 24-1. The intensity of illumina-

tion of this spot is extremely high, since it contains all the light flux striking the lens. As has already been stated, it is not desirable to subject a small spot on the cathode of a photocell to an extremely high intensity of illumination, but rather the light should be spread over as large an area of the cathode as possible. For this reason alone, the photocell should not be disposed at the focal spot of the lens. Suppose, therefore, that we move the photoelectric cell farther back but dispose in the focal plane of the lens a baffle plate of cardboard or sheet metal with a small hole at exactly the spot where the beam of light comes to a focus. This condition is shown in Fig. 24-4. The full-drawn lines represent the rays of light contained in a beam, which is intended to operate the photocell; these rays are seen to come to a focus at exactly the spot where the baffle plate has

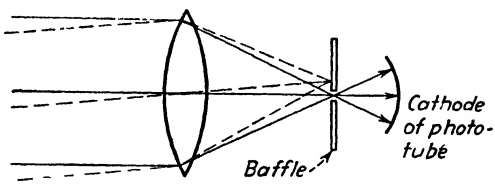

FIG. 24-4.—The combination of a lens and a baffle plate provides means to prevent the operation of a photocell from any but the desired light source.

its hole. Suppose that in the vicinity of the operating light source there is another source of light, the rays of which strike the lens as shown by the dotted lines. These rays also come to a focus in the focal plane, and without a baffle a large part of this light would strike the photocell too. But we see that with the baffle plate none of these rays strike the photocell, which will therefore be responsive only to the light source that is intended to operate it. Such an arrangement may be required, if it is desired to obtain operation of a cell from a distant light source where a large amount of stray light may originate near the light source.

24-7. Modulated Light Sources and Their Applications.—The system described in Sec. 24-6 of keeping unwanted light from operating the photocell is quite satisfactory for moderate distances, up to 100 ft, for instance, and in places where the ambient light is not subjected to too wide a variation. In outdoor installations, on the other hand, the conditions are usually such that sufficient discrimination cannot be obtained in this way. In such cases the use of so-called "modulated" light may solve the problem. A light source is said to be modulated when its light output varies in strength at a definite frequency. A light source may be modulated mechanically, simply by having a shutter interrupt the beam at a given frequency. Thus a disk with 20 slots and teeth at its circumference, driven by an 1,800-rpm motor and interposed between the light source (such as a headlight bulb) and the first lens, will evidently produce a light beam that is interrupted 600 times per second. If the photoelectric cell

is connected to an ac amplifier, slow variations of the stray light will not be amplified; if the amplifier in addition is tuned to 600 cycles, it is evident that there is no way of "fooling" the cell into responding to any other than the modulated light source.

Instead of mechanical modulation, we can use a gaseous discharge lamp such as a neon bulb. Since such lamps are as a rule poor "point" sources of light, it is usually rather difficult to focus them into a narrow beam. This remark does not refer to all gaseous discharge lamps. Special water-cooled high-power mercury-arc lamps have been constructed which are extremely small in size and which have been used in connection with parabolic reflectors or lenses to furnish a strong beam of modulated light. A rather interesting application of this modulated light is found in a device used for the determination of the height of clouds. The measurement of cloud heights during nighttime has been a very simple problem. A powerful searchlight shoots up a vertical beam of light which produces an illuminated spot when it hits a cloud. At a known distance from the searchlight, say, 1,000 ft, an observer determines the angle at which the illuminated spot appears; if the angle is, for instance, 45 deg from the horizontal, the cloud is obviously 1,000 ft high. This system fails during daytime because even the most powerful searchlight is a poor competitor when the sun is shining on a cloud. By shooting a modulated beam of light at the cloud and having a photoelectric cell, connected to an amplifier tuned to the modulation frequency, sweep through the meridian, it is possible to determine the desired angle, although the human eye would be utterly unable to determine the spot where the modulated beam hits the cloud. The ambient light may be several hundred thousand times as strong as the modulated light, but the photocell and its associated amplifier will, of course, respond only to the latter.

24-8. Life of Conventional Light Sources for Phototubes.—Wherever reliability of operation is concerned, the life of a light source is, of course, of prime importance. The life of an ordinary Mazda lamp is on the average 1,000 hours; projection lamps, on the other hand, may have a rated life of only 40 hours. Wherever possible, it is therefore desirable to operate the light source with a reduced voltage. Even a relatively small reduction of operating voltage will increase the life of a lamp considerably. A 10 per cent reduction in voltage, for instance, increases the life to over four times its average value. The light output, on the other hand, is reduced to approximately 70 per cent. This does not mean, however, that the output of a photoelectric cell would also be reduced to 70 per cent. When the filament of a lamp is operating at a lower temperature, the reduction of radiation is most pronounced in the shorter wave lengths of light; in other words, the light seems to become more reddish. Since the photocells generally used in conjunction with incandescent light sources have their maximum response in the infrared part of the spectrum, the current pro-

duced by the cell will not fall off so rapidly as the light input. Based on a "lumen sensitivity," the cell becomes more sensitive when the temperature of the light source is reduced; as a matter of fact, if the temperature of the light source is reduced so far that the human eye cannot perceive any radiation, the photocell will still have a response, which would mean that its "lumen sensitivity" is infinite! Automobile-headlight bulbs operated at 5 volts, or even lower, make excellent long-life light sources.

24-9. Use of Filters to Obtain Operation with Invisible Radiation.—The preceding discussions have obviously dealt with problems where the interruption of the light beam, or its existence, or nonexistence, was of prime importance. In problems of this kind, the spectral composition of the beam is of no importance. The fact that 90 per cent of the total radiation coming from a Mazda bulb is invisible is neither an advantage nor a disadvantage as long as we can couple it with a photocell that is responsive to this radiation, in other words, that makes use of this invisible radiation. As a matter of fact, for certain installations, such as burglar alarm systems, it is decidedly desirable to be able to hide the beam of light. Since in the case of an incandescent lamp as a light source and a cesium-coated photocell most of the action takes place in the infrared part of the spectrum anyway, only a relatively small loss of sensitivity occurs if we remove the radiation in the visible part of the spectrum by means of appropriate filters. Filters are devices that absorb radiation of certain wave lengths and permit others to pass freely. If we look at a scene through a green filter, for instance, everything appears green. This is not due to the fact that the filter converts all light into green light, but rather that it does not permit the passage of light with higher or lower wave lengths. A white sheet of paper (or any white surface) reflects light of all wave lengths equally well; if we illuminate it by white light, which is a mixture of radiation of all wave lengths, it will reflect them all equally well. But, by letting the light pass through a green filter before reaching our eye, all but the green radiation is absorbed in the filter. Incidentally, a green sheet of paper produces the result of appearing green by a trick similar to that of the filter except that it does it by reflection rather than by transmission. In other words, a green surface appears green because, with white light falling on it, it reflects only the green radiation and absorbs all other radiation. Figures 24-5*a*, *b*, and *c* show the transmission characteristics of a red, green, and blue filter commonly employed in the photography of colored objects. These filters, known as "Wratten" filters, consist of dyed gelatin; they are made by the Eastman Kodak Company. Observe that the vertical scale on these curves is logarithmic. Consider the transmission curves of the red and the blue filter. The red filter will not pass any radiation with a wave length less than 580 mμ, while the blue filter will not pass any radiation above 520 mμ. Consequently, if these two filters are placed on top of each other, whatever light is able to pass

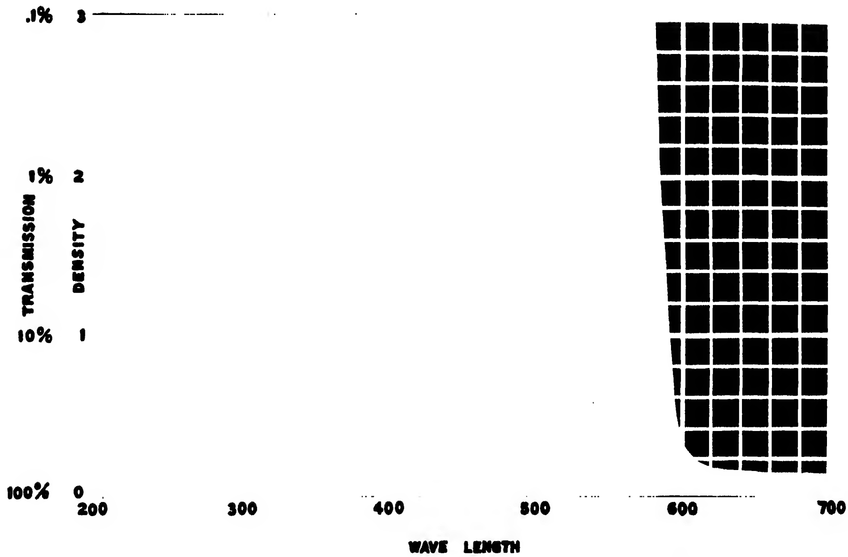

FIG. 24-5a.—The transmission characteristics of a standard Wratten red filter. (*Courtesy Eastman Kodak Co.*)

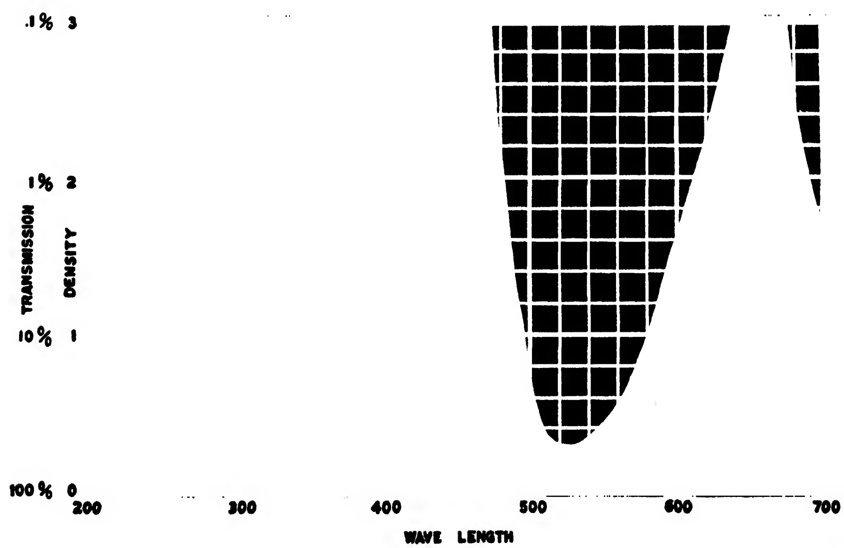

FIG. 24-5b.—The transmission characteristics of a standard Wratten green filter. (*Courtesy Eastman Kodak Co.*)

through one filter will be absorbed by the other, so that apparently no light whatsoever will pass through the combination of the two. However, if the spectral transmission curves shown in Fig. 24–5 are extended beyond the 700-mμ range, it would become apparent that all three filters pass infrared radiation quite freely. If we therefore place a combination of these two filters in front of a light source, practically all visible radiation will be absorbed; but a photoelectric cell operated from this light source will still respond to the infrared part of the radiation. Sometimes it is

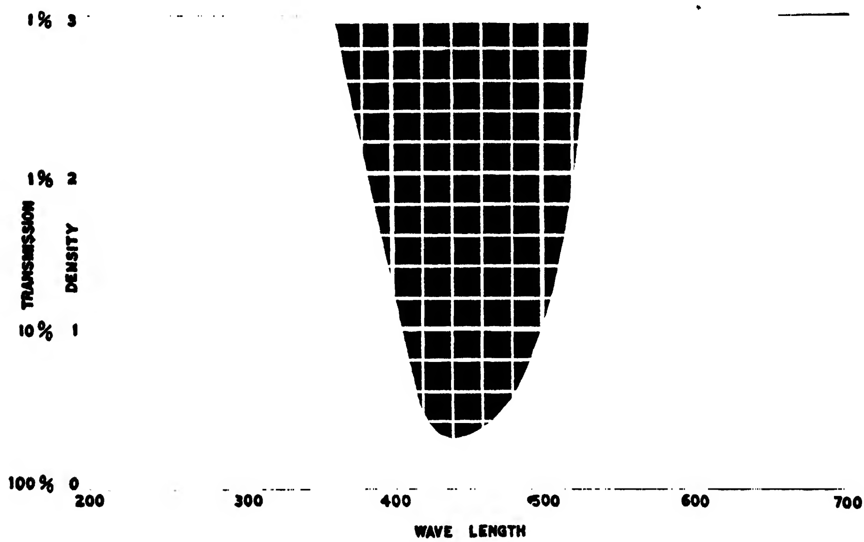

FIG. 24–5c.—The transmission characteristics of a standard Wratten blue filter. (*Courtesy Eastman Kodak Co.*)

stated that such an installation operates with "invisible light." This is, of course, a misnomer because there is no such thing as invisible light. There is invisible radiation, to be sure, but since the term "light" refers to radiation with a wave length to which the human eye responds, it is obvious that no such thing exists as invisible light. It is, of course, not necessary to employ two filters as outlined in the above example, since manufacturers of colored glasses are able to offer single filters that absorb practically all visible radiation while permitting the passage of infrared radiation freely.

24–10. Means to Approximate the Response of the Human Eye.—Obviously, the use of photocells is not restricted to problems involving the interruption of a beam of light. In many problems it is desired to let the photocell take the place of the human eye; for instance, in cases where it is desired to determine the sheen or gloss of a surface or where it is desired to measure the actual *light* output of a light source (not the *total* output of radiation). In such cases it is quite evident that the photocell should

have a response closely approximating that of the human eye. The first step in this direction is, of course, to make use of a cell having as high a response as possible in the visible part of the spectrum. In Chap. XXIII it was stated that the various materials used for the preparation of the cathode of a photocell have different response curves. A cell with a potassium cathode, for instance, has much higher response in the visible part of the spectrum than a cesium-type cell. At present the cathodes are usually prepared by a combination of various elements and, for this reason, the manufacturer usually refers to their surfaces by certain designations such as S-1 and S-2. The spectral response curves of the various surfaces may then be obtained from the manufacturer. Although the choice of the proper cathode will go far toward making the response of the cell more nearly like that of the human eye, there is nevertheless no photoelectric surface that has the same response as that of the human eye. By the correct use of filters, however, it is possible to modify the response curve of any given cell (provided it has any response at all in the visible part of the spectrum) to approximate that of the human eye. If a cell with a considerable response in the infrared part of the spectrum is to be used for such a purpose, the most important condition is to provide a filter to absorb the infrared radiation. A very effective filter for infrared radiation is a 2.5 per cent solution of cupric chloride. A solution of this chemical in water appears pale blue, and as far as the human eye is concerned, seems to absorb very little of the light. But when a flat cell containing the liquid is placed between an incandescent lamp and an ordinary photocell, the current through the cell will drop to 5 or 10 per cent of its original value, indicating that it was responding mostly to the invisible radiation. Although such a liquid filter is more effective in absorbing infrared radiation than any other type of filter, it is not a practical solution for industrial applications where evaporation of the liquid and loss of it due to spilling are serious handicaps. Infrared radiation is essentially heat radiation, and glass manufacturers have been able to produce glasses that are especially effective in suppressing this type of radiation. Such glasses are usually employed in projection apparatus where it is desired to protect the slide from the heat of the projection bulb. They will take out most of the infrared part of a given radiation.

24-11. Principle of Color Comparators.—Photocells have been used quite successfully in color comparators, even if their response is altogether different from that of the human eye. The reason for this is that the human eye, as marvelous an instrument as it is, can be fooled relatively easily. It will pronounce two fields as of equal color, even if the light falling on one of them is of entirely different composition from that falling on the other. Thus the familiar greenish-blue light of a mercury lamp is essentially a monochromatic light, which means that it is radiation of essentially only one wave length. The effect of this light as far as the eye

is concerned can, however, be duplicated by the mixture of red, blue, and green light in proper proportion. Conversely, if two objects give three equal responses on a photocell when it receives light from them at first through a red, then through a green, and finally through a blue filter, then we can definitely state that the two objects will appear to the human eye as of the same color. All color comparators are built on this principle. The standard sample is viewed by the photocell in succession through the three filters just mentioned, and each response is noted. The same procedure is then applied to the unknown sample and, if the three responses are identical, the sample is pronounced equal to the standard. It is evident that with such an arrangement it is not necessary that the photoelectric cell has a response exactly identical to that of the human eye, since it is not called upon to say how bright the object looks through a given filter; it has only to say whether the sample and the standard appear equally bright through a given filter.

It is hoped that this chapter has served to emphasize the fact that great care must be taken, if it is intended to use photoelectric cells in a role approximating that of the human eye. The spectral response of the cell, the spectral characteristics of the filter system, and those of the light source must be taken into account or else the results obtained from the photoelectric arrangement may seem utterly irrational.

SUGGESTED ADDITIONAL READING AND REFERENCES

See end of Chap. XXV.

CHAPTER XXV

AMPLIFIER CIRCUITS FOR PHOTOELECTRIC CELLS

25–1. Application of the Load-line Principle to Phototube Characteristics.—It is almost superfluous to insert a chapter on the amplification of photoelectric currents because this problem is in no way different from the amplification of any other minute currents. When a photoelectric cell in series with a resistance is placed across a source of direct voltage and light is permitted to fall on the cathode, current will flow in the series combination and a voltage will appear across the load resistor. As already mentioned, the amount of voltage across the resistor for a given amount of

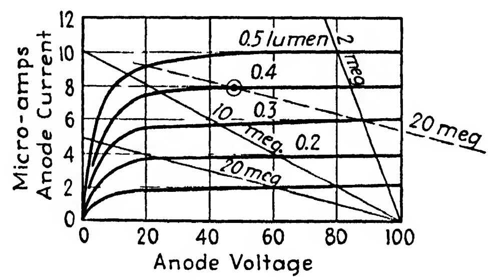

FIG. 25-1.—The application of the load-line principle to the combination of photocell and load resistor.

light falling on the cathode can be predicted, if the characteristics of the photocell, such as shown in Figs. 23–9 and 23–10, are available. It is only necessary to draw in a load line, just as was done in the case of an ordinary triode or pentode. In Fig. 25–1 are shown the characteristics of a type 917 photocell, and three load lines representing 2, 10, and 20 megohms are indicated. The combination is operated from a supply voltage of 100 volts. With a load resistor of 10 megohms and a light flux of 0.1 lumen falling on the cathode, the voltage across the cell will be about 80 volts, with 20 volts across the resistor; when the light is increased to 0.2 lumen, the voltage across the resistor will, of course, increase, with an equal decrease in the voltage existing across the cell. The voltage change across the resistor, as indicated by Fig. 25–1, is approximately 20 volts. This is considerably more than would be required for the operation of an ordinary amplifier tube, unless the latter is connected in a cathode-follower circuit. It is therefore evident that satisfactory operation can be obtained with a light change considerably less than 0.1 lumen. As in the case of a

pentode, the output appearing across the load resistor will be higher with a higher load resistance. On the other hand, the supply voltage must be increased if extremely high values of load resistance are used; otherwise, the intersection of the load line with the characteristics will fall in the curved part of the characteristics. Thus in Fig. 25–1, if it is desired to obtain operation when the light changes from 0.4 lumen to, say, 0.41 lumen, a resistor of 20 megohms would be unsuitable, unless the supply voltage is raised sufficiently so that the intersection of the load line representing 20 megohms cuts the characteristic for 0.4 lumen in the flat part of the characteristic. This is indicated by the dotted line in Fig. 25–1, which, if extended, would intersect the voltage axis at about 200 volts.

25–2. Basic Connections of Load Resistor to Amplifier Tube.—The voltage appearing across the load resistor is applied to the grid of an amplifier

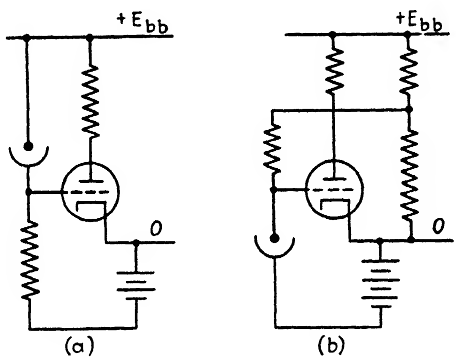

FIG. 25–2.—The voltage developed across the load resistor in series with the photocell may be connected to the grid of an amplifier tube in such a way as to result in either an increase or a decrease of plate current with an increase of light.

tube in the manner discussed in detail in Chaps. X and XI. There are, of course, two basic connections possible: the cell and its load resistor can be connected to the amplifier tube either in such a way that the voltage developed across the load resistor makes the grid of the amplifier tube more positive, or so that it makes it more negative. In the first case, the plate current will increase with an increase of light; in the second case it will decrease. The two basic connections are shown in Figs. 25–2*a* and *b*. The circuit of the former gives an increase in plate current with an increase of light, while the circuit shown in the latter acts in the opposite manner. The choice between the two depends, of course, on the problem to be solved. If both of the circuits serve the purpose, that in Fig. 25–2*a* is somewhat more convenient because it usually requires less bias voltage than that in Fig. 25–2*b*. Where high sensitivity is desired, it is evidently necessary to employ load resistors of high value. The remarks made in Secs. 11–15 to 11–19 apply to this case also, and the reader is urged to re-

fresh his memory on this subject by reading those pages. It will be remembered that the lead connecting the active end of the load resistor to the grid of the amplifier tube should be protected as carefully as possible from any leakage occurring between it and other terminals carrying either positive or negative potentials. This was discussed in detail in connection with Fig. 11–14. Since it is always easier to prevent leakage to a terminal that is brought out through the top of the tube, away from all other terminals carrying voltage, not only should the amplifier tube used in connection with high-sensitivity photoelectric circuits have its grid brought out through the top, but it would also be desirable if the active terminal of the photocell, *i.e.*, the terminal connected to the load resistor and also to the grid of the amplifier tube, were brought out on top of the tube where it could be insulated more effectively than the pins at the base of the tube. But this would mean that the photocells used in the two circuits shown in Figs. 25–2*a* and *b*, although electrically of the same characteristics, should be constructed differently. The photocell used in the circuit shown in Fig. 25–2*a* should preferably have its cathode brought out to a cap on top of the tube, while in the circuit shown in Fig. 25–2*b* the anode should preferably be brought out in this manner. The manufacturer has recognized the desirability of having these types available. The high-vacuum phototubes, types 917 and 919, are electrically identical, but the 917 has the anode brought out to a cap on top of the tube (as desirable for the circuit shown in Fig. 25–2*b*) while the 919 has the cathode brought out at the top and is therefore the choice for a circuit as shown in Fig. 25–2*a*. This choice is not available for gaseous phototubes but, since the latter are usually employed only for circuits where "On" and "Off" operation is desired and where a relatively large amount of light is available, this fact is of little importance. Since, in the case of high-sensitivity circuits requiring high values of load resistors, the precautions outlined in Sec. 11–18 should be taken into consideration, it is obvious that in such circuits relay operation or even meter operation is as a rule not obtainable in the plate circuit of the first amplifier tube; therefore a load resistor should be placed in the plate circuit of this tube and the voltage developed across it applied to another stage of amplification.

25–3. Influence of the Size of the Load Resistor on the Frequency Response of the Phototube.—As long as the light variations to which the photoelectric circuit is to respond are slow or if the speed of response is not critical, the load resistor employed in the circuits shown in Fig. 25–2 may be as high as desired, subject, of course, to the limitations dictated by grid current, leakage current, etc. If the circuit is to be used in connection with a modulated light source, however, or if speed of response is of importance, the load resistor cannot be made too high. It is evident that the capacitance of the phototube itself, the capacitance to ground of the wiring between the load resistor and the grid of the amplifier tube,

the grid-to-cathode capacitance, and the grid-to-anode capacitance of the latter are all effectively in parallel to the load resistor. As already mentioned in Sec. 14–15, the grid-to-anode capacitance under operating conditions is larger than the actual capacitance between these two electrodes, owing to the Miller effect. All these capacitances easily add up to a value in the neighborhood of 20 $\mu\mu f$. This may look like a small capacitance, but at a frequency of 1,000 cps it represents a reactance of 8 megohms. This would not cause a serious reduction of the frequency response if it happened to be parallel to a resistor of 100,000 ohms or even 1 megohm. If it is in parallel to a load resistor of 20 or 50 megohms, it will cause a reduction of response to a fraction of what it would be at a low frequency. This fact is often overlooked by the designer of photoelectric circuits, who is surprised to find the response to sudden light changes rather rounded off.

When a photoelectric cell is used for the recording of high-frequency phenomena, it is therefore not possible to use a load resistor of high value, but resistors of as low as 50,000 to 100,000 ohms must be used. The output voltage appearing across these load resistors is then only a fraction of what it would be with a high-value load resistor, and the loss of sensitivity must be made up by an increase of amplification following the phototube.

25–4. Operation of Phototube Relays from Ac Source.—In the circuits discussed so far, it was assumed that the photoelectric cell and the associated amplifier tubes were operated from a direct supply voltage. If relay operation only is required, the circuit may be operated from an ac source. It will, of course, be realized that the photoelectric cell and the amplifier tube will be inoperative for the half cycle during which the anode voltage is negative. Because ac operated circuits eliminate the need of a rectifier and its associated filtering system and because grid bias can be obtained very conveniently, ac operated circuits have become very popular. Figure 25–3 shows an example of an ac operated photoelectric circuit. Examination of it reveals the fact that the circuit is entirely equivalent to that shown in Fig. 25–2a. Since the relay in the plate circuit of the amplifier tube is an inductive load, it should be by-passed with a fairly large capacitor because, as will be remembered from the discussion of rectifier circuits, it is undesirable to have an inductive load in a half-wave rectifier circuit, and in this particular case chattering of the relay may result. With a capacitor across the relay, on the other hand, the capacitor will become charged during the half cycle when the amplifier

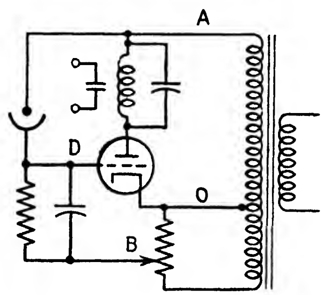

FIG. 25–3.—If relay operation only is required, alternating voltages obtained from a transformer may be used for the operation of phototubes and amplifier tubes.

tube is conducting—provided, of course, that it is made conducting by the action of the photocell on the grid—and current through the relay will be maintained by the capacitor during the half cycle when the tube is nonconducting. The diagram also shows a capacitor across the resistor in series with the photocell. This capacitor is usually rather small, in the order of 20 to 100 $\mu\mu$f, but various authors do not seem quite to agree on the reason for its presence. The most common explanation is that it serves the purpose of correcting a phase displacement of the grid voltage due to the capacitances of the phototube and the grid of the amplifier tube. That the capacitor in the grid circuit of Fig. 25–3 plays a very important role becomes clear when one is told that this circuit will operate *without* the grid resistor and with *only the capacitor* in the grid circuit. This is evidently a performance of which the dc circuit shown in Fig. 25–2*a* would be incapable. It has been stated that the circuit shown in Fig. 25–3 performs during the half cycle when point A is positive with respect to point O in exactly the same manner as the circuit shown in Fig. 25–2*a*. This statement is true as far as it goes, but an examination of what happens during the half cycle, when A is negative and when the tube can therefore not carry any plate current, will reveal a phenomenon of great importance to the performance of the circuit. During the half cycle when A becomes negative, B will evidently become positive. If no voltage exists across the capacitor in the grid circuit, D also becomes positive and current flows from the grid to the cathode of the tube. Just as in the case of grid-leak detection, the capacitor will consequently charge to a voltage equal to the peak voltage existing between O and B, with a polarity such that D will be negative with respect to B. Therefore, if no light is falling on the photocell, the bias voltage of the amplifier tube while its anode is positive will consist not only of the then negative half wave of the voltage between O and B, but also of the direct voltage existing across the capacitor due to the charging that has taken place during the preceding half cycle. If the photocell passes current, however, this current flowing through the capacitor will discharge it first to zero and then begin to charge it in a positive direction. By adjusting the voltage of B, the reduction of negative grid voltage, by the action of the current flowing through the photocell, may then serve the purpose of energizing the relay in the plate circuit of the amplifier tube. It is clear that a smaller capacitor will result in a more sensitive circuit, although this reasoning cannot be carried to a point where its capacitance becomes of the same order of magnitude as that of the photocell and the associated wiring.

25–5. Combination of Phototube and Grid-controlled Gaseous Tube.—The combination of a grid-controlled gaseous tube and a phototube represents at present the most popular circuit arrangement for a light relay. A gaseous triode is not well suited for this service, however, since it shows a grid current considerably larger than that encountered in vacuum-type

triodes. In Sec. 21–24 it was explained, however, that this objection can be overcome by the introduction of a shield grid. In present-day commercial light relays, we therefore usually find a combination of a phototube with a small shield-grid thyratron operating a small relay directly in the plate circuit of the thyratron. To obtain control of the thyratron, the circuit shown in Fig. 22–8 is usually employed, with the resistance R replaced by the photoelectric cell. This modification converts the circuit shown in Fig. 22–8 to the one shown in Fig. 25–4. Sometimes the operation of this circuit is simply explained by stating that the variable resistor of the circuit shown in Fig. 22–8 is now replaced by the variable resistance of the photoelectric cell. This explanation holds true only if the photoelectric cell is of the selenium type, *i.e.*, if it is truly a resistance varying with the light falling on it. The photoemissive type of cell, however, is essentially a rectifier, and it is quite evident that the circuit shown in Fig. 25–4 must again depend on the rectifying action of the grid-to-cathode path of the gaseous tube. Comparison of the circuit shown in Fig. 25–4 with the one in Fig. 25–3 reveals that there is not much difference between the two. Here too the grid capacitor will charge, during the half cycle when point B is positive, to a voltage equal to the peak voltage between O and B. The current flowing through the photoelectric cell will then cause a current through the capacitor in a direction opposite to that produced by the grid current and will therefore cause the gaseous tube to fire the earlier in the cycle, the higher the photoelectric current. Several other arrangements of phototubes in connection with gaseous tubes are possible, but, with the basic principle as shown in Fig. 25–4 in mind, the reader should have no difficulty in analyzing similar circuits.

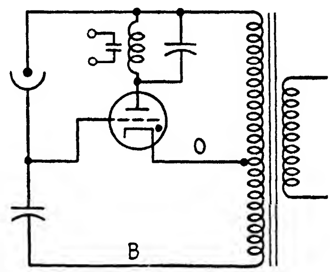

FIG. 25–4.—A very popular circuit consists of the combination of a phototube and a gaseous tube. The circuit can operate only by virtue of the rectifying action of the grid of the gaseous tube.

25–6. Circuits for the Comparison of Two Light Values.—There are a number of problems where light is made to strike two photoelectric cells and where operation is desired when the light falling on one of them exceeds the amount received by the other. Such circuits are obviously particularly well suited if it is desired to compare two amounts of light with each other, regardless of the actual amount of the two. This problem could, of course, be solved simply by duplicating the circuit shown in Fig. 25–2a or b and by connecting an indicating instrument between the two anodes of the two amplifier tubes. The circuit then becomes obviously a balanced circuit such as shown in Fig. 11–12a, but with the right-hand tube of this figure also connected to a photocell. The result of this modification is shown in Fig. 25–5. This circuit has the serious disadvantage,

however, that it will not work over a very wide range of light intensities. Evidently if the two light intensities to be compared with each other should vary over a range of 100 to 1, or even more, the voltage developed across the two grid resistors would also vary in this ratio. It would be a rather difficult task to make the two amplifier tubes operate over so wide a range of grid voltages.

A circuit with considerably higher sensitivity, and at the same time not suffering from the disadvantage just outlined, is shown in Fig. 25-6. Two photoelectric cells of the same type are connected in series across a source

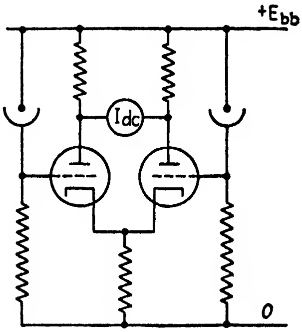

FIG. 25-5.—A balanced circuit. With equal amounts of light falling on the two phototubes, the current in the meter connecting the two plates will be zero.

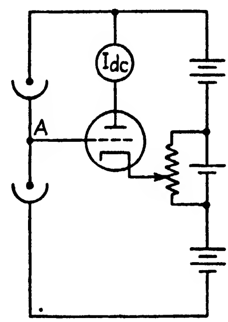

FIG. 25-6.—A circuit indicating equality of two amounts of light over a much wider range of light values than the circuit shown in Fig. 25-5.

of direct voltage. If the tubes have identical characteristics and the same amount of light falls on each, then for reasons of symmetry the junction between the two cells will be at a potential exactly halfway between the two ends of the direct supply voltage. If we place the cathode of the amplifier tube at a potential resulting in the proper operating bias of the tube, it is evident that the plate current will increase when the upper photocell receives more light than the lower one, and vice versa. If vacuum-type photocells with their flat characteristics are used in this circuit, an extremely high sensitivity will be achieved. The reason for this is obvious when we examine the series combination of the two photoelectric cells somewhat more in detail. When a resistor and a photoelectric cell are placed in series across a direct supply voltage the behavior of the combination can be predicted with the aid of the load line, as shown in Fig. 25-1. Now, when two photoelectric cells are placed in series across a direct voltage, one of them could be considered as the load of the other. The fundamental condition that the two voltages across the two cells must add up to the total supply voltage remains just as valid as when the combination consisted of only one cell and a resistor. In order to analyze the circuit

shown in Fig. 25–6, we therefore must plot the two characteristics of these photocells against each other as shown in Fig. 25–7. In order to show the essential point of the analysis more clearly, the characteristics are shown with a somewhat greater slope than they actually have. With an equal amount of light falling on both cells, the two characteristics are seen to intersect at point P_1, which is, of course, located exactly in the center between the 0- and 200-volt marks (assuming that the direct supply voltage across which the two photocells are connected is 200 volts). Let us assume now that the upper photocell in Fig. 25–6 receives somewhat less light; its anode characteristic will then drop a small amount. This is indicated in Fig. 25–7 by the line b', which is seen to intersect the characteristic a at P_2. The potential of point A in Fig. 25-6 has therefore shifted by an amount equal to the horizontal distance between P_1 and P_2 in Fig. 25–7; in this case about 52 volts. It is clear that the flatter the characteristics of the two photocells are, the larger will be the shift of potential of the junction with a small change of light. It is also clear that the operating point of the amplifier tube will not be shifted if the amount of light falling on the two cells is reduced an equal amount because, under such a condition, the potential of A will not change from its midway position.

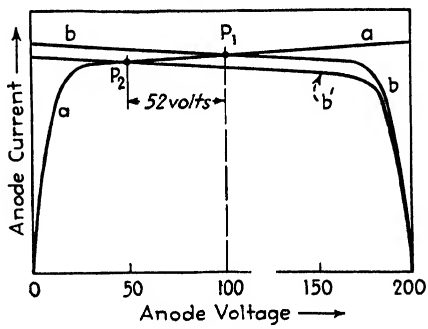

FIG. 25-7.—The operation of the circuit shown in Fig. 25–6 can be analyzed with the characteristics shown here.

In order to make full use of the high sensitivity that can be achieved by means of this circuit when it is desired to compare two amounts of light with each other, it is, of course, again necessary to take all precautions against any possible leakage between the junction point of the two cells and any other terminals. The amplifier tube to be used in connection with this circuit should have its grid terminal brought out through the top, and all measures to reduce the grid current to as low a value as possible should be taken. For the same reason, the upper photocell should have its cathode brought out on the top of the tube while the lower one should have its anode brought out in this manner; the upper one should therefore be a 919, and the lower one a 917. With the amplifier tube mounted close to the two photocells, the three grid caps can then be simply connected to each other.

25–7. Methods of Controlling Sensitivity of Circuit for Comparison of Light Values.—In many cases the sensitivity of the circuit shown in Fig. 25–6 is so high that it is next to impossible to keep a meter in the plate circuit of the amplifier tube at a steady reading. The slightest change of

light on either one of the cells will make it go either to zero or to a value determined by approximately zero grid bias, when the grid begins to draw current. Where this high sensitivity is not desired, it can be reduced by either one of the two methods shown in Fig. 25–8. The first method consists of placing a high resistance parallel to each phototube; obviously the parallel combination of phototube and resistance will now have a characteristic with a slope essentially equal to that of the resistance alone. (This would be exactly true if the characteristic of the cell itself were absolutely flat, *i.e.*, had zero slope.) As far as Fig. 25–7 is concerned, the result would be simply that the two characteristics are less flat.

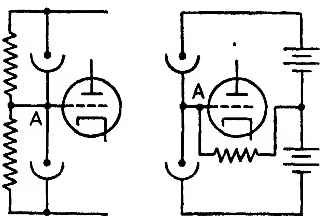

FIG. 25–8.—If it is desired to reduce the sensitivity of the circuit in Fig. 25–6, the two methods shown here may be used. Both circuits provide simply a load for the photoelectric cells.

In the second method a resistor of high value is placed between the junction of the phototubes and a center tap of the direct supply voltage. (Although shown as a tap of the batteries in Fig. 25–8, this tap may, of course, also be obtained by means of a center-tapped resistor.) One could consider this resistor as a "load" for point A. The reader will realize that from an electrical point of view the two methods are entirely identical, since application of Thévenin's theorem to the case of the two resistors placed across the two cells shows this connection to be equivalent to connecting a resistor of half the value of the two individual shunt resistors between the junction of the two photocells and a center tap of the dc source. The obvious advantage of the second method is that only one resistor of high value is required and that even this single resistor can achieve identical results with a value half as large as would be required in the case of two resistors. The sensitivity of the circuit can be changed over a wide range by using different values for this resistor.

25–8. Circuit for Measuring Light-intensity Ratios.—In the circuit shown in Fig. 25–6 the change of meter indication from the value shown when the circuit is in balance is essentially proportional to the actual difference between the two amounts of light falling on the cells. For instance, let both phototubes receive a light flux of 1 lumen; under this condition the junction between the two cells will be midway between the potentials of the dc source. Now let the light falling on one of the cells increase to 1.01 lumens. This is a change of 1 per cent, and the meter in the plate circuit of the amplifier tube will show a certain change. If the light falling on each tube is reduced to 0.1 lumen, the same change of meter reading will be obtained when the light on one of them changes to 0.11 lumen, or a 10 per cent change. In some problems this may be entirely satisfactory, but in others an indication proportional to the ratio (or rather its devia-

tion from unity) may be desired. A circuit giving this result makes use of a principle shown in Fig. 25-9. Two phototubes are connected in a back-to-back connection and are supplied from a source of alternating voltage. In series with them is placed a capacitor. Let us assume at first that the two tubes receive the same amount of light. During the half cycle when A is positive, current will flow through tube I and through the capacitor, making G positive with respect to O. During the half cycle when A is negative, an equal amount of current will flow through tube II, so that the net charge accumulated on the capacitor during one cycle will be zero. Now let the light falling on tube I be increased to twice the amount falling on tube II. This destroys the equality of charges accumulated on the capacitor during the half cycles of conduction of the two tubes, and G will therefore become more positive with every cycle of the supply voltage. This evidently cannot keep on forever, however, and the question therefore arises, how far G will go "up." Assuming that the potential of A with respect to O is of sinusoidal wave shape, it is evident that, as G becomes positive with respect to O, the time during which A will be positive with respect to G will become less than one half cycle, while the time that it is negative with respect to G will become more than one half cycle. One could also say that the direct voltage appearing across the capacitor shifts the zero line of the alternating voltage. An equilibrium will evidently be established, when the conducting period of the tube carrying the larger current has been decreased, and the conducting period of

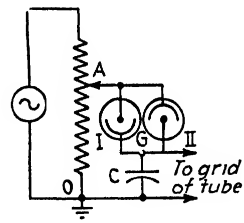

FIG. 25-9.—The basic principle involved in a circuit by means of which the ratio of two light values may be measured.

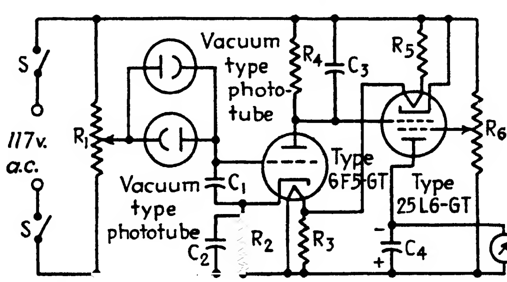

FIG. 25-10.—A complete ac operated circuit embodying the principle shown in Fig. 25-9 (*Courtesy Radio Corporation of America.*)

the tube with the smaller current has been increased so that the net change of charge of the capacitor during one cycle has again become zero because if this condition were not fulfilled, the voltage across it would keep on rising. The equilibrium voltage will be essentially a function of the ratio of the two light values, not of their difference, as in the circuit shown in

Fig. 25–6. The diagram of a complete circuit, incorporating this principle and operating directly from the ac line, is shown in Fig. 25–10.

25–9. Circuits for Multiplier Phototubes.—When it is necessary to obtain operation of a relay, meter, or amplifier from an extremely small

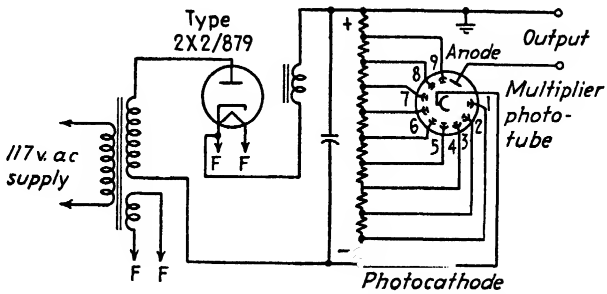

Fig. 25–11.—Power supply for a multiplier phototube circuit. (*Courtesy Radio Corporation of America.*)

amount of light, the use of a multiplier phototube, as described in Sec. 23–15, is indicated. Since each stage of such a tube requires from 75 to 100 volts for its operation, with the last stage operating with an even higher voltage (approximately 250 volts), the total voltage required is from 1,000 to 1,250 volts. If reproducibility of results is a factor, it is quite important to keep the voltages of the intermediate stages as constant as possible. When operating with 100 volts per stage, for instance, the current amplification of a type 931A is 200,000. A drop of voltage per stage to 90 volts,

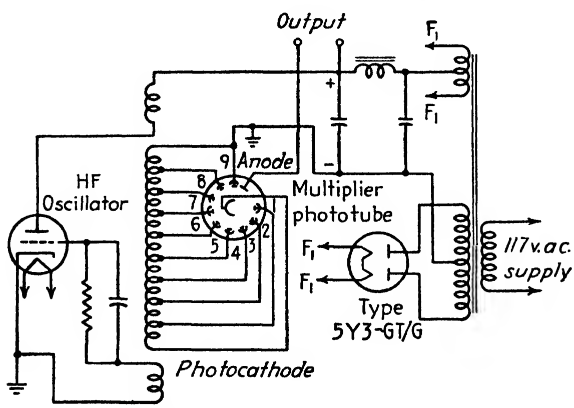

Fig. 25–12.—The multiplying stages of a multiplier phototube may be operated by means of a high-frequency alternating current. The last stage only is operated from a dc supply. (*Courtesy Radio Corporation of America.*)

or 10 per cent, causes the current amplification to drop to 100,000, or one-half. This is not surprising when one considers that the action of the tube is a multiplication process. For instance, let the figure 100,000 be reached by multiplying the figure 1 five times with the factor 10; in other words,

$100{,}000 = 10^5$. If this factor is now reduced to 9 (a reduction of only 10 per cent), the result of the successive multiplications will not be 90,000, but $9^5 = 58{,}959$, or a reduction of about 41 per cent.

If dc operation is required, a half-wave or full-wave rectifier circuit with a bleeder across the total output of the rectifier may be used. A circuit making use of such an arrangement is shown in Fig. 25–11. In order to eliminate the influence of varying load conditions of the last stage on the voltage distribution of the intermediate stages, a separate dc supply for the last stage is sometimes provided.

When only relay operation is required, the multiplier phototube may be operated from a transformer provided with taps that furnish the desired voltages for the intermediate stages. These tubes have also been operated with high-frequency alternating voltage as a supply of the intermediate stages, with only the final stage operated with direct current. Such an arrangement permits their use when the light variations occur with an audio frequency, for instance, while the supply voltage is of much higher frequency. A circuit operating on this principle is shown in Fig. 25–12.

25–10. Summary.—In keeping with the purpose of this book, no attempt is made to describe any particular application of phototubes. The use of something as intangible as a beam of light to obtain reliable control over certain processes has, of course, fired the imagination of many inventors, and the list of applications of photoelectric tubes is an ever-growing one. The reader will have a better concept of the many still hidden possibilities when he recalls that a photoelectric tube is not only a light-sensitive device (although at present most of its applications are based on this property alone) but also a detector of radiation of frequencies beyond the limit of the human eye. Its ability to detect infrared radiation, for instance, makes it a very sensitive device to control the temperature of objects being rapidly heated. Thus in the process of heat-treating, small objects, such as valve stems for automobile engines, are often heated by passing an electric current through them. The process takes only a matter of seconds, but the temperature to which they are heated must be kept within very close limits. Since the heating effect varies as the square of the applied voltage, it is clear that line voltage fluctuations may cause considerable variations in the final temperature. Any temperature-controlling device depending on contact with the heated body would, of course, be too slow to cut off the current at the right instant. On the other hand, by letting the radiation from the heated object, collected by a suitable optical system, strike a photoelectric cell, the heating current can be interrupted at the exact instant when the desired temperature has been reached. The future will probably bring many surprising applications of photoelectric cells when their characteristics differing from that of the human eye have been fully appreciated.

PROBLEMS

25-1. A 32-cp automobile-headlight bulb is mounted with its filament 3 in. from a sheet-metal plate. Opposite the lamp a slot $\frac{1}{32}$ in. wide and $\frac{1}{2}$ in. long is cut in the plate; behind the slot a type 917 phototube is mounted so that it receives the light coming through the slot. The phototube is in series with a 10-megohm resistor across a dc source of 100 volts. See Fig. 23-9 for characteristics. What voltage change will occur across the resistor when the slot is covered and uncovered?

25-2. A type 917 phototube is to give a 2-volt change across its load resistor when the light falling on it changes from 0.5 to 0.52 lumen. Give suitable values for the load resistor and the supply voltage. (Assume the light to be coming from a Mazda lamp operating under conditions for which the curves and data given in Sec. 23-12 apply.)

25-3. The average characteristics of a type 868 gas phototube are given in Fig. 25-13. The projected area of the cathode is 1 sq in. The cell is placed in series with a 5-megohm

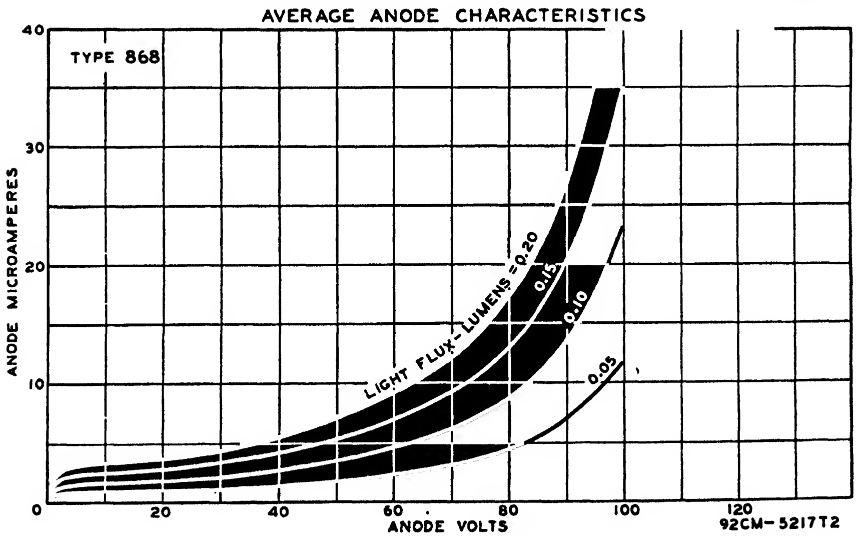

FIG. 25-13.—Characteristics of a type 868 gas phototube. (*Courtesy Radio Corporation of America.*)

resistor across 90 volts dc. A 60-watt Mazda lamp is placed 12 in. from the cell and is then moved away from the cell in steps of 2 in., until it is 24 in. away. Plot the voltage appearing across the load resistor as a function of the distance.

25-4. A photoelectric circuit is to operate when a 300-watt lamp located 20 ft from the phototube is turned on. For reliable operation of the circuit a light-flux change of 0.1 lumen is required. Assume that conditions do not permit the placing of a lens in front of the light source (a street lamp, for instance, and its being turned on is to operate the lights in a store window). What size of lens will be required in front of the cell?

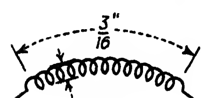

FIG. 25-14.—Approximate dimensions of the filament of a Mazda A-87 lamp.

25-5. The filament of a 15-cp automobile dome light bulb (Mazda A-87) consists of a spiral approximately $\frac{3}{16}$ in. long and 0.020 in. in diameter, as shown in Fig. 25-14. For the purpose of calculations consider the filament as a straight spiral $\frac{3}{16}$ in. long.

a. A 2 in. diameter, 3 in. focal length lens is used in front of the bulb to direct a beam of light toward a photocell 15 ft

away. A 3 in. diameter, 4 in. focal length lens is used ahead of the cell. Assume that each lens causes a 25 per cent loss of useful radiation. What light flux can be expected on the cell?

b. What light flux would the cell receive without any lens system, if its cathode area is 1 sq in.?

25-6. In the circuit shown in Fig. 25-15 a 6SN7 double triode is used in connection with a type 917 vacuum phototube.

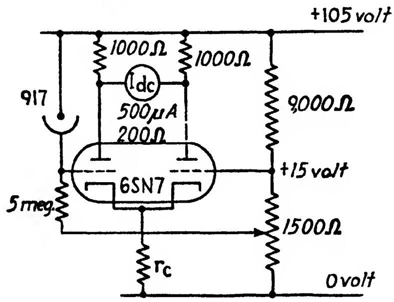

Fig. 25-15.—Phototube circuit of Prob. 25-6.

a. What light flux will be required on the 917 to make the meter read zero, when the arm of the 1,500 ohm potential divider is at its lowest (*i.e.*, most negative) position?

b. What cathode resistor r_c will be required, if the plate current of each tube at balance (*i.e.*, zero meter current) is to be approximately 2 ma?

c. What light-flux change will be required to make the meter read full scale? (Meter: 500 μa full-scale, 200 ohms resistance.)

25-7.

a. Check the resistor values in the circuit shown in Fig. 25-16 for (1) correct filament current; (2) maximum voltage across the 868, which must not exceed 90 volts.

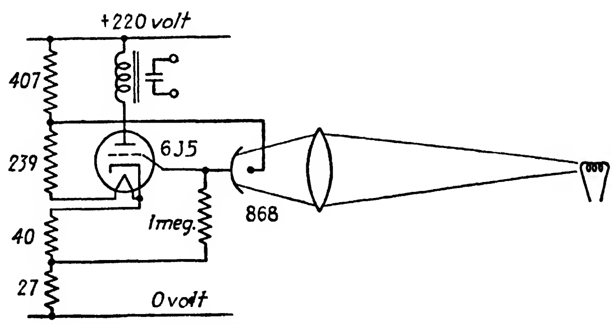

Fig. 25-16.—Dc operated phototube relay of Prob. 25-7.

b. The relay requires 8 ma for operation. The coil resistance is 2,500 ohms. A 4 in. diameter, 6 in. focal length lens is used to concentrate the light of a 100-watt lamp on the cell. How far can the lamp be from the lens so that relay operation results when the lamp is turned on and off?

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CHAPTER XXVI

CATHODE-RAY TUBES AND CATHODE-RAY-TUBE PATTERNS

26-1. Fundamental Principle of the Cathode-ray Tube.—Probably no other single electronic tube has become of more importance to the electrical engineer than the cathode-ray tube, and with television just "around the corner," it will become of great importance to the general public also. Complete instruments incorporating this tube, known under the name "cathode-ray oscilloscopes," are often referred to as "oscilloscopes" or even simply as "scopes" and have become an indispensable tool for the electrical engineer, especially for the electronic engineer. In the observation and analysis of the phenomena taking place in high-frequency circuits, as well as for the observation of transients, the oscilloscope is a most convenient tool, having no substitute.

Cathode-ray tubes are among the oldest members of the family of electron tubes. The first one was made in 1897 by Braun in Germany, who used it for oscillographic purposes. The tube has already been mentioned in Chap. VI, where the description of its operating principle was used for the purpose of explaining the mechanism of acceleration of electrons in an electric field. The reader should refresh his memory on this subject by referring back to Figs. 6-1 and 6-4, and Secs. 6-8 to 6-11. The cathode-ray tube, as shown in this earlier discussion, consists of a cathode furnishing electrons, and a circular-shaped anode with a hole in the center. When voltage is applied between the two electrodes, an electric field is established between the cathode and the anode, and the force acting on the electrons causes them to accelerate, according to the fundamental laws of mechanics. When they reach the anode, they have a speed given by Eq. (6-1). Most of the speeding electrons will strike the anode, but some of them will pass through the hole in the center of it and keep on going with this speed on the other side of the anode until they strike the opposite wall of the tube. In a cathode-ray tube this wall is covered with fluorescent material which lights up at the point where the electrons strike it; thus a luminous spot is produced at the point of impact. Figure 6-4 shows the mechanical analogy of the phenomenon discussed so far.

26-2. Completion of the Circuit for the Beam Current.—The first question which may arise in connection with the two figures and which has not been touched on in the previous discussion is the following. Traveling electrons represent an electric current. Now it is clear that those electrons striking the anode in Fig. 6-1 will return to the positive terminal of

the source of voltage, in this case the battery, making up for those electrons leaving the negative terminal of the battery and going to the cathode for emission. In other words, for these electrons the circuit is closed. But how about those that scoot through the hole in the anode and finally reach the screen? Since they represent a current too, we seem to have here the astounding phenomenon of a current going to a point (the screen), without returning from it! Has Kirchhoff's law been abrogated?

If the battery in Fig. 6–1 gives out more negative charges on its negative terminal than it receives on its positive terminal, and if the screen on the other hand receives continuously negative charges without losing any, it is clear that the potential of the screen will become more negative while the potential of the combination consisting of the supply battery and the electrode system as a whole will become more positive. This means that an electric field will appear between screen and anode. This field will continue to increase in strength until the screen is as negative with respect to the anode as the cathode is. The electrons passing through the hole in the anode would then find themselves in an electric field opposing their motion, or, in the mechanical equivalent shown in Fig. 6–4, the right-hand board would be raised to a level exactly as high as the cathode. Under this condition the marbles passing through the gate in the center would have to roll uphill, and if the hill is just as high as the one that they came down, they will not be able to reach its top. The conclusion therefore is that the cathode-ray tube cannot work, or at least only for a very short instant, until the screen is charged up. It is very fortunate that the inventor of the tube did not know about this; otherwise, we would have no cathode-ray tube. The phenomenon that saves the situation here is secondary emission. The high-speed electrons contained in the beam striking the screen liberate electrons by secondary emission; of course, if there were no electric field at all between screen and anode, these emitted secondary electrons would have no tendency to move. Therefore the first high-speed electrons arriving at the screen actually cause the latter to be charged up negatively. In the electric field thus established between the screen and the anode the secondary electrons will now move toward the anode and an equilibrium will be reached when the number of secondary electrons arriving at the anode is exactly equal to the high-speed electrons contained in the beam. The mechanical analogy of the cathode-ray tube is then shown in Fig. 26–1. In the space between screen and anode we now have marbles moving in both directions, the high-speed ones rolling uphill and striking the screen and the secondary low-speed ones traveling in the opposite direction. Since the brightness of the spot on the screen depends on the speed with which the electrons strike it, it is evident that a loss of brilliancy occurs, owing to the fact that the screen assumes a negative potential with respect to the anode. Measurements on actual tubes indicate that this voltage developed between the screen and the

anode is approximately 50 volts. In modern tubes this loss is usually avoided by providing a graphite coating on the inside walls of the tube between the screen and the anode. This provides a conducting path between the screen and the anode, and a glance at Fig. 6–1 indicates that

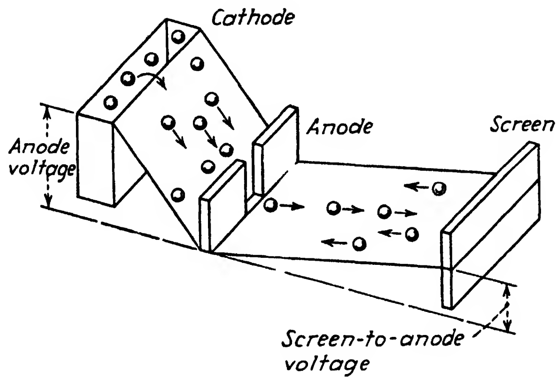

FIG. 26-1.—A mechanical analogy of the cathode-ray tube. Note electrons traveling in opposite directions in the region between the anode and the screen.

under such a condition the "lost" current going to the screen now returns to the anode in exactly the same way as those electrons striking the anode itself.

26–3. Electron Gun; Focusing the Beam of Electrons.[1]—The electrode system producing the beam of high-speed electrons (in the simple case of Fig. 6–1, the cathode and the anode) is sometimes referred to as the "electron gun," which is admittedly a very descriptive term, since the arrangement has no other purpose than that of shooting electrons in a beam at the screen. The electron gun in a modern cathode-ray tube bears little resemblance to the simple arrangement shown in Fig. 6–1. For reasons that will become clear later in this chapter, it is desirable to have the spot as small and at the same time as brilliant as possible. With the simple arrangement shown in Fig. 6–1 the size of the spot obviously depends on the size of the circular hole in the anode and the size of the emitting spot of the cathode. Even if the electrons were to emit in a perfectly parallel beam through this hole, the interaction of the electric fields surrounding each one would cause a spreading of the beam. (According to the old way of explaining it, they would repel each other.) Therefore a simple arrangement, as shown in Fig. 6–1, would result in a rather dim circle of light on the screen. Much effort was therefore spent to reduce the size of the spot. Evidently, in order to have a small spot, it is not necessary that the beam has a small cross section all the way along but it would be just as satisfactory if the beam converged at only one spot at the instant of striking the screen. The similarity of this problem with the one faced by an optical designer, trying to make a lens for a camera, is striking.

The light rays coming from one point of the subject strike the whole surface of the camera lens, and the action of the lens is to collect these rays and bring them to a focus at the spot where the film is disposed. In a similar way, it would be entirely satisfactory to start with a divergent beam of electrons coming from the cathode if it is only possible to bring the beam to a focus at the place of the screen. From the equivalent term in the optical field, this procedure is called "focusing the beam." There are several methods available for focusing the electron beam. In the first method, which is due to van der Bijl, a small amount of gas is included in the tube and the method is therefore called "gaseous focusing." The action taking place is not quite clear, depending on the density of the electron beam and the gas pressure in the tube. It seems that the high-speed beam of electrons ionizes the gas, and the ions thus produced, which find themselves in an essentially field-free region and have therefore no tendency to move, form some kind of channel and keep the beam of electrons together.

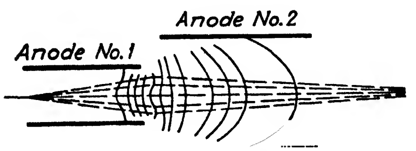

FIG. 26-2.—A divergent beam of electrons may be focused by applying proper voltages to two cylindrical anodes.

Since the gas pressure in the tube cannot be changed, once it has been sealed, focusing is accomplished by adjusting the density of the electron beam until the desired result of a sharp spot is obtained. The density of the electron beam is controlled simply by adjusting the temperature of the cathode (and therefore controlling its emission), which in turn is accomplished by regulation of the filament current. Gas focusing is no longer used but has been replaced by either electrostatic or electromagnetic focusing. The focusing system of the electron gun in a modern cathode-ray tube usually consists of two or more concentric cylinders as shown in Fig. 26–2; they are sometimes referred to as anode 1 and anode 2. When the voltages applied to these two cylinders are in a certain relation to each other, the electric field produced by this arrangement causes the divergent beam of electrons to be bent similarly to the way in which a beam of light rays passing through a lens system is bent. If one of these voltages is made adjustable, usually the one applied to anode 1 (which is at a potential somewhere between the emitting cathode and the main anode 2), the spot can be brought to a sharp focus on the screen of the tube. The ability to produce a small and brilliant spot is, however, not the only improvement offered by the modern electron gun over the arrangement shown in Fig. 6–1. For the investigation of certain transient phenomena, as well as for the purpose of timing, it is desirable to interrupt the beam completely and quickly. Although this could theoretically be done simply by removing the anode-to-cathode voltage, it can be achieved with much less trouble by inserting a control grid between the cathode

and the first anode. This acts then just as the grid in any triode or other vacuum tube would act. All cathode-ray tubes manufactured at present are equipped with such a control grid, which has been found a very valuable improvement over the older types.

It is also possible to achieve focusing by a magnetic field parallel to the axis of the beam. Although this method has been used by other devices operating with electron beams, such as the electron microscope, it has not been used in connection with cathode-ray tubes.

26-4. Deflection of the Electron Beam by Electrostatic or Magnetic Fields.—The usefulness and value of the cathode-ray tube are due to the fact that the traveling electrons making up the beam produced by the electron gun can be acted on by electric or magnetic fields while they are in flight between the muzzle of the gun and the screen. If the force acting on them is at right angles to the direction of their flight, the beam will be deflected and will hit the screen at a spot displaced from the position where it hit without such action. The most convenient way of deflecting the beam consists of letting it pass between plates, called "deflection plates," to which a voltage is applied. The electric field between these two plates is at right angles to the flight of electrons, and the electrons will therefore be accelerated toward the positive plate during the time of their flight between the two plates. When they leave the field between the two deflection plates, they therefore have a velocity at right angles to the original direction of their flight and the spot will therefore be displaced by the distance h, as indicated in Fig. 26-3. Let the length of the deflect-

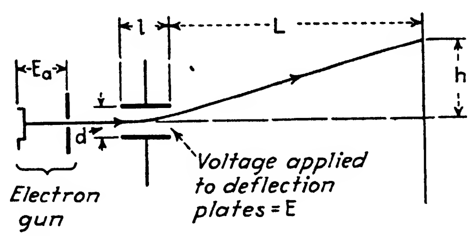

FIG. 26-3.—The amount of deflection of the beam depends on the geometry of the tube and the applied voltages.

ing plates in the direction of the flight of electrons be l and the distance between the deflection plate and the screen be L. The deflection h produced by the application of a voltage E to the two deflection plates will then evidently be

1. Proportional to the electric field existing between the two deflection plates, and therefore also to the voltage applied to these plates, because this will determine the acceleration and also the speed at right angles to the original flight that the electrons have when emerging from the region between the two deflection plates.

2. Proportional to the time that the electrons spend between the two deflection plates. This time will clearly be proportional to the length l of the deflection plates and

inversely proportional to the speed with which the electrons traverse the region between the deflection plates. If no force has been acting on them in the direction of their flight in the time between leaving the end of the electron gun and entering the region between the deflection plates, this speed will evidently be the same as the one they had when leaving the electron gun.

3. Proportional to the time that the electrons spend in the space between the deflection plates and the screen, because during this time the crosswise velocity of the electrons acquired in the region between the deflection plates has a chance to move the electrons sideways. The time for the electrons to traverse the distance L between the deflection plates and the screen is evidently proportional to this distance and again inversely proportional to the speed of the electrons.

Let V be the speed with which the electrons emerge from the electron gun; this speed is proportional to the square root of the voltage existing between cathode and final anode and is given by Eq. (6–1), when the final anode voltage E_a is substituted into this equation. If the distance between the two deflection plates is d, then the electric field existing between the two plates with the voltage E applied to them is given by E/d volts per cm. With these relations the deflection h is given by

$$h = c_1 \frac{E}{d} \frac{l}{V} \frac{L}{V} \text{ cm} \tag{26–1}$$

where c_1 is a proportionality factor, and the three fractions following it are the values given above under paragraphs 1, 2, and 3. If we now insert in Eq. (26–1) the relation given by Eq. (6–1), *i.e.*, that the speed V of the electrons is proportional to the square root of the accelerating voltage, Eq. (26–1) will take the form

$$h = c_2 \frac{ElL}{dE_a} \tag{26–2}$$

where c_2 is another proportionality factor due to the substitution of E_2 for the two V's in Eq. (26–1).

It is worth while to investigate Eq. (26–2) because it shows the influence of the various factors on the amount of deflection obtainable with a given tube. The quantities l, L, and d are, of course, given by the geometry of the tube itself; they show that the longer the tube and the longer the deflection plates in the direction of the travel of the electrons, the larger will be the deflection for a given voltage applied to the deflection plates. The user has, of course, no control over these factors, but it can be assumed that for a given size of tube the manufacturer has chosen the best combination of them, *i.e.*, the one giving the highest deflection sensitivity.

Equation (26–2) indicates that the deflection obtained with a given voltage applied to the deflection plates will be inversely proportional to the accelerating voltage E_a. The sensitivity of a given cathode-ray tube can therefore be increased by decreasing the voltage applied to the final anode which, as we have seen, determines the speed of the electrons. An

increase of sensitivity by this method is not obtained without a price. The brilliancy of the spot produced on the screen depends on the speed with which the electrons strike it, and it is therefore quite evident that a reduction of anode voltage will also be accompanied by a reduction of the brilliancy of the spot.

26-5. Deflection Sensitivity of Standard Commercial Cathode-ray Tubes.—The engineer who wishes to use a cathode-ray tube for his problems must know how much voltage is required on the deflection plates in order to obtain a given deflection. The maximum voltage that could possibly be of use is the one that will deflect or move the spot from the center of the screen to its edges. For reasons to be seen later, it is usually not possible to make use of the full possible deflection; therefore a voltage capable of moving the spot approximately 70 per cent of the distance from the center to the edge of the screen can be considered as the practical maximum value for any particular problem. For present-day commercially available cathode-ray tubes, operating with rated voltages on the various electrodes, it takes in the order of 120 to 180 volts applied to the deflection plates to move the spot 70 per cent of the distance from the center to the edge of the screen. These figures cover cathode-ray tubes with 1 to 5 in. diameter screens.

26-6. Equivalence between a Cathode-ray Tube with One Set of Deflection Plates and a String Oscillograph.—If a cathode-ray tube had only one set of deflection plates, the spot could obviously be moved in only one direction. We could orient the tube, for instance, in such a way that an application of voltage to the set of deflection plates would move the spot horizontally to either the right or the left, depending on the polarity of the voltage applied to the deflection plates. Those who are familiar with an ordinary string oscillograph will remember that such an instrument produces a spot of light moving horizontally when a current passes through the string galvanometer. When an alternating current passes through the string, the spot of light moves rapidly back and forth. An observer will obtain the impression of a solid line, owing to the persistency of vision. The length of the line is, of course, determined by the positive and negative peak values of the alternating current flowing through the string. If a string oscillograph is to be used for the analysis of ac waves,

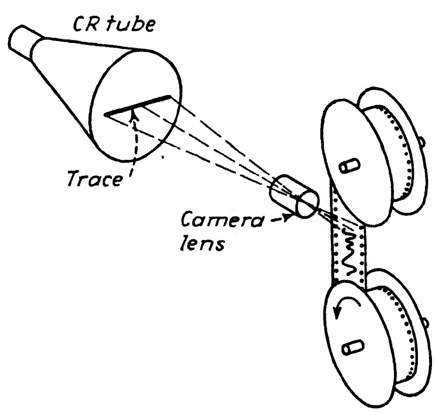

FIG. 26-4.—A cathode-ray tube with the beam deflected in only one direction will serve the same purpose as an ordinary string oscillograph.

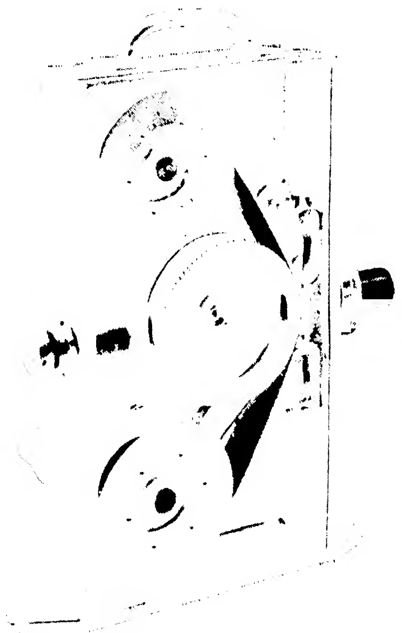

FIG 26-5a —The principle shown in Fig 26-4 can be put to practical use by means of this type of camera (*Courtesy General Radio Company, Cambridge, Mass.*)

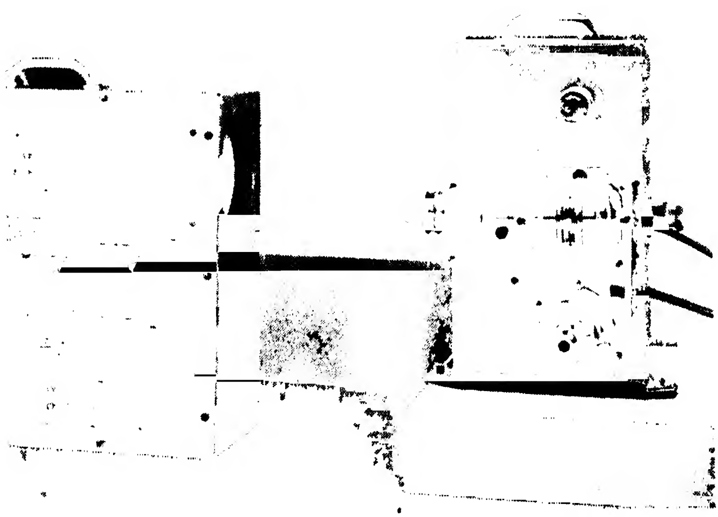

FIG 26-5b —Arrangement of camera and C-R oscilloscope Horizontal deflection plates only are used. (*Courtesy General Radio Company Cambridge, Mass.*)

a "time axis" must be provided. For visual observation, a rotating mirror is usually employed, while, for the purpose of obtaining a record or so-called "oscillogram," the light beam is made to strike a film moving with constant speed at right angles to the deflected beam. Evidently a cathode-ray tube with only one set of deflection plates would already be a very useful device, because it is entirely equivalent to a string oscillograph. We can set up in front of the screen a camera that produces a picture of the spot on a film moving with constant speed. The principle of such a system is illustrated schematically in Fig. 26–4, and Fig. 26–5 shows the photograph of a commercially available camera for this purpose. The main advantage of this combination of cathode-ray tube and moving film-type camera over the conventional magnetic string oscillograph is the much higher frequency response obtainable with it; its disadvantage is the difficulty in obtaining the records of several phenomena on the same film simultaneously. One prominent manufacturer has released a cathode-ray tube with two electron guns and two sets of deflection systems, which makes the recording of two phenomena possible. Experimentally cathode-ray tubes with three systems have been made, so that it is quite possible that instruments incorporating such tubes will in the near future invade the field where the mechanical oscillograph was used.

26–7. Cathode-ray Tube with One Set of Deflection Plates as an Indicating Instrument.—With only one set of deflection plates, the cathode-ray tube can obviously be employed as a voltmeter if the relation between the peak value and the rms value of an alternating voltage is known. A sinusoidal voltage of 100 volts rms has an amplitude of 141 volts. If such a voltage is applied to the deflection plates of a cathode-ray tube and the sensitivity is such that the application of 100 volts dc to the deflection plates would cause the spot to move 1 in. from its zero position, then evidently the application of the alternating voltage will produce a line 2.82 in. long, since the spot will then move 1.41 in. in one direction during the half cycle of a given polarity of the alternating voltage but an equal distance in the opposite direction during the half cycle of opposite polarity. Such a cathode-ray tube voltmeter has two distinct advantages over an ordinary voltmeter: (1) Its impedance is extremely high since it consists practically only of the small capacity existing between the two deflection plates. (2) Its indication (*i.e.*, the length of the produced line) is independent of the frequency of the alternating voltage, up to frequencies as high as 20 megacycles. These remarks apply, of course, only if the voltage to be measured is of sufficient magnitude to be connected directly to the deflection plates. If one has to observe much smaller voltages, it is necessary to amplify them before they can be applied to the deflection plates. In such a case it is evident that the frequency characteristic of the amplifier and its input impedance may be the limiting factors. This subject will be discussed in somewhat more detail later.

26-8. Deflection of the Beam in Two Directions at Right Angles to Each Other; Magnetic Deflection.—Although a cathode-ray tube with only one set of deflection plates is a valuable device, as discussed in Sec. 26-7, its usefulness is tremendously increased by the addition of a second set of deflection plates arranged in such a way as to deflect the beam at right angles to the deflection produced by the first pair. All commercially available cathode-ray tubes are therefore equipped with two sets of deflection plates, or other means of producing deflections in two directions perpendicular to each other.

The closing statement of the preceding paragraph intimated that the beam may be deflected by means other than an electrostatic field. The beam of traveling electrons represents an electric current. When a conductor carrying an electric current is placed in a magnetic field oriented at an angle of 90 deg with respect to the current (such as is the case with the conductors disposed in the slots of the armature of an electric motor), a force will act on the conductor at right angles to the direction of the magnetic field and of the electric current. In a similar way, a beam of traveling electrons will be acted upon by a magnetic field. Cathode-ray tubes are available in which either one or both sets of deflection plates have been omitted so that magnetic coils can be placed near the neck of the tube. This method of deflecting the beam may be convenient for some particular problems, but most problems are solved by means of electrostatically deflecting tubes.

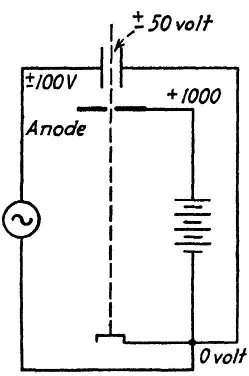

FIG. 26-6.—This circuit would be inoperative because, with the potential of the deflection plates near cathode potential, a strong retarding field would exist between them and the anode.

26-9. Methods of Applying the Deflecting Voltage to the Deflection Plates.—The method of applying the voltage to be investigated to the deflection plates deserves a little more attention than it usually receives. Suppose that we intend to place a voltage of 100 volts dc on the deflection plates. The question now arises whether this voltage must have any particular relation to the voltages of the other electrodes, or whether it can be entirely independent of them. In the discussion of vacuum triodes or pentodes it was our custom to consider the potential of the cathode as the reference point. Suppose we retain this custom for a moment, calling the potential of the cathode of the electron gun zero and assuming that the final anode operates at a potential of +1,000 volts. Now let us assume that we connected one of the deflection plates to zero, *i.e.*, to the cathode, which will make the other plate either +100 or −100, depending on the polarity of the voltage under observation. The potential along a line midway between the two deflection plates will then be either +50

or −50 volts. This condition is indicated in Fig. 26–6. What would such a potential distribution do to the beam of electrons? A glance at Figs. 6–1 and 26–1 reminds us that, in order for the electrons to keep on traveling with the original speed with which they emerge from the gun, they must not be forced to run uphill; in other words, no electric field must exist *in the direction of their flight*. This condition is obviously not satisfied with an arrangement shown in Fig. 26–6. At the point of emergence from the electron gun, the potential is +1,000, while in the region between the deflecting plates it has dropped to +50 or has even become negative. In the latter case, the electrons would evidently come to a standstill before ever entering the region between the deflection plates; in the former case, practically all the speed that they had at the point of emergence from the electron gun would be lost. In other words, with the anode at a potential of 1,000 volts and the deflection plates at an average potential of only a few volts (either positive or negative), a very strong electric field exists between the anode and the deflection plates *in the direction of the flight*, and with such a polarity as to decelerate the traveling electrons. In order to avoid this condition, a connection as shown in Fig. 26–7 is obviously much more satisfactory. In this figure one of the deflection plates is seen to be connected directly to the anode; with a deflecting voltage of plus or minus 100 volts between the two plates, the potential at a point midway between the two plates is now seen to vary between +950 and +1,050; or if we consider only what happens to the electrons *after* they emerge from the second or final anode, they will be subjected to an additional average voltage of 50 volts, accelerating or decelerating them, depending on the polarity of the applied voltage. Equation (26–2) showed that the deflection produced by a given voltage between the deflection plates was inversely proportional to the square of the speed of the traveling electrons. With the arrangement shown in Fig. 26–7 the electrons go faster when the left-hand deflection plate is positive with respect to the right-hand plate than when it is negative. The two deflections produced in opposite directions with the same voltage applied will therefore not be the same. Although this does not mean that the tube cannot be calibrated properly, it is a decided handicap when it is intended for precision observations.[2]

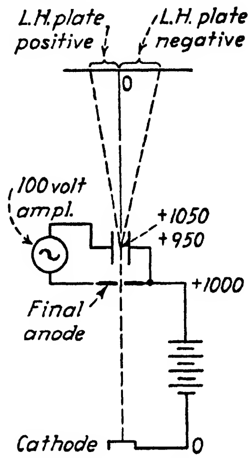

FIG. 26–7.—With one deflection plate connected to the anode, the average potential in the center region between the deflection plates will vary, with a consequent change in speed of the electrons in the beam. This results in unequal sensitivity in opposite directions of deflection.

If the two cases shown in Figs. 26–6 and 26–7 are clearly understood,

it is not very hard to see that the most desirable state of affairs is obtained when the center point between the two deflection plates has the same potential as the anode. The most desirable—but not always obtainable—application of the voltage to the deflection plates is therefore to provide a center tap of the deflecting voltage and to connect this point to the anode. Such an arrangement will make one deflection plate negative, the other one an equal amount positive with respect to the anode, with their average potential with respect to the anode being zero. A circuit showing this condition is illustrated in Fig. 26–8. Earlier cathode-ray tubes, and even today's less expensive tubes, do not permit the connection shown in Fig. 26–8 because the connection of one deflection plate of each set to the anode as shown in Fig. 26–7 is accomplished inside the tube. In the case of such tubes, we must, of course, be satisfied with the results obtainable with the circuit shown in Fig. 26–7. It may be of interest to point out that the circuit arrangement shown in Fig. 26–8 is a true push-pull arrangement, since the voltages of the two deflection plates with respect to the anode are 180 deg out of phase (*i.e.*, when one of them is positive, the other is negative).

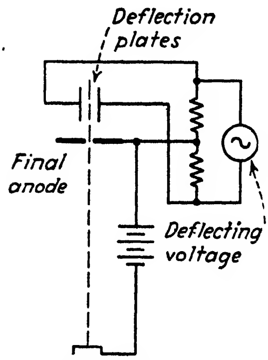

Fig. 26–8.—The most satisfactory way of applying a signal to the deflection plates is shown here. The average potential of the two deflection plates is equal to the anode potential.

26–10. Point of Grounding a Cathode-ray-tube System.—In amplifier circuits the cathode is usually grounded. This is evidently a very logical procedure because the signal is applied to an amplifier tube between cathode and grid, and the grounding of the cathode permits the signal to be grounded at one terminal, or at least to be very close to ground potential. In the case of the cathode-ray tube the attempt to introduce the signal with one side connected to the cathode, as shown in Fig. 26–6, was a failure. As shown in Figs. 26–7 and 26–8, it is necessary to introduce the signal with one side (or its center tap) connected to the anode. In order to avoid dangerous voltages between the voltage to be connected to the deflection plates and ground, it is therefore common practice to connect the *anode* of the cathode-ray tube to *ground.* This will evidently place the cathode and the filament of the tube at a high negative voltage with respect to ground. Since, as a rule, no control voltages have to be introduced with respect to the cathode, this requirement presents no great handicap, except that the transformer winding supplying the heating current for the filament of the cathode-ray tube must be insulated from ground for a voltage equal to the anode-operating voltage of the tube. In contrast to the usual arrangement, it is now the *positive* end of the dc supply system that is grounded, while the negative end is "way down in the basement."

26-11. Typical Power-supply System for a Cathode-ray Tube.—With the foregoing discussion in mind, the reader will have little difficulty in understanding a typical oscillograph circuit, as shown in Fig. 26-9, incorporating a 3AP1 cathode-ray tube. Since only a small amount of current is necessary, a half-wave rectifier circuit is entirely satisfactory. A single capacitor is usually sufficient to provide satisfactory filtering. This particular tube is an inexpensive one, and one plate of each pair of deflection plates is internally connected to anode 2, and the circuit therefore has the

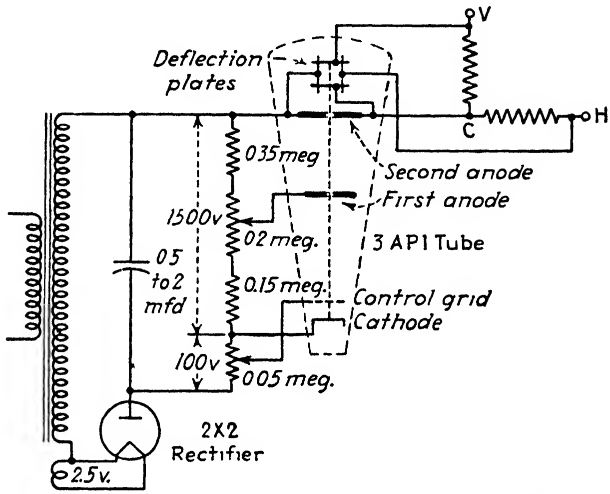

FIG. 26-9.—The only equipment needed to place a cathode-ray tube into operation is a transformer, a rectifier tube, a capacitor, and a few resistors.

objectionable features discussed in connection with Fig. 26-7. If the voltage applied to the deflection plates does not offer a path for direct current, the two deflection plates should be connected through a resistance. This is necessary because stray electrons of the beam may strike the deflection plates and cause them to charge up. A charge accumulating thus will cause the spot to drift to one side or, if the pair of plates as a combination begins to charge, to affect the speed of the electron beam. For a tube with all four deflection plates brought to separate terminals, the circuit would look like Fig. 26-9, except that the two input circuits are modified as shown in Fig. 26-8.

A circuit as shown in Fig. 26-9 is all that is needed to put a cathode-ray tube into operation. It will cause a spot to appear at or near the center of the screen, and application of a voltage between terminals C and V will move the spot in a vertical direction while application of a voltage between terminals C and H will move the spot in a horizontal direction. Whether the spot is moved up or down, and right or left, respectively, will depend on the polarity of the voltage applied to terminals V and H, respectively.

Commercially available cathode-ray oscilloscopes contain more circuit elements than shown in Fig. 26–9. This is due to the fact that the circuit shown in Fig. 26–9, although entirely sufficient in itself and ready for operation, can have its usefulness increased by the addition of a number of auxiliary circuits. These will be discussed in Chap. XXVII.

26–12. Screen of a Cathode-ray Tube as an Electrical Curve-plotting System.[3]—A cathode-ray tube with proper accelerating and focusing voltages applied represents a device that furnishes a small brilliant spot on a circular field, *i.e.*, the screen of the tube. With no voltage applied to the input terminals *CV* and *CH* of Fig. 26–9, the spot will be at or near the center of the field. In future discussions this position will be referred to as the "zero" position. Application of voltage to the two sets of input terminals will move the spot vertically or horizontally, respectively.

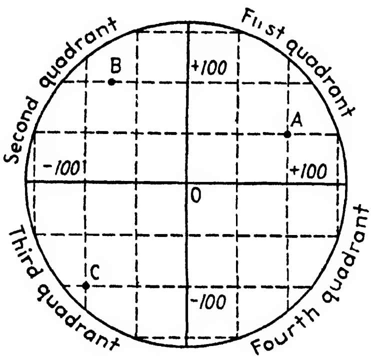

FIG. 26–10.—The screen of a cathode-ray tube can be considered as an electrical plotting system. The spot can be placed anywhere on the screen by the application of proper voltages to the horizontal and vertical deflection plates.

When we plot a curve on cross-section paper, every point on the curve is completely defined by its abscissa and its ordinate. (These two values are also called the "coordinates" of a point.) It is quite apparent that a cathode-ray oscillograph is in reality an electrical plotting system since the luminous spot can be placed at any point of the screen by the application of appropriate voltages to the deflection plates. Let us assume for simplicity's sake that we have a tube in which the application of 100 volts to the deflection plates causes a deflection of 1 in., and let Fig. 26–10 represent the view of the screen of this tube. Making terminal *H* 100 volts positive and terminal *V* 50 volts positive with respect to terminal *C* will place the spot at point *A* in Fig. 26–10, because this point has an abscissa of 100 volts and an ordinate of 50 volts. To place the spot at *B* would require −75 and +100 volts on input terminals *H* and *V*, respectively; the spot would appear at *C* with −100 volts applied to both terminals. Let us suppose that we want to obtain a graph showing the relation between the voltage applied to a given diode and the current flowing through it. The usual procedure is to vary the applied voltage and note the current values belonging to several voltage values and then to plot corresponding pairs on cross-section paper. But, if we desire, we can obtain this curve directly on the screen of a cathode-ray tube by the following procedure. The voltage applied to the diode is also applied to the horizontal deflection plates. The spot will therefore move horizon-

tally to a position determined by the voltage applied to the diode; in other words, its abscissa is the voltage applied to the diode. The current through the diode is made to flow through a resistance (a shunt), and the voltage across this resistance is therefore proportional to the current. The voltage appearing across the resistance is applied to the vertical deflection plates and determines therefore the ordinate of the point on the screen. A circuit accomplishing this is shown in Fig. 26–11, although it is probable that the voltages may have to be amplified first before being applied to the deflection plates. When we vary the voltage applied to the series combination of diode and resistor, the spot on the screen will describe the desired curve. If we vary the applied voltage slowly, we shall see the spot traveling slowly through the curve; if the applied voltage is changing rapidly—say, when we simply apply a 60-cycle alternating voltage to the combination—the spot will travel back and forth through the curve 60 times a second. Now it is a well-known fact that the human eye retains a light impression for a period of approximately $\frac{1}{20}$ sec. If a point of light, such as a glowing cigarette or a flashlight, is moved quickly through a circle for instance, the observer will obtain the impression of a line rather than that of a point. With a fast-changing voltage applied to the circuit shown in Fig. 26–11, the human eye will therefore see on the screen, not a spot, but the entire curve or graph at once. When the quick-moving spot produces what appears as a line on the screen of the cathode-ray tube, it is said to produce a "pattern." We shall now investigate some of the more common patterns appearing on the screen of the cathode-ray tube when the two voltages applied to the two pairs of deflection plates bear certain relations to each other.

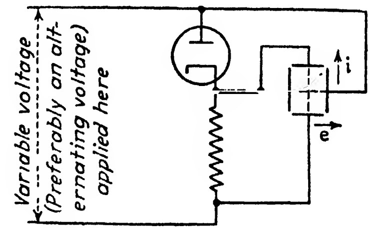

Fig. 26–11.—A circuit that will plot the volt-ampere characteristics of a diode on the screen of the tube.

26–13. Conditions for a Straight-line Pattern.—The simplest case will obviously result when the terminals H and V are connected together, and a voltage is applied between them and the common terminal C. No matter whether the applied voltage varies slowly or rapidly and no matter what its wave shape may be, with this connection there will always be the same voltage applied to the vertical and to the horizontal set of deflection plates. If the deflection sensitivity of the two sets of deflection plates is equal, this means that the ordinate of any given point that the spot may occupy is equal to its abscissa. This evidently means that the spot will move along a 45-deg line. How far the line will extend into the first and third quadrants will depend on the magnitude of the positive and the negative peak of the voltage applied to H and V. If the applied voltage is an alternating voltage with a frequency in excess of approxi-

mately 20 cps, the human eye will perceive a 45-deg line as already explained. We shall not be able, however, to form an opinion as to whether the spot is traveling through the 45-deg line with equal speed over all parts of it. The speed with which the spot moves will obviously depend on the wave shape of the applied voltage. If the applied voltage is of triangular wave shape, the spot will move with equal speed over all parts of the pattern. But if it is of sinusoidal wave shape, the spot will move fastest as it passes through the zero point, and travel slower at the two ends of the pattern. Where extremely high speeds are involved, this phenomenon may make itself felt by the fact that various parts of the pattern appearing on the screen are of different brightness. Those parts of the pattern over which the spot travels extremely fast will appear as faint lines, while those traveled at a lower speed will appear more brilliant.

A 45-deg line resulted when the voltages applied to the two sets of deflection plates and the sensitivity of the two sets were equal. If the voltage applied to the vertical plates is always exactly one-half of that applied to the horizontal plates, or if the deflection sensitivity of the vertical plates is adjusted to one-half, but with equal voltages applied to both sets, we shall evidently still obtain a straight line as a pattern, but the angle will no longer be 45 deg but approximately 26 deg (tan 26 deg $= 0.5$). If we apply to the horizontal set of deflection plates a sinusoidally varying voltage with an amplitude of 100 volts, while applying to the vertical set an alternating voltage of the same frequency with an amplitude of 50 volts, and if these two voltages are in phase with each other, *i.e.*, if they reach their maximum at the same instant and pass through zero at the same instant, the conditions just outlined are obviously satisfied and a straight line at an angle of approximately 26 deg will result. It should be noted, however, that the two voltages do not have to be of sinusoidal wave shape; they can be of any wave shape, provided only that it is the same for both and that they are in phase with each other.

What pattern would result if the two voltages were equal or proportional to each other, but of opposite polarity? Obviously, we would again obtain a straight line, but this time it would extend into the second and fourth quadrant. Two alternating voltages of the same wave shape 180 deg out of phase with each other (such as would be obtained between the center tap and the two outside terminals of a center-tapped transformer) would give us such a line, for instance. Therefore if two alternating voltages of unknown characteristics result in a straight-line pattern when applied to the two sets of deflection plates, we can definitely state that they are of equal frequency, equal wave shape, and in phase or 180 deg out of phase with each other, depending on whether the straight line lies in the first and third quadrants or in the second and fourth quadrants, respectively. If the characteristics of one of the two voltages are known, the other one will then also be determined.

26-14. Two Voltages of Sinusoidal Wave Shape and 90 Deg Phase-displaced Applied to the Deflection Systems.—Let us now try to predict the pattern that appears on the screen when the two voltages applied to the deflection plates are two sinusoidal voltages of equal amplitude, but 90 deg displaced with respect to each other. One fact stands out immediately: whatever pattern results, it cannot pass through the zero point. This is evident because, for the spot to be at the zero position, the two voltages must be zero at the same instant. But with two voltages 90 deg

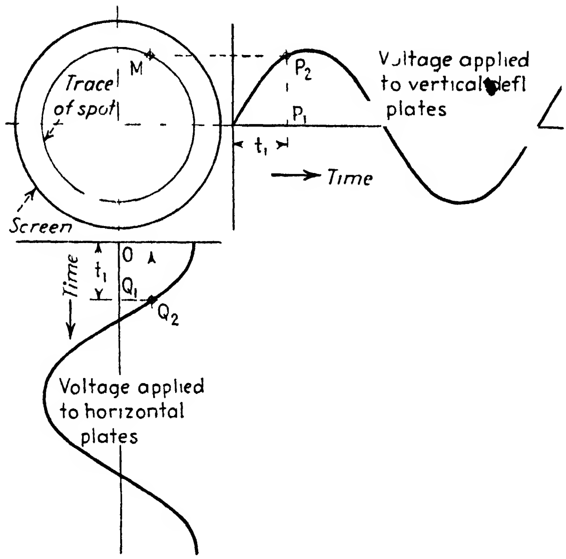

FIG. 26-12.—If the two voltages applied to the two deflection systems are given, it is an easy matter to construct the path that the spot will travel through on the screen.

out of phase applied to the deflection plates, one voltage will be at a maximum when the other one is zero, and vice versa. At the instant, for example, when the spot is deflected a maximum amount to the right, the vertical deflection will be zero; at the instant when the spot is deflected a maximum amount in the upward direction, its horizontal deflection will be zero. In order to obtain the complete pattern that will be produced by the two voltages, the procedure outlined in Fig. 26-12 may be employed. The two voltages are plotted along two axes perpendicular to each other and intersecting at the zero point of the screen. It is clear that with such a method the ordinates of corresponding points on the two curves when projected into the area of the screen will intersect at a point giving the position of the spot on the screen for the particular instant. Thus, at time t_1 the vertical deflection of the spot is given by the length of the line P_1P_2 while the horizontal deflection is given by the length Q_1Q_2, resulting in point M_1 on the screen. If this procedure is repeated for a

number of additional points, the successive positions occupied by the spot on the screen will be seen to lie on a circle. It is quite evident that an ellipse with its main axes in the horizontal and vertical directions will result if the two voltages are of sinusoidal wave shape and 90 deg out of phase, but if their amplitudes are not alike. It means simply that the ordinates of the circle shown in Fig. 26–12, for example, are all reduced in a certain ratio, which is known to convert the circle into an ellipse.

26–15. Determination of Phase Displacement of Two Sinusoidal Voltages from Pattern on Screen.[4]—We have now seen that two sinusoidal alternating voltages of equal frequency applied to two sets of deflection plates will produce a straight line, when they are in phase; and that the pattern will be a circle or an ellipse with its major axes falling in the horizontal and vertical directions when the two voltages are exactly 90 deg out of phase. How will the pattern look when the two voltages have a phase displacement somewhere between the two extreme cases? Let us assume that the amplitudes of the voltages applied to the horizontal and vertical deflection plates are 150 and 100 volts, respectively. If the two voltages are in phase, a straight line *a* will result; in the case of 90-deg phase displacement we shall obtain the ellipse marked *b*. If the voltage applied to the vertical deflection plates lags 30 deg behind the voltage applied to the horizontal deflection plates and we use the method outlined in Fig. 26–12 to find the various positions occupied by the spot on the screen, an ellipse, marked *c* in Fig. 26–13, will be obtained. Suppose now that two sinusoidal voltages of equal frequency when applied to the deflection plates result in an ellipse similar to the one marked *c* in Fig. 26–13. Can we tell the phase displacement of the two voltages with respect to each other from this ellipse? Drawing a vertical tangent to the ellipse marked *c*, and intersecting this line with a horizontal line drawn through the center of the ellipse, give us the length *OQ*, which evidently is the amplitude of the horizontal deflection because it is equal to the farthest excursion of the spot in the horizontal direction. Now consider point *P*. When the spot is at this point, the voltage applied to the vertical plate is obviously zero. If the two voltages were in phase, the horizontal deflection would also be zero at this instant; but in Fig. 26–13 the horizontal deflection at this instant is *OP*, and its ratio to the amplitude is *OP*:*OQ*. Since for a sine wave the ratio of the instantaneous value to the amplitude is

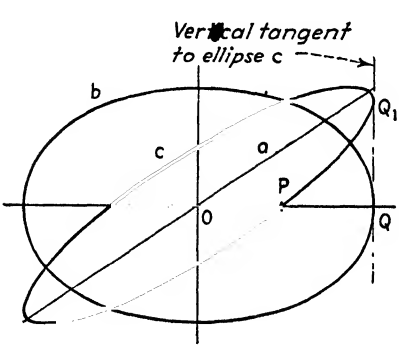

FIG. 26–13.—With two sinusoidal voltages applied to the deflection systems, the phase angle between the two can be determined from the pattern appearing on the screen.

equal to the sine of the angle, the ratio $OP:OQ$ is equal to the sine of the phase angle between the two voltages.

26–16. Voltages of Slightly Different Frequency Applied to the Deflection Plates.—Suppose we apply to the deflection plates two voltages of 100 and 101 cps, respectively. When discussing beat frequencies, we saw that two such voltages can be considered as getting into and out of phase with the "beat" frequency, *i.e.*, with the difference of the two original frequencies. Each successive cycle of the one voltage is one-hundredth of a cycle more displaced from the other wave than the preceding one The pattern on the screen will therefore change slowly from a straight line in the first and third quadrants to an ellipse (or circle), then to a straight line in the second and fourth quadrants, again to an ellipse, and finally back to the original straight line. The pattern will go through a complete cycle of change of appearance with a frequency equal to the beat frequency between the two applied voltages. This phenomenon is the basis of a simple method of determining the frequency of a given alternating voltage by comparing it to a known frequency. A calibrated variable-frequency oscillator must be available for this purpose. The voltage of the unknown frequency is applied to one set of deflection plates, and the variable-frequency oscillator is connected to the other set. The frequency of the latter is then varied until a straight line or an ellipse appears on the screen. or until these two patterns drift slowly from one to the other. At this instant we then know that the frequency of the unknown voltages is equal to that produced by the calibrated oscillator. The method is not confined to the case where the two voltages are of sinusoidal wave shape. One or both of them may deviate from a sine wave, and although under this condition the resulting pattern may not be a straight line or an ellipse, it will nevertheless be a single closed line with no part of it crossing any other part.

26–17. Voltages with Frequencies in a Ratio Such as 1:3 and 3:5 Applied to the Deflection Plates; Lissajous Figures and a Mechanical Method of Visualizing Them.[5]—An interesting subject is the analysis of the pattern that results when the frequencies of two voltages applied to the deflection plates are in certain ratios to each other, such as 1:3, 1:5, or 3:7. The patterns produced on the screen are then known as "Lissajous" figures. Whenever one is faced with problems involving frequency comparisons, a detailed study of this subject, about which much has been written, will be profitable. In this book we shall confine ourselves to presenting a mechanical model, the operation and visualization of which may help toward the interpretation of patterns as they may appear in such cases on the screen of a cathode-ray tube.

In Fig. 26–14 a disk D, which can be rotated, is mounted as shown in front of a screen. The disk carries near its circumference a vertical rod R, which for the purpose of this discussion should be considered as made of transparent material. A ball B of opaque material is carried by the rod

but can slide up and down along the rod. Consider the arrangement illuminated by a light source to the right and far away from the figure. The light rays can then be considered as parallel and B will throw a shadow on the screen indicated by B'. R, being transparent, will not throw any shadow. Now let the disk rotate, but consider the ball as remaining at a certain point on the rod. The shadow on the screen will then describe a horizontal line, the distance of the shadow from the center at any given instant being proportional to the sine of the angle φ, indicated in Fig. 26–14. Of course, an observer of the shadow, seeing it going back and forth, would not be in a position to give the direction in which the disk is rotating or to say whether it is describing at any instant the half circle nearer the screen or the half circle farther away from it. Now assume that, by some mechanism not shown in the figure, B is made to slide up and down along R while the disk is rotating. Let this motion also follow a sine law. If the ball slides up and down once while the disk completes one revolution, the shadow will describe a straight line or one of the ellipses shown in Fig. 26–13, depending on the phase relationship between the motion of the ball along the rod and the horizontal motion produced by the rotation of the disk. Space does not permit a more detailed discussion of this subject, but the reader is invited to give the matter more thought, which should be profitable as well as interesting. As the next step, a mental picture should be obtained of what happens if the ball slides up and down two times, three times, four times, etc., while the disk completes one revolution. After this a more complex relation, such as having the ball slide up and down five times while the disk completes two or three revolutions, may be analyzed. In all these cases the pattern is, of course, again determined not only by the frequency relation of the two voltages but also b· their instantaneous phase relation. With a voltage of 60 cps applied to the horizontal deflection plates and a voltage of 180 cps applied to the vertical plates, for instance, the pattern may look as shown in either Fig. 26–15a or b, with any number of patterns between these two, depending on the phase relationship between the two voltages. The pattern will look the same as the path described by the shadow of the ball in Fig. 26–14, when the disk in this figure is made to rotate with a speed of 60 rpm, for example, while the ball slides up and down on the rod with a frequency

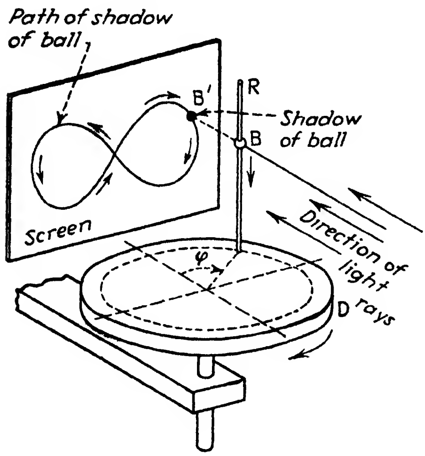

FIG. 26 14.—A mechanical device which will help in visualizing the pattern that will appear on the screen with the application of voltages of different frequencies to the deflection systems.

three times as high. The reader is invited to visualize the pattern shown in Figs. 26–15*a* and *b* with the aid of the model shown in Fig. 26–14. Since problems of this kind do not confront the electrical engineer very often, the short discussion presented here must be considered sufficient.

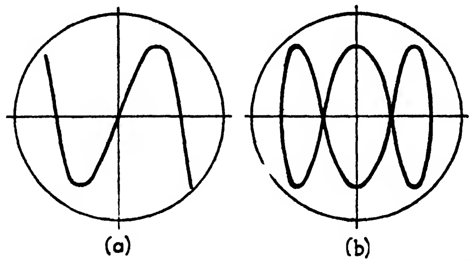

FIG. 26–15.—If the sinusoidal voltage applied to the vertical deflection plate has a frequency three times as high as that applied to the horizontal plate, the patterns shown here may appear on the screen, depending on the phase relationship between the two voltages.

26–18. Linear Time Bases.—Up to now we have applied voltages of sinusoidal wave shape to both sets of deflection plates. The patterns appearing on the screen were straight lines, ellipses, and circles, but we have not yet had a sine wave appear on the screen. It is therefore evident that, although the arrangements discussed so far may be valuable for the determination of phase relationships or for frequency comparisons, they are not suitable for studying the wave shape of a given alternating voltage. The true wave shape of a current or voltage is obtained when we plot its instantaneous values against uniformly progressing time values; in other words, equal time intervals must be represented on the X axis by equal distances. If a 60-cycle sinusoidal voltage, for instance, is plotted in such a way that a complete cycle is equivalent to 12 in., then every inch of length of this graph represents 30 deg, or $\frac{1}{12}$ cycle, or $\frac{1}{720}$ sec. If a 60-cycle sinusoidal voltage applied to the vertical deflection plates of a cathode-ray tube is expected to produce a single sine wave on the screen of the tube, say, 3 in. in length, it will be necessary to move the spot horizontally from a position approximately 1.5 in to the left of the center through a distance of 3 in. with uniform speed. If during this time the spot is moved simultaneously in the vertical direction according to a sine law, it will describe a true sine wave on the screen of the tube. What will have to happen after the spot has reached the extreme right-hand position? We have seen that the straight line, ellipses, etc., were produced by the spot retracing again and again the same path. Therefore, if we expect the next picture of the sine wave to coincide with the preceding one, it is evidently necessary for the spot to return from the extreme right-hand position to the starting point in an infinitely short time. Although this is evidently a physical impossibility, we might be willing to sacrifice the last portion of the sine wave to permit the spot to return to the starting point. In other words, instead of letting it go from left to right for a full $\frac{1}{60}$ sec, as required in order to

record the *complete* sine wave, we might stop its motion from left to right just short of this complete cycle and snitch a small fraction of the cycle to return it to the starting point. From an electrical point of view, it will be necessary to produce a voltage which changes linearly with time for a certain length of time and which is then brought back to the original value in as short a time as possible. The graph of such a voltage against time will evidently look like a saw tooth and is often referred to as a "saw-tooth" wave shape. Circuits that provide saw-tooth voltages for the movement of the spot on a cathode-ray tube in the horizontal direction are often called "sweep circuits." They are discussed in Chap. XXVII.

In the foregoing paragraph we saw that, in order to make a 60-cycle voltage appear as a single wave with its true wave shape, it was necessary to operate the sweep circuit with the same frequency. It is evident that, if we should operate the sweep circuit with 30 or 20 cps, there will appear on the screen two or three complete waves of the 60-cycle voltage. These waves appear to be standing still if the frequency of the sweep circuit is exactly 30 or 20 cps. If this condition is only approximately fulfilled, every succeeding picture produced on the screen will be slightly displaced with respect to the preceding one. It is this type of displacement that produces the illusion of motion in a motion picture. For the same reason, the waves or the pictures on the screen of the cathode-ray tube will appear to be moving if each wave is slightly displaced with respect to the preceding one. In some problems, this slow motion is not objectionable; in others, it is desired to operate the sweep at a frequency equal to, or in a fixed relation to, the phenomenon to be observed.

PROBLEMS

26-1. A neon lamp with a breakdown voltage of 65 volts and an extinction voltage of 55 volts is connected in a circuit shown in Fig. 26-16. Assuming that the deflection sensitivity of both sets of plates is 50 volts dc per in., plot the pattern that will appear on the screen of the tube.

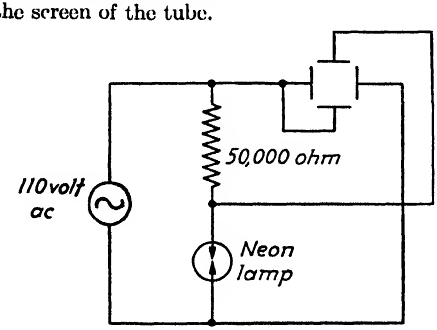

FIG. 26-16.—Diagram of circuit for Prob. 26-1.

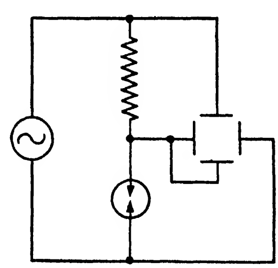

FIG. 26-17.—Diagram of circuit for Prob. 26-2.

26-2. The cathode-ray tube of Prob. 26-1 is reconnected to the circuit as shown in Fig. 26-17. Plot the pattern that will now appear on the screen.

26–3. A voltage with a frequency of 60 cps and a triangular wave shape is connected to the horizontal set of deflection plates of a cathode-ray tube, and a voltage with a frequency of 150 cps and of sinusoidal wave shape is connected to the vertical set of deflection plates. Let the phase relation be as indicated in Fig. 26–18. Assume further that both have the same peak value and that the deflection sensitivity of both sets of deflection plates is the same. Plot the pattern that will appear on the screen.

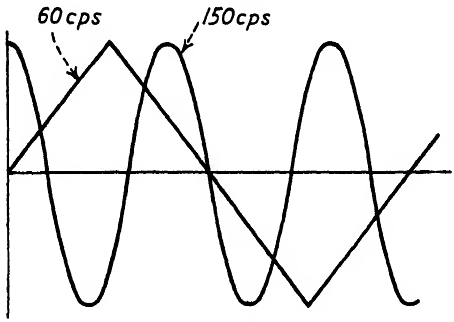

FIG. 26–18.—Wave shape and phase relation of voltages in Prob. 26–3.

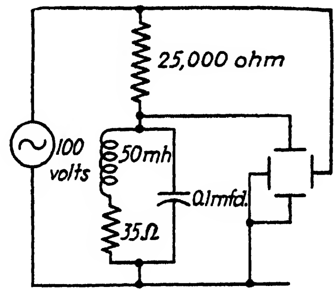

FIG. 26–19.—Diagram of circuit in Prob. 26–4.

26–4. An inductance of 50 mh with a resistance of 35 ohms is combined with a capacity of 0.1 μf in a circuit as shown in Fig. 26–19.

a. At the resonant frequency of the circuit, how will the pattern on the screen look, if the vertical deflection sensitivity is twice as high as the horizontal?

b. How will the pattern look at a frequency 5 per cent above the resonant frequency?

SUGGESTED ADDITIONAL READING

M.I.T. Staff: "Applied Electronics," Chap. 1, pp. 19–35, John Wiley & Sons, Inc., New York, 1943.

Reich, H. J.: "Theory and Application of Electron Tubes," Chap. 15, pp. 628–634, McGraw-Hill Book Company, Inc., New York, 1944.

Terman, F. E.: "Radio Engineers' Handbook," Sec. 16, pp. 322–344, McGraw-Hill Book Company, Inc., New York, 1943.

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2. DuMont, A. B.: Elimination of Distortion in Cathode Ray Tubes, *Electronics*, January, 1935, p. 16.
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4. Bagno, S., and A. Barnett: Cathode Ray Phase Meter, *Electronics*, January, 1938, p. 24.
5. Moss, H.: Cathode Ray Tube Traces, *Electronic Eng.* (London), June, 1944, p. 21; July, 1944, p. 65; August, 1944, p. 109; September, 1944, p. 153; October, 1944 p. 188; December, 1944, p. 285; January, 1945, p. 329.

CHAPTER XXVII

AUXILIARY CIRCUITS; EQUIPMENT FOR CATHODE-RAY OSCILLOSCOPES

27–1. The Two Most Important Additions to Increase the Usefulness of the Cathode-ray Tube.—The circuit shown in Fig. 26–9 illustrates a complete cathode-ray oscillograph. Application of voltages to the two sets of input terminals results in a motion of the luminous spot on the screen. For many problems, such as frequency comparisons, the simple arrangement shown in this figure is entirely sufficient, but most commercial oscilloscopes are equipped with auxiliary circuits, which make it possible to use the instrument in the study of problems that could not be solved with the simple circuit shown in Fig. 26–9.

The most important auxiliary circuit is, of course, an arrangement furnishing a "sweep" voltage or a time base as described in Sec. 26–18. It was stated there that, in order to obtain an alternating voltage in its true wave shape on the screen of a cathode-ray tube, it was necessary to make the spot move in the horizontal direction with a uniform speed, or linearly with time. Most modern cathode-ray oscilloscopes contain a generator producing such a voltage. A switch is provided which permits the connection of the horizontal set of deflection plates to this generator. This does not mean that one has to use the sweep circuit; the two sets of input terminals are brought out and the above-mentioned switch also permits the plates to be connected to the outside terminals for such problems as frequency comparisons.

The second important addition to the circuit shown in Fig. 26–9 is made necessary by the fact that a relatively high voltage is required to produce a satisfactory deflection of the spot. It has been stated that voltages in the order of 100 volts or more are required to deflect the spot over the useful range of the screen on the ordinary cathode-ray tube. To overcome this handicap, it is evidently necessary to provide amplifiers when it is required to study voltages smaller than the values given above. Amplifiers for cathode-ray tubes constitute a problem all by themselves and have received much attention. The important requirements for them will be discussed later in this chapter.

Some of the more refined instruments provide additional auxiliary features such as provision for beam blanking and for the production of a single sweep for the study of transients.

27-2. Fundamental Problem of the Sweep Circuit and a Mechanical Solution.—Returning to the discussion of the sweep circuit, we have seen that this problem is essentially one of producing an alternating voltage with a saw-tooth wave shape, *i.e.*, an alternating voltage that will change linearly with time for a given period and then return as quickly as possible to the starting value of the voltage. When the first cathode-ray tubes were commercially available, a mechanical solution of this problem was proposed. Although this scheme is no longer used, it is worth our while to study it because it shows better than anything else the basic simplicity of the requirement. In Fig. 27-1*a* let a direct voltage E be connected to a center-tapped resistance R_1 and to a potential divider R_2. The horizontal deflection plates of a cathode-ray tube are connected to the two resistors as shown. When the arm of the potential divider is in its lowest position, the right-hand deflection plate will be negative with respect to the left-hand plate, and the spot will be deflected to the left. When the arm is moved from its lower position to the upper position with a uniform speed, the spot will travel with a uniform speed across the screen of the tube. After the arm has reached the upper position, it should be brought back to the lower position in an infinitely short time, or at least in a very short time if we wish to obtain a saw-tooth voltage. By constructing the potential divider R_2 in a circular shape, as shown in Fig. 27-1*b*, and by leaving as small a gap between the start and the finish of the resistance winding as is convenient, a linear sweep may be obtained simply by driving the arm of the potential divider with a constant rotational speed. With the direction of rotation as indicated, it is seen that the arm of the potential divider will become progressively more positive until it reaches the positive end of the resistance wire. It then jumps the small gap between the positive and the negative end of the potential divider, which will bring the spot back to its starting position. This method of producing a linear sweep, although straightforward and very simple from a theoretical point of view, has very severe limitations. If we place a 60-cycle voltage on the vertical deflection plates and wish it to appear as a single stationary wave on the screen, then the sweep will have to pass over the screen once every $\frac{1}{60}$ sec. The arm of the potential divider shown in Fig. 27-1*b* would, therefore, also have to operate with this speed, which means that it would have to be driven by a synchronous motor running at a speed of 3,600 rpm. How long could a sliding contact be maintained in operating condition at this speed? If a sweep circuit constructed along these lines and operating with a frequency of 60 cps presents a problem, the difficulties become in-

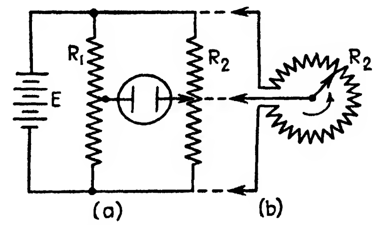

FIG. 27-1.—The spot can be made to sweep from one edge of the screen to the other with a uniform speed by means of the mechanical device shown here.

surmountable when we wish to operate the sweep with a higher frequency. In the study of problems where the cathode-ray oscillograph finds most use, 60 cps is considered a very low frequency. If the sweep circuit had to operate at 600 cps, the motor driving the arm of the potential divider would have to operate at a speed of 36,000 rpm! Even this is still considered a low frequency. It is therefore evident that the mechanical arrangement shown in Figs. 27–1*a* and *b* does not represent a practical solution of the problem.

27–3. Use of Relaxation Oscillators for the Production of the Time Sweep.—When discussing relaxation oscillators, we saw that a resistor-capacitor combination, with a neon lamp shunted across the capacitor, produced oscillations of a triangular wave shape. This fundamental principle is employed in most commercial cathode-ray oscilloscopes equipped with sweep circuits. A neon-tube oscillator gives only a relatively small output; furthermore it cannot be controlled any too easily over a wide range of frequencies. In a practical arrangement, it is therefore always replaced by a gaseous tube, such as the 884 or the 885. The basic circuit is shown in Fig. 27–2. The capacitor C is charged over resistance R from a source of direct voltage E. The voltage across the capacitor will therefore increase along an exponential curve until it reaches the breakdown voltage of the gaseous tube, which will then discharge it to a voltage equal to the arc drop of the gaseous tube. The breakdown voltage of the gaseous tube can be adjusted to any desirable value by means of the grid bias e_c. (In actual circuits this bias voltage is usually obtained either from a bleeder circuit or by a self-biasing resistor.) In the commercially available oscilloscopes the tube is made to fire when the voltage across the capacitor has reached from 50 to 70 volts, which means that the "swing" of point A is around 35 to 55 volts (assuming the voltage across the tube while conducting as 15 volts). The *amplitude* of the triangular wave thus produced is therefore about 17 to 27 volts (the amplitude is one-half of the total swing), which is not enough to operate the deflection plates directly, but more than enough when additional amplification of the triangular wave is used before it is applied to the deflection plates. The following considerations throw some light on the choice of this voltage value.

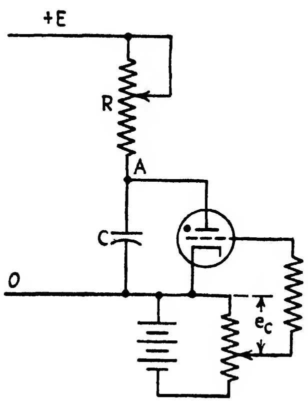

FIG 27–2.—A linear sweep is usually obtained by means of a gaseous tube.

27–4. Factors Affecting the Linearity of the Sweep Voltage.—For a perfect linear sweep the voltage across the capacitor C in Fig. 27–2 would have to rise along a straight line. Now we know that with a resistor-capacitor combination this is not the case but that the voltage will rise

along an exponential curve. Only if we can break off the charging process when the voltage across the capacitor is still small compared to the voltage of the supply, will the exponential curve be reasonably linear. It will be remembered that, according to the fundamental law of the capacitor, the rate of change of voltage across it is proportional to the current flowing through it. At the beginning of the charging process the full supply voltage will be across the resistor and the current flowing through the resistor and capacitor will be given by E/R. When the capacitor has charged to 10 per cent of the supply voltage E. the voltage across the resistance will be only 90 per cent of the supply voltage, with a corresponding reduction in current flowing through the resistor and the capacitor. The rate at which the voltage across the capacitor changes at this instant will therefore be only 90 per cent of the rate at which it was changing at the beginning. When the voltage across the capacitor is used for the operation of a sweep circuit in a cathode-ray oscillograph, the spot will travel 10 per cent slower near the end of the sweep than near the start. This shows that it is desirable to employ a supply voltage as high as possible and to limit the breakdown voltage to a low value. If this is true, why not let the gas tube operate between, say, 25 volts as upper limit and the arc drop of 15 volts as the lower limit? It is evident that under such a condition a 1-volt change in either the upper or the lower limit would cause a 10 per cent change in the time taken by one sweep. Such voltage variations in the breakdown or extinction point must, however, be reckoned with, in the case of gaseous tubes. It is clear, that, if we let the capacitor charge to 50 or 60 volts, on the other hand, a voltage variation of 1 volt will not cause so big a relative change in the frequency of the sweep circuit. The choice of breakdown voltage therefore must represent a compromise between linearity of the sweep and stability of operation. In the usual commercial equipment, the charging voltage runs from 300 to 800 volts, with the above-mentioned values for the breakdown voltage. These circuits, especially with the lower values of supply voltage, are therefore not too satisfactory from a point of linearity. For many practical problems the departure from linearity is not a serious handicap, but wherever exact time relations must be determined, it is not possible to assume that equal distances along the X axis represent equal time intervals. It is then necessary either to improve the linearity of the sweep circuit or to provide other means of timing the trace on the screen. Means to improve the linearity will be discussed later in this chapter.

27-5. Synchronization of the Sweep Voltage with the Voltage Applied to the Vertical Plates.—When a signal with a frequency of 100 cps, for example, is applied to the vertical deflection plates and we wish to have it appear as a stationary single wave, it is necessary to operate the sweep circuit also at a frequency of exactly 100 cps. If we wanted to have two complete waves of the signal appear on the screen, the sweep would have

to operate at 50 cps.' Variations of the supply voltage and variations in the grid bias and the breakdown voltage make it almost impossible to keep the circuit shown in Fig. 27–2 operating at a frequency *exactly* equal to that of the applied signal, or at a submultiple of it. In a certain way the circuit shown in Fig. 27–2 is similar to that of a multivibrator; there, too, operation depended on the time when the discharge process of a capacitor, following an exponential curve, reached a certain point. In the discussion of multivibrators it was pointed out that the so-called "free running" period of the multivibrator could be synchronized by means of a

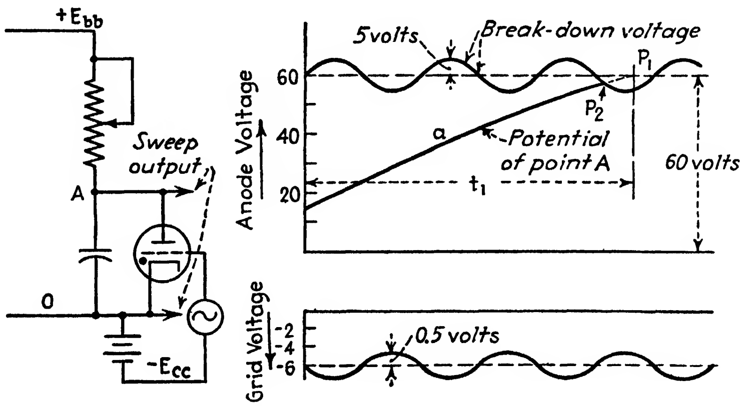

FIG. 27–3.—The sweep can be synchronized with a submultiple of the frequency of the voltage under observation by applying the latter to the grid of the gaseous discharge tube.

voltage injected into the grid circuit. A similar result can be achieved with the circuit shown in Fig. 27–2 by injecting into its grid circuit a synchronizing voltage. Most commercial oscilloscopes permit injection of such a voltage. How a synchronizing voltage operates can easily be understood with the aid of Fig. 27–3. Suppose that with a bias voltage of 6 volts the breakdown voltage of the particular tube used in the circuit shown in Fig. 27–2 is exactly 60 volts. Let the line *a* in Fig. 27–3 represent the rise of voltage across the capacitor after it has been discharged to 15 volts by the preceding operation of the gaseous tube; this line is shown to cross the 60-volt line at point P_1, with the time t_1 therefore representing the free running period of the sweep circuit. Now let us place in series with the fixed bias of 6 volts an alternating voltage with an amplitude of approximately 0.5 volt. The breakdown voltage of the tube will then no longer be a constant value of 60 volts, but it will be higher when the grid is more negative, and lower than 60 volts while the grid is less negative than 6 volts. This is indicated in Fig. 27–3 by the sine wave superimposed on the constant value of 60 volts. The capacitor voltage now reaches the breakdown point at P_2 which causes a reduction of the time interval between two successive discharges. By adjusting the free

running period or by changing the amplitude of the synchronizing voltage introduced in the grid circuit, it is easy to correlate the two adjustments in such a manner that breakdown of the gaseous tube occurs in intervals that are exact multiples of one period of the synchronizing voltage. The synchronizing voltage can be introduced into the grid circuit either by a small transformer, or it may be capacitively coupled to the grid of the gaseous tube in a manner similar to that employed when amplifier stages are coupled by this method. In commercially available equipment a three-position switch is usually provided which permits the synchronizing voltage to be obtained either from the line voltage or internally directly from the signal being applied to the vertical plates, or finally it may be obtained from any suitable voltage source externally connected to a binding post provided for this purpose. Offhand, it would seem that synchronization from the signal applied to the vertical plates would in all cases be entirely satisfactory and that the provision of two additional methods would be an unnecessary complication, but in problems involving the study of phase relationships of nonsinusoidal voltages, it is desirable to synchronize from an external voltage.

27-6. Use of a Pentode to Improve the Linearity of the Sweep Circuit.—It has been pointed out that the circuit shown in Fig. 27-2 does not give an entirely linear sweep owing to the fact that the charging current flowing through the capacitor decreases as the voltage across the capacitor increases. Where it is of importance to have the sweep as linear as possible, the resistance R in Fig. 27-2 must be replaced by a constant-current device, which suggests the use of a pentode in place of the resistor R. The modification of the circuit resulting from this substitution is shown in Fig. 27-4. From a practical point of view, this circuit is not very satisfactory. The constant-current characteristics of a pentode are obtained only when the screen voltage and the control-grid voltage are held constant. In the circuit shown in Fig. 27-4 the cathode of the pentode swings up and down as the capacitor charges and discharges, respectively. The screen voltage can therefore not be obtained from the direct voltage supply furnishing the charging current for the capacitor but must be obtained by separate batteries connected to the cathode of the pentode. This is evidently a very unsatisfactory circuit arrangement. The difficulty can be overcome by exchanging the position of the capacitor and the pentode; the circuit rearranged in this manner is shown in Fig. 27-5. The screen voltage can now be obtained from a

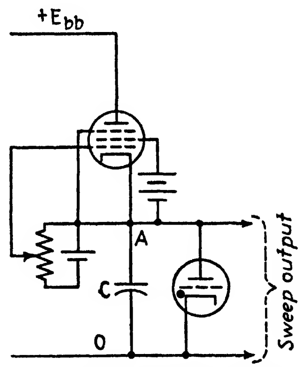

FIG. 27-4.—A higher degree of linearity is obtained if the resistance of Fig. 27-3 is replaced by a constant-current device, such as a pentode.

bleeder placed across the supply voltage, and the bias voltage for the control grid can be obtained by a cathode resistor. The output of the circuit shown in Fig. 27-5 is opposite to that of the circuit shown in Fig. 27-4. Point A in the latter circuit swings positive with the charging of the capacitor and is then suddenly made negative by the discharge, while point B in the circuit shown in Fig. 27-5 swings negative during the charging process and will be yanked suddenly positive with a discharge by the tube. The two circuits, if connected identically, therefore cause the sweep to operate in opposite directions. Reversal of the connection to the deflection plates, or rotating the cathode-ray tube itself 180 deg, usually permits the operation of the sweep in the desired direction which in most commercial models is from left to right.

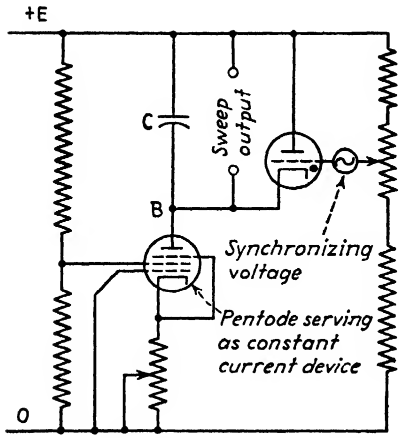

FIG. 27-5.—Exchanging the position of the pentode and the capacitor in Fig. 27-4 results in a more practical circuit.

27-7. Frequency Range of Sweep Circuits in Commercial Oscilloscopes; Use of Vacuum-tube Sweep Circuits.—In the more common arrangement shown in Fig. 27-2, the sweep frequency is evidently determined by the time constant of the capacitor C and the resistor R. It is, of course, desirable to have as wide a range of frequencies for the sweep circuit as possible, so that low- and high-frequency waves may be studied with equal ease. The sweep frequency can usually be adjusted over a range as low as 2 cps in certain models to approximately 30,000 cps. Since this tremendous range could not be covered conveniently simply by having a variable resistor as shown in Fig. 27-2, a tap switch is usually provided, which permits selection of various capacitor values, ranging from 0.005 to about 0.5 μf in several steps. The fine adjustment between these steps is then accomplished by varying the resistor R. Essentially the same system is used when the circuit shown in Fig. 27-5 is employed except that the fine adjustment is obtained by varying the plate current of the pentode by means of the variable resistor in the cathode circuit of this tube.

Although the gaseous tube is by far the most convenient type to use in connection with the design of sweep circuits and satisfies almost all normal requirements, it has a definite frequency limit owing to the ionization process involved in its operation. For the study of very high-frequency phenomena, sweep frequencies in excess of those obtainable with gaseous tubes are sometimes required. Circuits employing vacuum tubes for the

discharge of the capacitors have been designed and used successfully. Since a vacuum tube cannot carry so much current as a gaseous tube, the discharge of the capacitor (sometimes called the "fly back") usually takes a larger percentage of the total cycle than it does in the case of gaseous tubes. As the loss of 5 or even 10 per cent of the total time of one sweep cycle for the return of the spot is usually of little consequence, these circuits are practical means of extending the frequency range, if this should be necessary.

27-8. Limitations Caused by the Introduction of Amplifiers between Signal and Deflection Plates.—As stated at the beginning of this chapter, another feature almost indispensable for the operation of a cathode-ray oscilloscope is an amplifier for the convenient study of smaller voltages. All commercially available oscilloscopes are equipped with amplifiers, at least for the vertical deflection plates, and most of them provide amplification for the horizontal deflection plates as well. The introduction of an amplifier between the actual voltage to be observed and the deflection plates is obtained, however, at a rather stiff price, as we shall see presently. When we apply an alternating voltage directly to the deflection plates and increase the frequency of the applied voltage, the deflection caused by it will remain constant until we reach very high frequencies. The upper frequency limit is determined by the time that the electron spends between the deflection plates. If the frequency of the applied voltage is, for instance, 1,000,000 cps and the time of flight for an electron through the region between the deflecting plates is one-millionth of a second, then obviously the electron during its flight between the deflection plates will be pulled toward one plate during one-half of the time and toward the other for the remainder, and it will therefore emerge without having acquired a crosswise velocity. In ordinary cathode-ray tubes the frequency limit due to this factor is in the order of 10 to 20 megacycles; for high-voltage tubes it exceeds even these high values. On the other end of the frequency scale the cathode-ray tube has, of course, no limitation. If we apply a voltage of zero frequency, *i.e.*, a direct voltage, to the deflection plates, the spot will remain deflected as long as the deflecting voltage is applied.

27-9. Considerations Pertaining to the Frequency Response of Amplifiers Used in Cathode-ray Oscilloscopes.—No chain is stronger than its weakest link. When we insert an amplifier between the signal and the deflection plates, the frequency response can, of course, be no better than that of the amplifier. Cathode-ray oscilloscopes are usually employed in the study of ac phenomena. This, coupled with the well-known difficulties of designing a highly sensitive amplifier for direct voltages, is probably the reason why no manufacturer of oscilloscopes has yet seen fit to incorporate dc amplification in them. This is rather to be deplored because the loss of the dc component of a voltage makes it sometimes hard

to obtain a proper picture of the relative size of the ac component of a given rectified voltage, for instance. However, although the available oscilloscopes will not amplify direct voltages, some manufacturers have succeeded in pushing the lower frequency limit to about 2 cps. This is accomplished by the use of large coupling capacitors and other modifications of the circuit.

At the upper end of the frequency range, the less expensive types usually begin to fall off rather rapidly at about 100,000 cps. At first glance this seems rather strange when we remember that every radio set amplifies voltages with frequencies fifteen to twenty times as high without any apparent difficulty. If the problem were only to amplify a given high-frequency voltage, it would be relatively simple indeed. In a radio receiver this is accomplished by using tuned circuits as a load in the plate circuit of the amplifying stages. An amplifier for a cathode-ray tube, on the other hand, must be flat down to the lowest frequency. This, of course, rules out the use of tuned circuits. Although the load in the plate circuit is therefore usually a resistor, a small inductance is sometimes included which begins to come into play as the higher frequencies are reached. Recently one manufacturer placed on the market a model which is claimed to have a flat response up to 1,000,000 cps and which will undoubtedly serve a very useful purpose in the solution of problems involving those frequencies. For the solution of problems encountered by the industrial electrical or electronic engineer who, until a few decades ago, had to be satisfied with string oscillographs with a frequency response not exceeding 5,000 or 6,000 cps, an instrument with an upper frequency limit of 50,000 to 100,000 cps will be entirely satisfactory. A warning should be given at this point, however. If we wish to study voltages with a frequency of 20,000 cps, for instance, and if the wave shape of these voltages differs appreciably from a sinusoidal wave shape, then the amplifier system must be flat up to frequencies many times as high as 20,000. Thus, a square wave of a frequency of 20,000 cps would not appear in this shape on the screen of a cathode-ray tube if the amplifier did not have a flat frequency response up to at least 200,000 cps; even then it would appear with rounded corners. In other words, in order to reproduce an irregular wave shape correctly, an amplifier must be capable of amplifying all the upper harmonics contained in the irregular wave. A square wave is particularly rich in harmonics, the ninth harmonic, for instance, having an amplitude of one-ninth the fundamental wave. It is therefore quite obvious that the sharp corners of such a wave will be completely lost if the frequency range of the amplifier does not extend to at least ten to twenty times the frequency of the square wave itself.

27–10. Output Voltage Required from Amplifier; Single-ended Amplifier Operation; Positioning of Spot on Screen.—An amplifier driving the deflection plates of a cathode-ray tube obviously works into a load of very

high impedance. The load consists of only the small capacitance existing between the two deflection plates. The amplifier therefore does not have to furnish any power but it must be capable of furnishing a relatively high output voltage. When the cathode-ray tube has one plate of each set of deflection plates tied to the anode, the use of a push-pull connection is not possible and the full signal must be applied to the other plate. A simple capacitance-coupled amplifier serving in such a case is shown in Fig. 27–6.

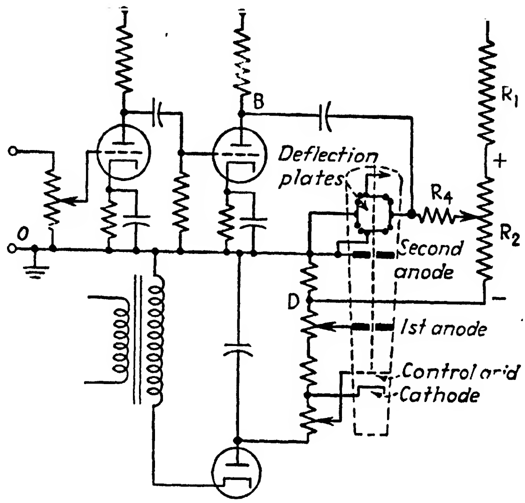

FIG. 27–6.—The diagram for a complete oscilloscope with a two-stage amplifier for one set of deflection plates. Potential divider R_2 permits positioning of the spot on the screen in the horizontal direction.

Although, for simplicity's sake, the amplifier tubes are shown as triodes, in practice pentodes are used because of their higher voltage amplification. The diagram shows another feature found in all commercial oscilloscopes, namely, means for positioning the spot on the screen. With zero volts applied to the deflection plates (*i.e.*, the plates short-circuited) the beam should hit the exact center of the screen. The electron gun is a structure sealed to the bottom of the tube, and its performance cannot be checked until the tube has been evacuated and all possibility of adjustment is gone. It is therefore impossible for the manufacturer to guarantee that the spot will appear in the exact center of the screen with zero volt applied to the deflection plates, and it is therefore necessary to provide means to apply a direct bias voltage to the deflection plates, which permits placing the spot at any point of the screen. Sometimes a deliberate displacement of the zero point is even desirable because it permits a more detailed study of small parts of a pattern appearing on the screen. In the circuit shown in Fig. 27–6, the anode of the cathode-ray tube is placed at the zero potential of the amplifier. The rectifier system furnishing the high direct volt-

age for the operation of the cathode-ray tube must therefore produce a high negative voltage with respect to the zero point. Potential divider R_2 is connected with its lower end to point D, and it is evident that by the proper choice of circuit constants we can have its upper and lower ends equal amounts positive and negative with respect to the zero point. The free deflection plate is connected over the high resistance R_4 (usually from 5 to 10 megohms) to the arm of the potential divider R_2. Obviously this permits us to apply a positive or negative dc potential to the right-hand deflection plate with respect to the left-hand plate and therefore makes it possible to place the spot to the right or left of the position it would have with zero volts applied to the deflection plate. (The diagram shows, of course, only one set of amplifiers and deflection plates; an exact duplicate for the other set is required.)

Now, suppose that the tube used in Fig. 27–6 requires a voltage of 150 volts to be applied to the deflection plates in order to move the spot from the center to the edge of the screen. If an ac signal is to produce a pattern filling the whole screen, it must swing the free deflection plate alternately 150 volts positive and negative with respect to the fixed plate. The total voltage swing of the anode of the final amplifier tube, *i.e.*, point B in Fig. 27–6, must therefore be fully 300 volts. Not only does this mean that the plate-supply voltage for the operation of the amplifier must be rather high, but it is also clear that linearity over so wide a range of output voltage will be none too good. Where more than mere observation of a pattern is required, the circuit shown in Fig. 27–6 can therefore not be recommended.

27–11. Push-pull or Balanced Operation of the Deflection Plates.—A much more desirable circuit arrangement is obtained if the plates of the cathode-ray tube are brought out separately and a push-pull arrangement is used for their operation. The conversion of a single-ended signal into a push-pull signal can be obtained by the use of phase-inversion circuits as shown in Figs. 15–2 and 15–4, discussed in Sec. 15–6, or by other similar circuits producing equivalent results. The usual method of coupling the output of the push-pull circuit to the two deflection plates is shown in Fig. 27–7, which presents only the essential points of this arrangement. The total voltage swing that each amplifier tube has to furnish now is only one-half of that required for the circuit shown in Fig. 27–6. The reader should convince himself of the truth of this statement by determining what voltages the amplifier tubes would have to furnish to make one deflection plate 150 volts positive with respect to the other, and vice versa. The direct bias voltage control needed for positioning the spot on the screen will now consist of a dual control connected in the manner shown in Fig. 27–7. Moving the two arms of the two potential dividers in an upward direction, for instance, evidently makes the right-hand deflection plate more positive while the left-hand deflection plate becomes more

negative. The circuit is arranged in such a way that the mid-point of the two potential dividers is at the same potential as the anode of the cathode-ray tube. In this way the average potential of the deflection plate is equal to the anode potential, a condition that we have seen to be the most desirable one.

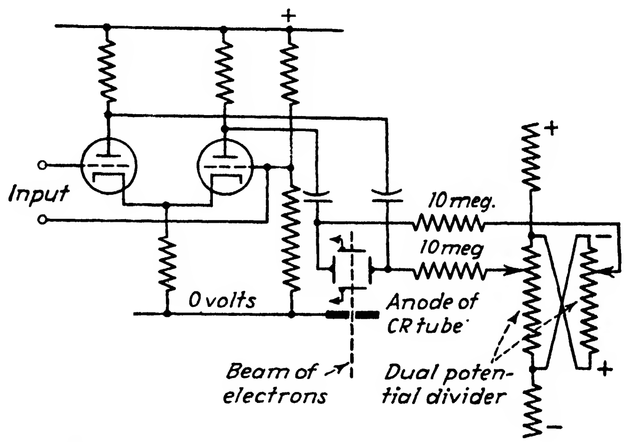

FIG. 27-7.—If the signal is applied to the deflection plates in a push-pull circuit, the positioning control must also provide a push-pull signal.

27-12. Amplifiers Capable of Indicating Dc Component of Signal.—The two circuits shown in Figs. 27-6 and 27-7 are not capable of recording the presence of a dc component in the input signal applied to the amplifier. The application of a direct voltage to the input terminal of these circuits will deflect the spot momentarily on the screen, but it will return to the zero position in a time depending on the time constants of the coupling capacitors and resistors. In the study of some problems, the loss of the dc component is a decided disadvantage; in such cases it is desirable to operate the deflection plates from a dc amplifier. A two-stage amplifier, based on the cathode-inversion circuit described in Chap. XV (Fig. 15-4), is shown in Fig. 27-8. The anode of the cathode-ray tube is connected to a point on a potential divider placed across the direct supply voltage for the amplifier having the same potential as the two anodes of the amplifier tubes connected to the deflection plates. This requirement is not too critical, however; the anode may have a potential difference of from 50 to 75 volts with respect to the zero signal level of the amplifier plates without causing any ill effects. Since it is impossible to introduce an additional direct bias voltage directly to the deflection plates of the cathode-ray tube with this circuit, positioning must be obtained in some other way. In the circuit shown in Fig. 27-8 positioning control is achieved by providing a variable bias for the grid of the first phase-inversion tube.

Figures 27-6 to 27-8 show straightforward capacitive or resistance coupling. It has been mentioned that an attempt is usually made to improve the low- and the high-frequency response of these amplifiers by the addition of inductances in the plate circuits and other additional compensating arrangements. The discussion of the details is considered as outside the scope of this book and will be found adequately treated in the literature.

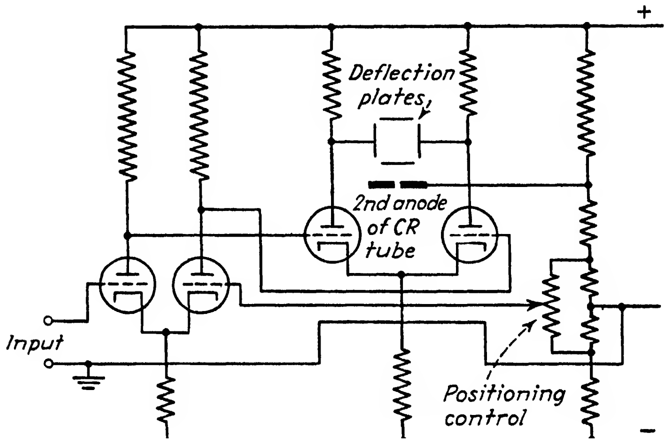

FIG. 27-8.—A dc push-pull amplifier circuit. The anode of the cathode-ray tube should be connected to a point on the bleeder circuit at or near the average potential of the plate of the final amplifier tube.

27-13. Characteristics of Various Screen Materials; Photography of Traces.—The screen of cathode-ray tubes may be prepared with a number of materials of different characteristics. When the instrument is to be used only for visual observation, a material giving a green trace is most satisfactory. Not only is the human eye most sensitive to the green part of the spectrum, but the screen may be observed for long periods without any fatigue. If it is desired to take pictures of the pattern appearing on the screen, conditions are different, however. When the pattern appearing on the screen is stationary, a time exposure may be made; under this condition, it is not necessary to have a particularly fast lens or a very bright and actinic trace. Entirely satisfactory pictures can be made with even an inexpensive camera, provided it is brought close enough to the screen (usually a portrait attachment can be obtained for fixed-focus cameras for close-ups). An exposure of ½ to 1 sec will give satisfactory results with a box-type camera. A black cloth should be used to cover the camera and the cathode-ray tube in order to minimize the effect of stray light.

If it is desired to photograph a transient, the usual procedure is to set the camera up in front of the screen and open the shutter. The transient

is then initiated, causing the spot to describe a trace on the screen. In this case the impression made on the film evidently depends on the brilliancy of the spot and the speed with which it sweeps over its path; how long the shutter remains open is of no consequence. In a similar way, if a moving film is used, as shown in Fig. 26–4, the brilliance of the spot is the determining factor. In these cases it is necessary to employ a fast lens, to make the spot as brilliant as possible (by operating the cathode-ray tube with a high anode voltage), and finally to use a screen material giving a radiation to which the photographic material is most sensitive. Screen materials satisfying the last-mentioned condition usually give a blue trace which, although not so bright to the human eye as the green trace, produces a darker line on the average film material. The subject of photographing traces on the screen of a cathode-ray tube has received much attention, and the interested reader will find an extensive literature covering it.

27–14. Persistency of Image on Screen.—Another characteristic of the screen, which is sometimes of importance, is its persistency. The beam of electrons impinging on the screen of a cathode-ray tube produces a luminous spot, but the radiation of light from this spot does not cease immediately when the beam is interrupted or moved to a different place. With the commonly employed screen material, the brightness of the spot decreases to about 1 per cent of full value in a period approximately $\frac{1}{20}$ sec. This means that the human eye cannot perceive the decay of brightness, since its persistency in turn is higher than that of the screen material. When it is desired to photograph the spot by means of a moving film, as shown in Fig. 26–4, this persistency is decidedly undesirable because it results in a widening of the trace on the film.

For certain problems, on the other hand, a long persistency is very desirable. Within recent years screen materials have been developed that retain the image of the trace as long as a few seconds. Tubes of this kind are particularly useful for the visual observation of transients. If it is desired to study a transient, such as the current in a solenoid while it operates, for example, the transient can be made to appear on a long-persistency screen; by placing a tracing paper over the screen of the tube an approximate picture can be obtained immediately. The persistency of the image on the screen of a cathode-ray tube is also being used in so-called "memory" oscillographs. In order to study the causes of disturbances on a transmission line, for example, it is necessary and desirable to have a recorded oscillogram, not only of the disturbance itself, but also of a few cycles just preceding it. It is clear that such a record may throw valuable light on just what initiated the disturbance. The voltage, or whatever quantity it is desired to record, is placed on the screen of a cathode-ray tube and a high-speed camera with the shutter closed is set up in front of the screen. When a disturbance occurs, it trips the shutter of the camera

and a picture of the screen is taken. Owing to the persistency of the screen, the picture will show not only the trace of the spot during the time that the shutter is open, but also the path described by the spot during the few cycles preceding the opening of the shutter.

27-15. Recording of Nonelectrical Quantities; Transducers.—The cathode-ray tube is a device for recording and observing fast-varying electrical voltages. It is evident that any physical quantity or value that can be converted somehow into an electrical voltage can be recorded by means of the cathode-ray oscillograph. Cathode-ray tubes have therefore become today a tool not only of the electrical engineer but also of the physicist and the mechanical engineer as well. Devices that convert nonelectrical quantities into electrical voltages so that they may be applied to a cathode-ray oscillograph are called "transducers." One of the earliest applications of the cathode-ray tube in the study of mechanical phenomena had to do with the investigation of the characteristics of fuels for internal-combustion engines. The problem of recording the pressure in their cylinders during and following the explosion has received much attention, and several methods of converting that pressure into an electrical quantity have been proposed. In one of the earlier methods the pressure was made to act on a stack of carbon disks, also referred to as a "carbon pile"; the resistance of such a carbon pile is known to vary with the pressure applied to it. This was not a very satisfactory solution, since the stack does not give the same resistance values with repeated applications of pressure.

When pressure is applied to a quartz crystal in a certain relation to its axis, a voltage will appear on opposite faces of the crystal. This property has been used in many transducers. If the pressure in the cylinder of an internal-combustion engine is permitted to act through a diaphragm on a quartz crystal, the voltages produced by the crystal may be used for the operation of a cathode-ray oscillograph. Circuits making use of this property of quartz crystals must be shielded carefully because the quartz crystal represents a high-impedance source of voltage and any high-voltage sources near by (such as the voltage delivered to the spark plug) are liable to produce disturbances. Another method makes use of a highly polished diaphragm on which a parallel beam of light is thrown, which is reflected by the diaphragm as a parallel beam as long as the diaphragm is flat. When pressure acts on the diaphragm, it dishes a certain amount, and the reflected beam becomes divergent. A photoelectric cell placed into the reflected beam will therefore receive an amount of light that varies with the pressure applied to the disk. The light variations on the photocell can, of course, be converted by a suitable amplifier into voltage variations that may be used for the operation of the cathode-ray tube. Within the past few months another device for the purpose of recording pressures on the screen of an oscillograph has appeared on the market. In it a diaphragm, on one side of which acts the pressure to be recorded, is placed

opposite a fixed disk. These two surfaces form a capacitor which changes when the diaphragm is brought closer to the fixed disk by the application of the pressure. This variable capacitance forms one arm of a Wheatstone bridge, operated with a frequency of 100,000 cps. The unbalanced voltage is recorded on the oscillograph.

It is obvious that the cathode-ray oscillograph is ideally suited for the study of vibration problems. Various types of transducers may be used, depending on what characteristic of the vibration is to be recorded. Naturally, the type of transducer varies, depending on whether the displacement, velocity, or acceleration of the vibrating part is to be recorded. Of the three characteristics mentioned, the last one is the easiest one to record because it does not require a fixed reference point. Crystal pickups are available which have only to be fastened to the vibrating part and which will furnish a voltage proportional to the acceleration of the vibrating part. Magnetic pickups are also available, which produce a voltage proportional to the velocity of the vibrating part.

The examples cited in the preceding paragraphs show the versatility of the cathode-ray oscillograph, and it can be confidently predicted that it will find even wider application in the future.

PROBLEMS

27–1. In the circuit shown in Fig. 27–9 the bias of the gas tube is adjusted to a value causing breakdown of the tube at an anode voltage of 70 volts. The arc drop of the tube is 15 volts.

a. What will be the frequency of the saw-tooth voltage produced by this circuit?
b. If this voltage is to be used as the time base of a cathode-ray tube, how many per cent slower will the spot travel at the end of the sweep than at the start?

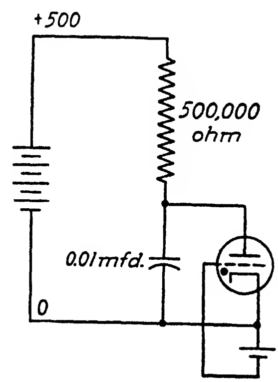

Fig. 27–9.—Diagram of circuit in Prob. 27 1.

27–2. A 6SN7 tube (two 6J5's in one envelope) is to be used in a circuit as shown in Fig. 15–4 to furnish a push-pull signal to the deflection plates of a cathode-ray tube.

The supply voltage E_{bb} is 300 volts, the two load resistors R have a value of 25,000 ohms each, and the cathode resistor has a value of 6,250 ohms.

a. If the quiescent current in each tube is to be 4 ma, what bias voltage (battery shown in Fig. 15–4) will be required?
b. One hundred twenty volts applied to the deflection plates of the particular cathode-ray tube move the spot from the center of the screen to the edge (horizontally or vertically, depending on which pair of deflection plates the voltage is applied to). If the deflection plates are connected to points A and B of the circuit shown in Fig. 15–4, what ac signal will be required on the grid of tube 1 to make the spot produce a straight line from one edge of the screen to the other?

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